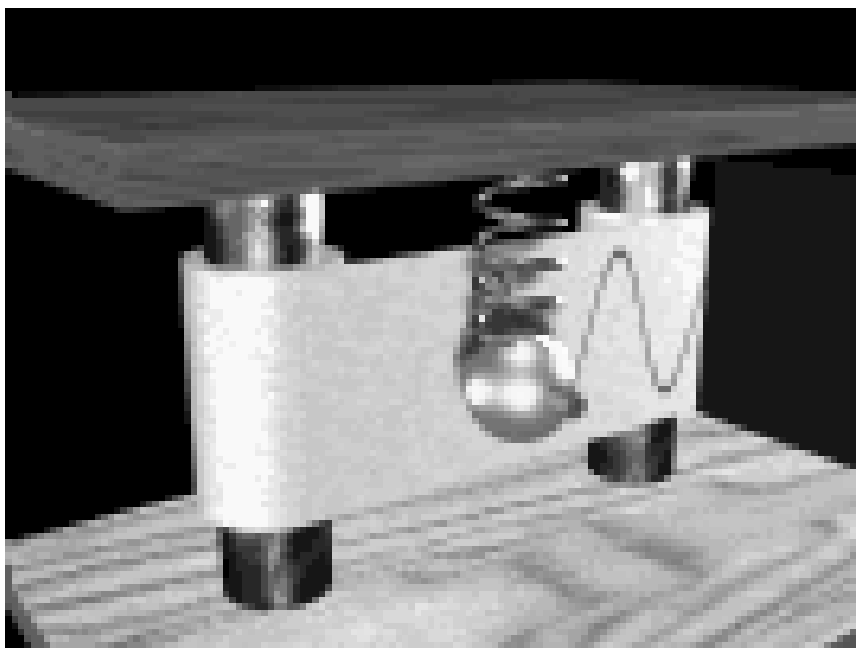
Physics 101: Lecture 19
Elasticity and Oscillations



Overview

- Springs (review)
  - ➔ Restoring force proportional to displacement
  - ➔ $F = -k x$ (Hooke's law)
  - ➔ $U = \frac{1}{2} k x^2$

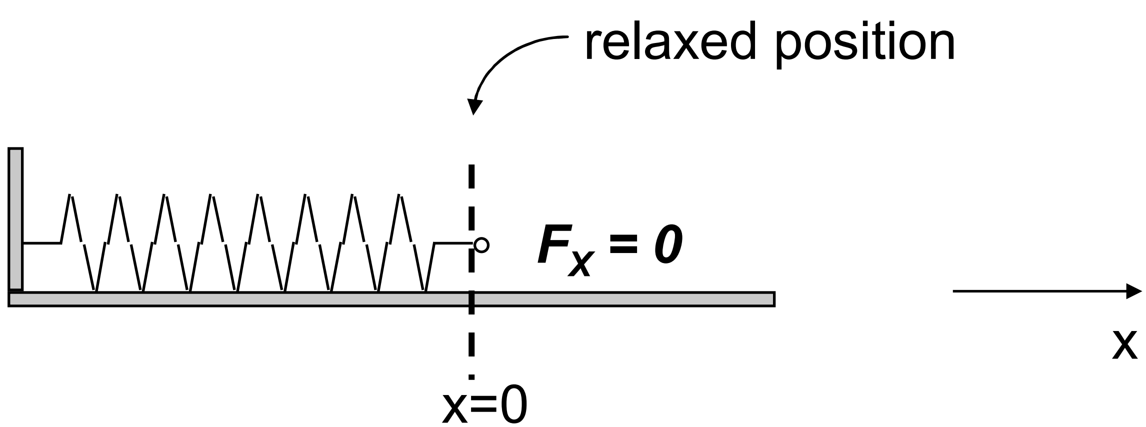
- Today
  - ➔ Simple Harmonic Motion
  - ➔ Harmonic motion vs. circular motion

Springs

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

➔ $F_X = -k\,x$

Where x is the displacement from the relaxed position and k is the constant of proportionality.



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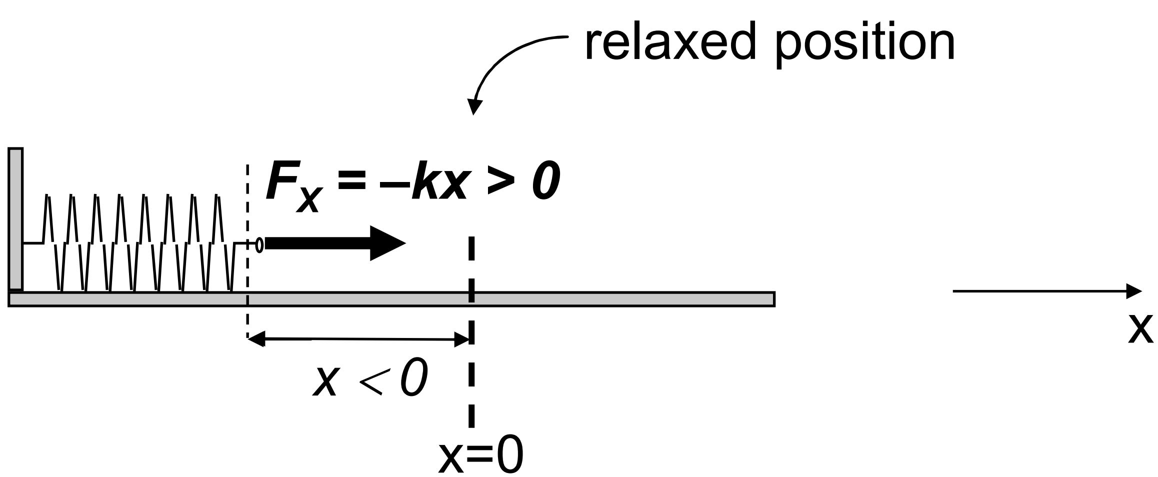
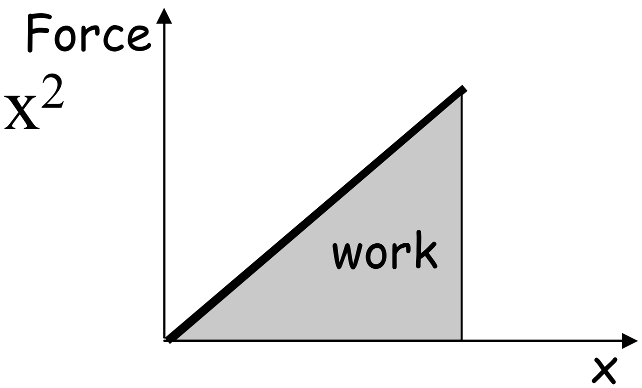


Potential Energy in Spring

- Force of spring is Conservative
  - ➔ $F = -k\,x$
  - ➔ W (by spring) = $-\,1/2\,k\,x^2$



  - ➔ Work done only depends on initial and final position
  - ➔ Define Potential Energy $U_{spring} = \frac{1}{2}\,k\,x^2$

Springs ACT

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.
  - ➔ $F_X = -k\,x$ Where x is the displacement from the relaxed position and k is the constant of proportionality.

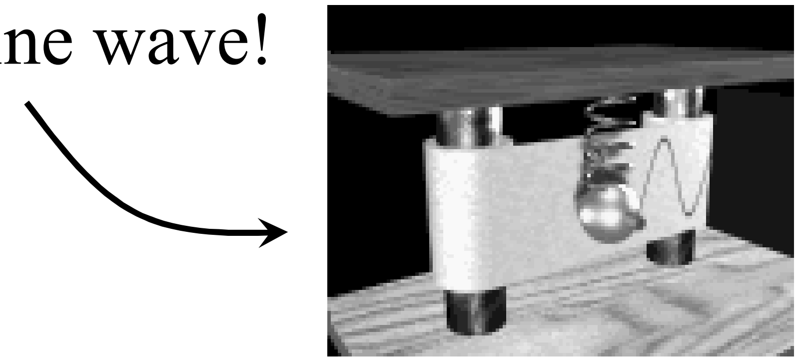
What is force of spring when it is stretched as shown below.

A) $F > 0$ B) $F = 0$ C) $F < 0$

Simple Harmonic Motion

- Vibrations
  - ➔Vocal cords when singing/speaking
  - ➔String/rubber band

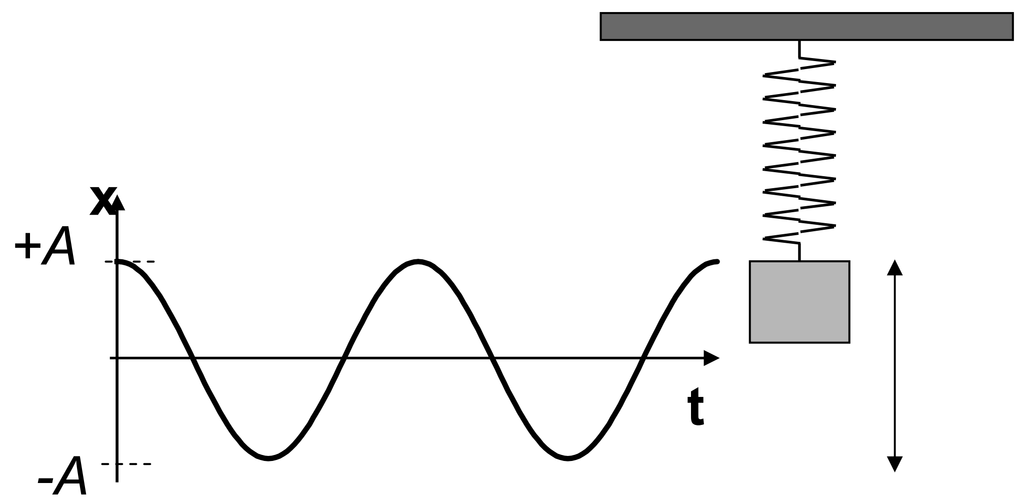
- Simple Harmonic Motion
  - ➔Restoring force proportional to displacement
  - ➔Springs $F = -kx$
  - ➔Motion is a sine or cosine wave!



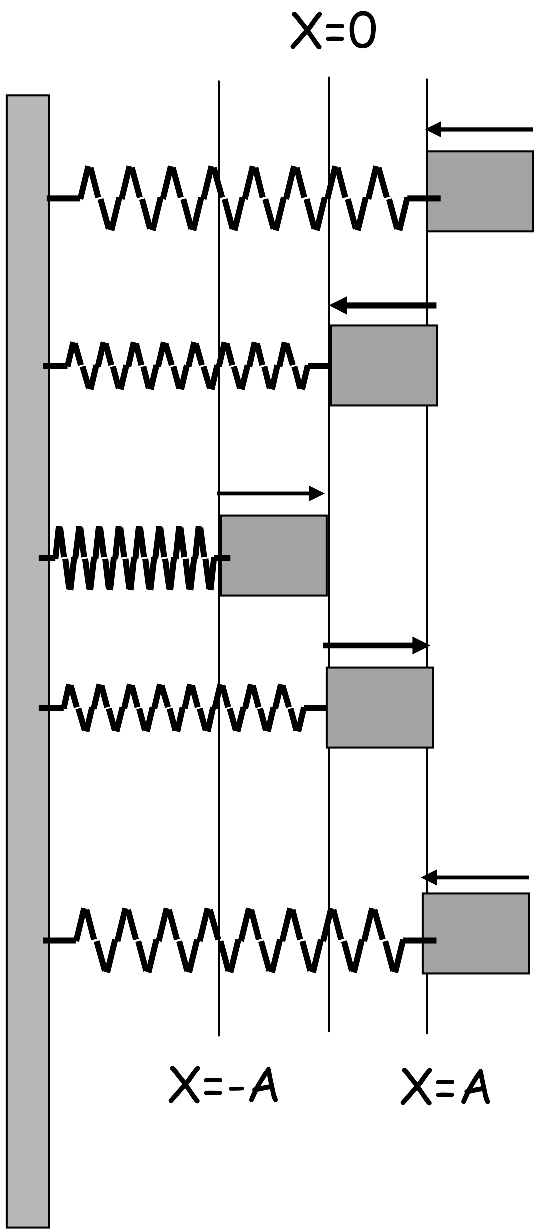
Spring ACT II

A mass on a spring oscillates up & down with simple harmonic motion of amplitude A. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the magnitude of the acceleration of the block biggest?

1. When x = +A or –A (i.e. maximum displacement)
2. When x = 0 (i.e. zero displacement)
3. The acceleration of the mass is constant



Simple Harmonic Motion: Anatomy



$X=A$; $v=0$; $a=-a_{max}$

$a = F/m = -kx/m$

$X=0$; $v=-v_{max}$; $a=0$

$X=-A$; $v=0$; $a=a_{max}$

$X=0$; $v=v_{max}$; $a=0$

$X=A$; $v=0$; $a=-a_{max}$

Energy

- A mass is attached to a spring and set to motion. The maximum displacement is x=A

  ➔ $\Sigma W_{nc} = \Delta K + \Delta U$

  ➔ $0 = \Delta K + \Delta U \Rightarrow$ total energy is constant

  Energy = $\frac{1}{2} k x^2 + \frac{1}{2} m v^2$

  ➔ At maximum displacement x=A, v = 0

  Energy = $\frac{1}{2} k A^2 + 0$

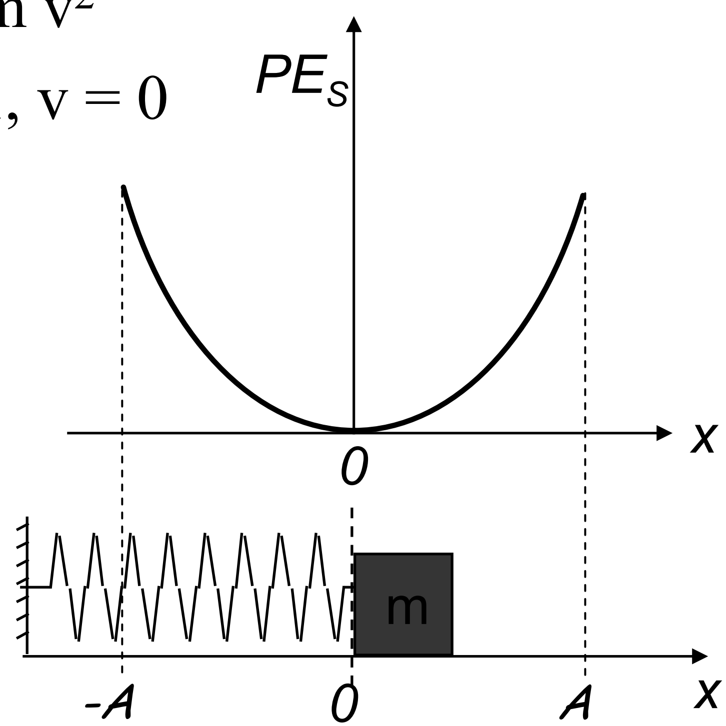
  ➔ At zero displacement x = 0

  Energy = $0 + \frac{1}{2} m v_m^2$

  Since Total Energy is same

  $\frac{1}{2} k A^2 = \frac{1}{2} m v_m^2$

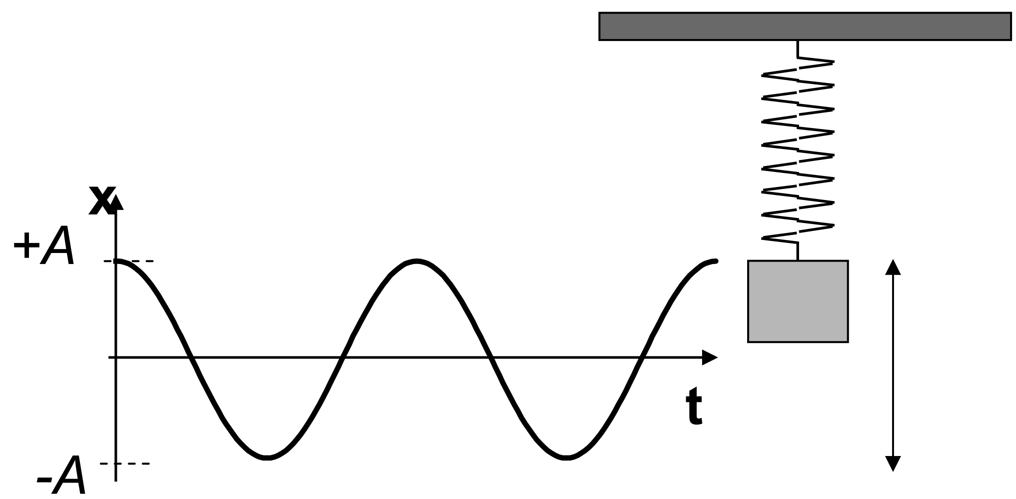
  $v_m = \sqrt{(k/m)}\, A$



Preflight 1

A mass on a spring oscillates up & down with simple harmonic motion of amplitude A. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the speed of the block biggest?

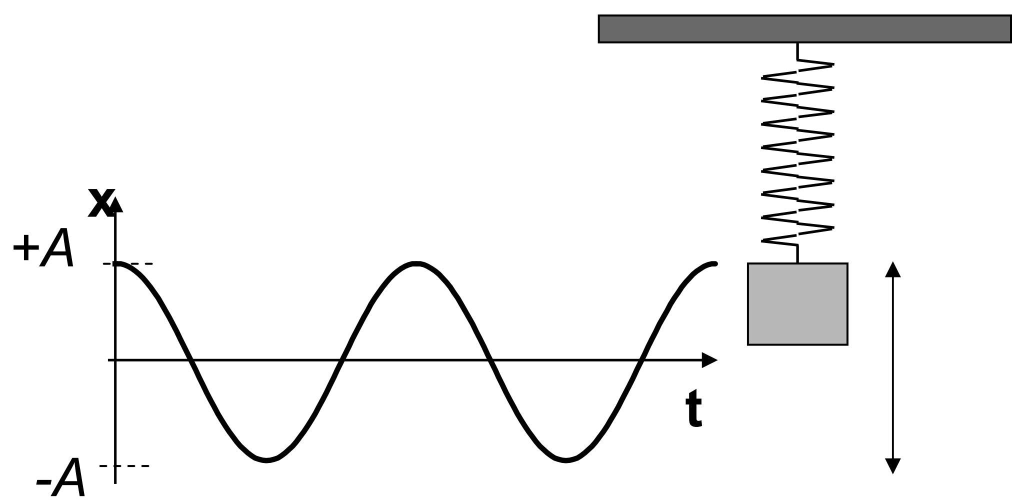
1. When x = +A or -A (i.e. maximum displacement)
2. When x = 0 (i.e. zero displacement)
3. The speed of the mass is constant



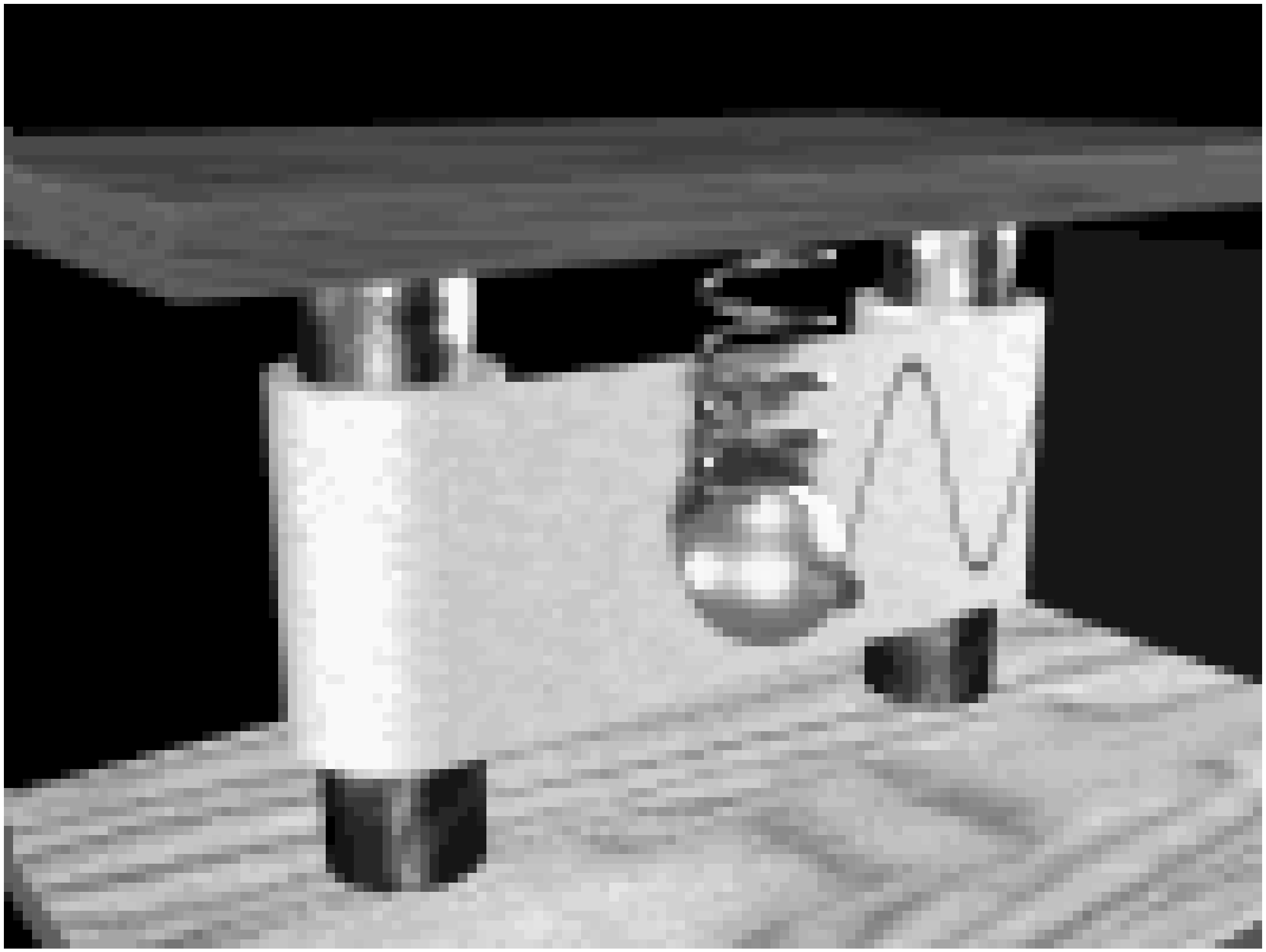
Preflight 3

A mass on a spring oscillates up & down with simple harmonic motion of amplitude *A*. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the total energy (K+U) of the mass and spring a maximum? (Ignore gravity).

1. When x = +A or -A (i.e. maximum displacement)
2. When x = 0 (i.e. zero displacement)
3. The energy of the system is constant.



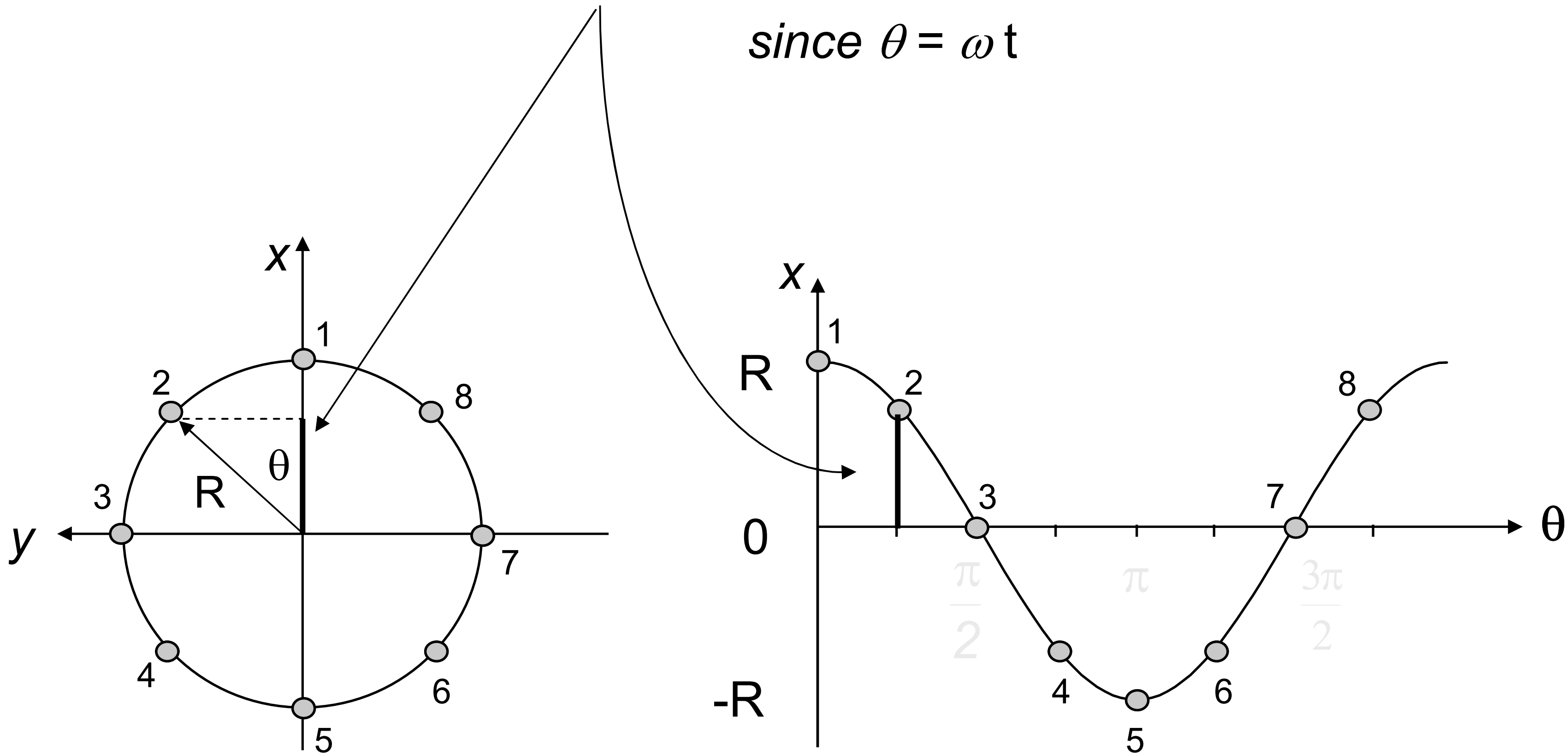
SHM and Circles



What does *moving in a circle* have to do with moving back & forth *in a straight line* ??

$$x = R\cos\theta = R\cos(\omega t)$$

since $\theta = \omega t$



Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$		$x(t) = [A]\sin(\omega t)$
$v(t) = -[A\omega]\sin(\omega t)$	OR	$v(t) = [A\omega]\cos(\omega t)$
$a(t) = -[A\omega^2]\cos(\omega t)$		$a(t) = -[A\omega^2]\sin(\omega t)$

$x_{max} = A$

$v_{max} = A\omega$

$a_{max} = A\omega^2$

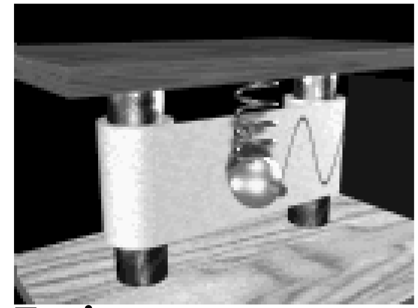
Period = T (seconds per cycle)

Frequency = f = 1/T (cycles per second)

Angular frequency = $\omega = 2\pi f = 2\pi/T$

Natural freq. for spring: $\omega = \sqrt{(k/m)}$

Example



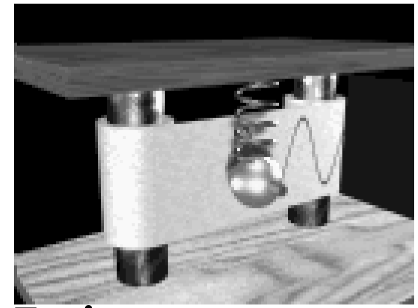
A 3 kg mass is attached to a spring (k=24 N/m). It is stretched +5 cm. At time t=0 it is released and oscillates.

Which equation describes the position as a function of time x(t) =

A) 5 sin(ωt) B) 5 cos(ωt) C) 24 sin(ωt)

D) 24 cos(ωt) E) -24 cos(ωt)

Example

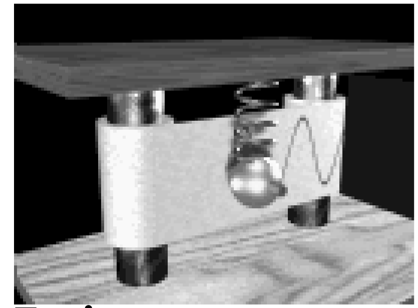


A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

What is the total energy of the block spring system?

A) 0.03 J B) 0.05 J C) 0.08 J

Example

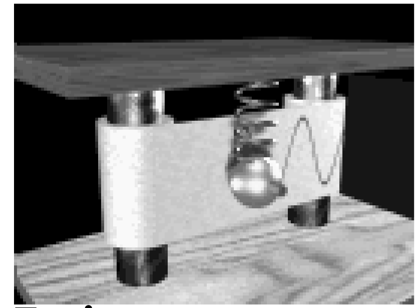


A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

What is the maximum speed of the block?

A) 0.45 m/s B) 0.23 m/s C) 0.14 m/s

Example



A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

How long does it take for the block to return to x=+5cm?

A) 1.4 s B) 2.2 s C) 3.5 s

Summary

- Springs
  - ➔ $F = -kx$
  - ➔ $U = \frac{1}{2} k x^2$
  - ➔ $\omega = \sqrt{(k/m)}$

- Simple Harmonic Motion
  - ➔ Occurs when have linear restoring force $F = -kx$
  - ➔ $x(t) = [A] \cos(\omega t)$ or $[A] \sin(\omega t)$
  - ➔ $v(t) = -[A\omega] \sin(\omega t)$ or $[A\omega] \cos(\omega t)$
  - ➔ $a(t) = -[A\omega^2] \cos(\omega t)$ or $-[A\omega^2] \sin(\omega t)$

Young's Modulus

- Spring $F = -k x$
  - ➔ What happens to "k" if cut spring in half?
  - ➔ A) decreases B) same C) increases
- k is inversely proportional to length!
- Define
  - ➔ Strain = $\Delta L / L$
  - ➔ Stress = F/A
- Now
  - ➔ Stress = Y Strain
  - ➔ $F/A = Y \ \Delta L/L$
  - ➔ $k = Y A/L$ from $F = k x$
- Y (Young's Modulus) independent of L