Exam I

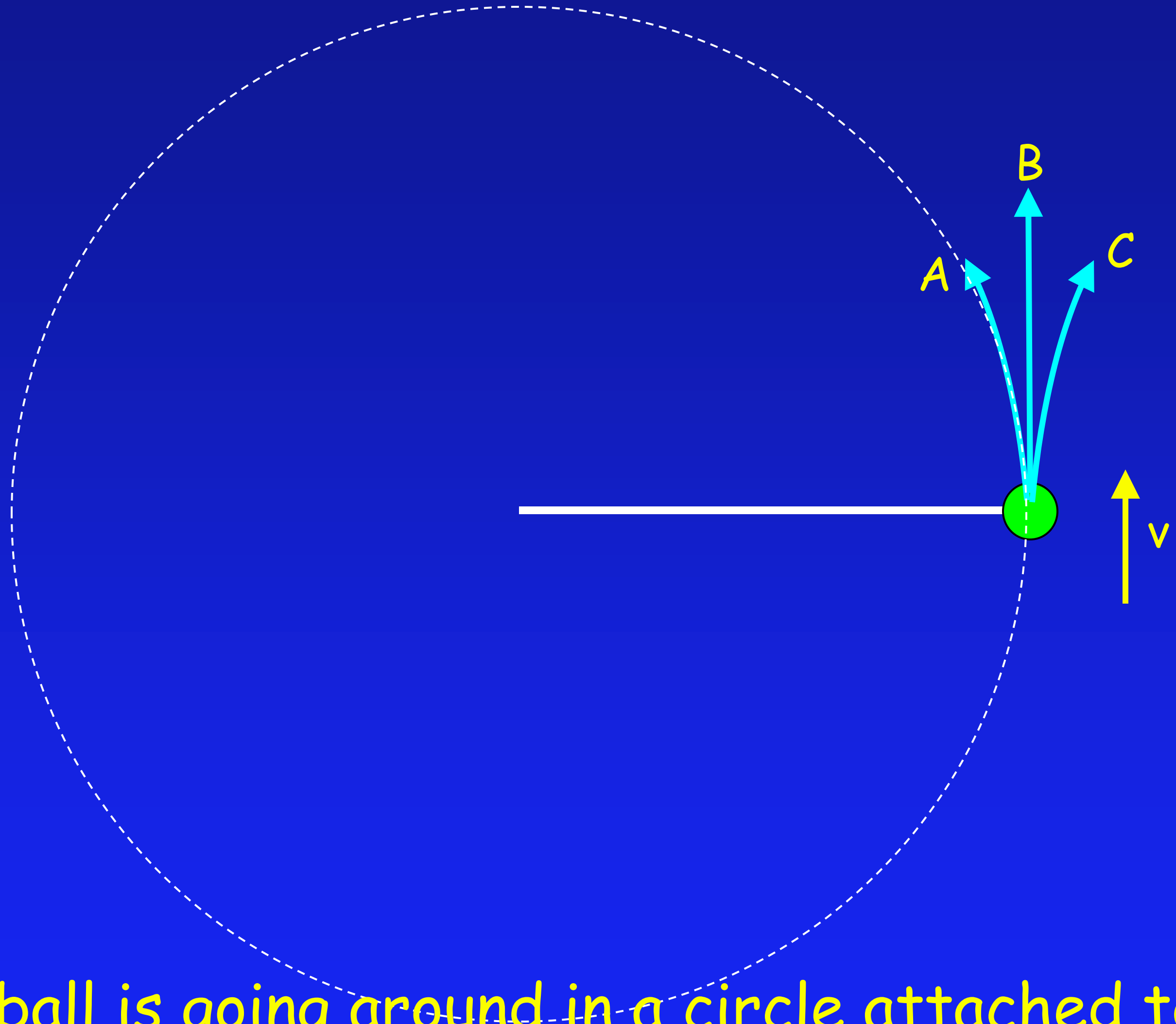
Physics 101: Lecture 08

Centripetal Acceleration and Circular Motion

http://www.youtube.com/watch?v=ZyF5WsmXRaI

- **Today's lecture will cover Chapter 5**

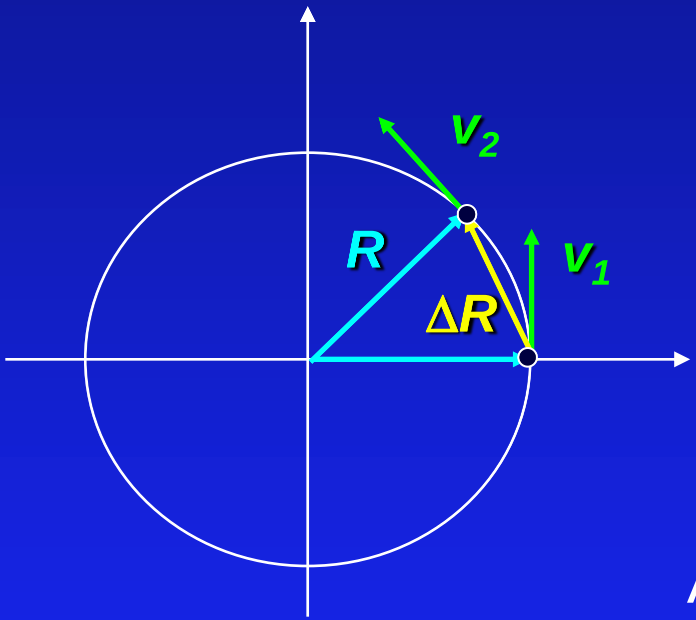
Circular Motion Act

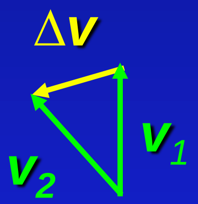


Answer: B

A ball is going around in a circle attached to a string. If the string breaks at the instant shown, which path will the ball follow (demo)?

Acceleration in Uniform Circular Motion





$$a = \frac{v^2}{R}$$

centripetal acceleration

$\mathbf{a}_{ave} = \Delta\mathbf{v} / \Delta t$

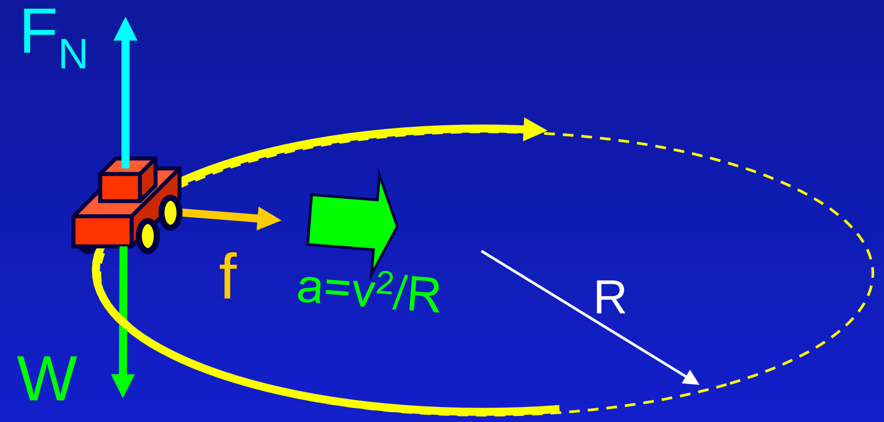
Acceleration inward

Acceleration is due to change in direction, not speed. Since turns "toward" center, must be a force toward center.

Preflights

Consider the following situation: You are driving a car with constant speed around a horizontal circular track. On a piece of paper, draw a Free Body Diagram (FBD) for the car. How many forces are acting on the car?

1% A) 1
24% B) 2
38% C) 3 ← correct
23% D) 4
14% E) 5



$\Sigma \mathbf{F} = m\mathbf{a} = mv^2/R$

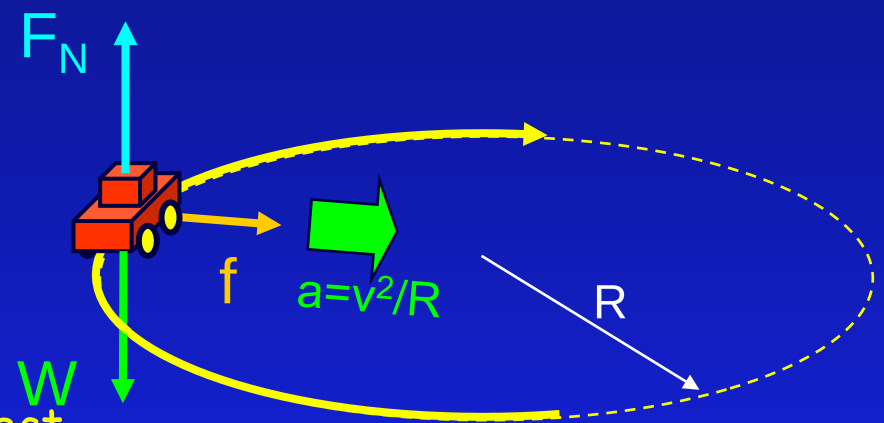
"Friction, Gravity, & Normal"

Common Incorrect Forces

- Acceleration: $\Sigma F = ma$
  - ➔ Centripetal Acceleration
- Force of Motion (Inertia not a force)
  - ➔ Forward Force,
  - ➔ Force of velocity
  - ➔ Speed
- Centrifugal Force (No such thing!)
  - ➔ Centripetal (really acceleration)
  - ➔ Inward force (really friction)
- Internal Forces (don't count, cancel)
  - ➔ Car
  - ➔ Engine

Preflights

Consider the following situation: You are driving a car with constant speed around a horizontal circular track. On a piece of paper, draw a Free Body Diagram (FBD) for the car. The net force on the car is



14% A. Zero

81% B. Pointing radially inward ← correct

5% C. Pointing radially outward

$$\Sigma \mathbf{F} = m\mathbf{a} = mv^2/R$$

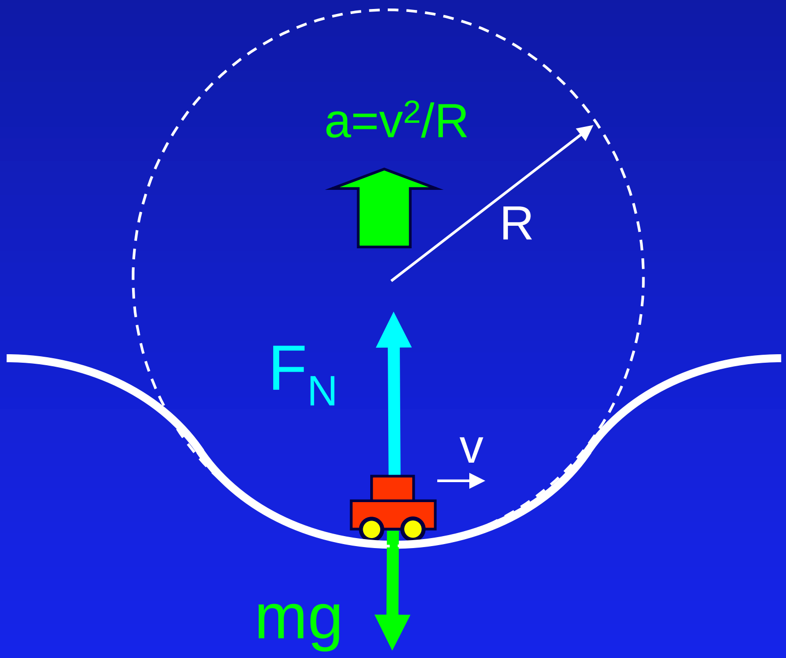
"Centripetal acceleration (the net force) is always pointing INWARD toward the center of the circle you are moving in.."

"must be a net force drawing the vehicle inward otherwise the cars direction would not change, and so the car would drive off the track and crash and burn"

ACT

Suppose you are driving through a valley whose bottom has a circular shape. If your mass is m, what is the magnitude of the normal force F_N exerted on you by the car seat as you drive past the bottom of the hill

A. $F_N < mg$
B. $F_N = mg$
C. $F_N > mg$ ⟵ correct



$\Sigma F = ma$

$F_N - mg = mv^2/R$

$F_N = mg + mv^2/R$

Roller Coaster Example

What is the minimum speed you must have at the top of a 20 meter roller coaster loop, to keep the wheels on the track.

Y Direction: F = ma

$-N - mg = m\,a$

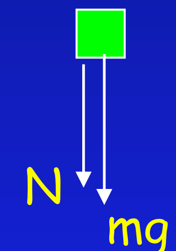
Let N = 0, just touching

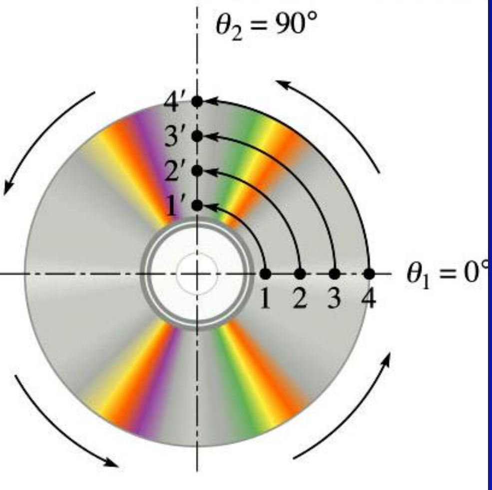
$-mg = m\,a$

$-mg = -m\,v^2/R$

$g = v^2 / R$

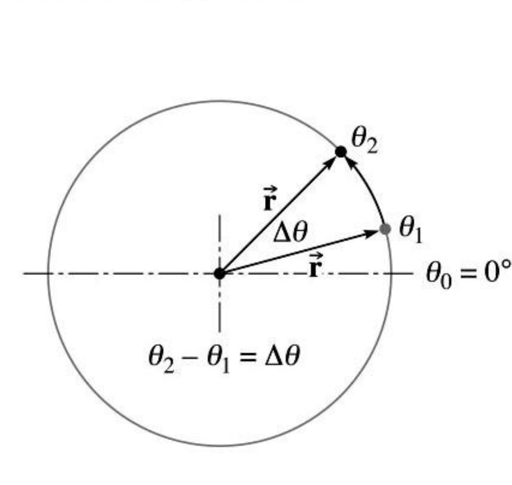
$v = \mathrm{sqrt}(g*R) = 9.9$ m/s





(a)

Circular Motion



(b)

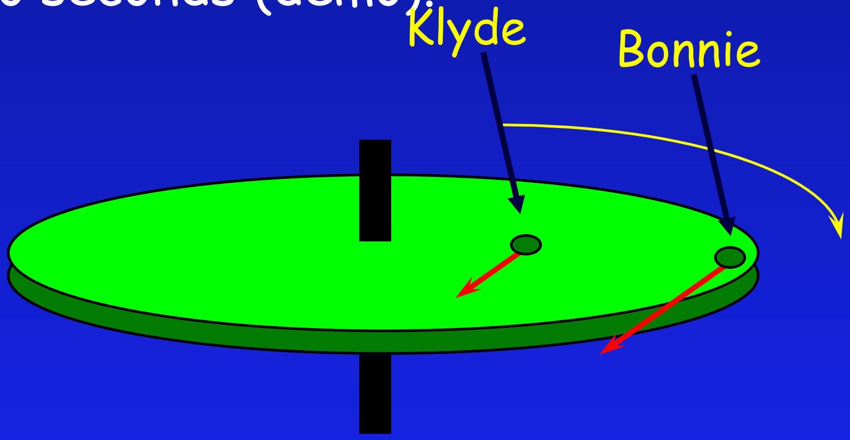
- Angular displacement $\Delta\theta = \theta_2 - \theta_1$
  - ➔How far it has rotated
- Angular velocity $\omega = \Delta\theta/\Delta t$
  - ➔How fast it is rotating
  - ➔Units radians/second 2π = 1 revolution
- Period =1/frequency $T = 1/f = 2\pi / \omega$
  - ➔Time to complete 1 revolution

Circular to Linear

- Displacement $\Delta s = r\,\Delta\theta$ (θ in radians)
- Speed $|v| = \Delta s/\Delta t = r\,\Delta\theta/\Delta t = r\omega$
- Direction of v is tangent to circle

Merry-Go-Round ACT

- Bonnie sits on the outer rim of a merry-go-round with radius 3 meters, and Klyde sits midway between the center and the rim. The merry-go-round makes one complete revolution every two seconds (demo).
  - ➔ Klyde's speed is:



(a) **the same as Bonnie's**

(b) **twice Bonnie's**

(c) **half Bonnie's**

$$V_{Klyde} = \frac{1}{2} V_{Bonnie}$$

Bonnie travels $2\pi R$ in 2 seconds $v_B = 2\pi R / 2 = 9.42$ m/s

Klyde travels $2\pi (R/2)$ in 2 seconds $v_K = 2\pi (R/2) / 2 = 4.71$ m/s

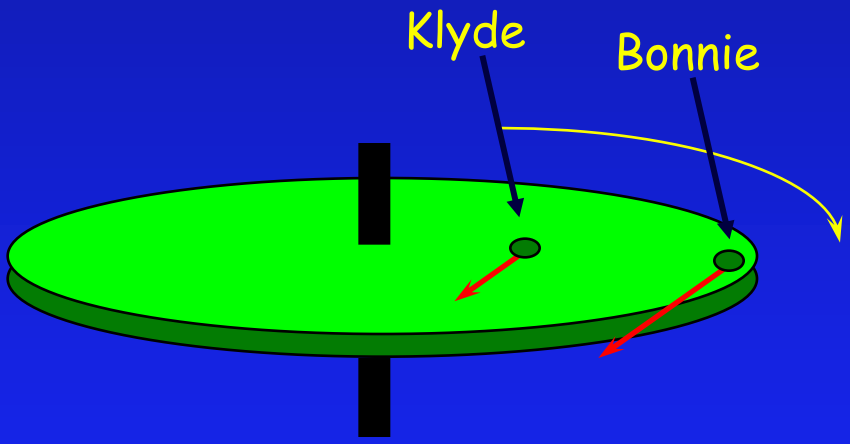
Merry-Go-Round ACT II

- Bonnie sits on the outer rim of a merry-go-round, and Klyde sits midway between the center and the rim. The merry-go-round makes one complete revolution every two seconds.
  - ➔ Klyde's angular velocity is:

(a) **the same as Bonnie's**

(b) **twice Bonnie's**

(c) **half Bonnie's**



- The angular velocity ω of any point on a solid object rotating about a fixed axis is the same.
  - ➔ Both Bonnie & Klyde go around once (2π radians) every two seconds.

Angular Acceleration

- Angular acceleration is the change in angular velocity ω divided by the change in time.

$$\overline{\alpha} \equiv \frac{\omega_f - \omega_0}{\Delta t}$$

- If the speed of a roller coaster car is 15 m/s at the top of a 20 m loop, and 25 m/s at the bottom. What is the cars average angular acceleration if it takes 1.6 seconds to go from the top to the bottom?

$$\omega = \frac{V}{R} \qquad \omega_f = \frac{25}{10} = 2.5 \qquad \omega_0 = \frac{15}{10} = 1.5$$

$$\overline{\alpha} \equiv \frac{2.5 - 1.5}{1.6} = 0.64 \text{ rad/s}^2$$

Summary
(with comparison to 1-D kinematics)

Angular	Linear
$\alpha = constant$	$a = constant$
$\omega = \omega_o + \alpha t$	$v = v_o + at$
$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$	$x = x_o + v_o t + \frac{1}{2}at^2$

And for a point at a distance R from the rotation axis:

$x = R\theta$ $v = \omega R$ $a = \alpha R$

CD Player Example

- The CD in your disk player spins at about 20 radians/second. If it accelerates uniformly from rest with angular acceleration of 15 rad/s^2, how many revolutions does the disk make before it is at the proper speed?

$$\omega^2 - \omega_0^2 = 2\alpha\Delta\theta$$

$$\frac{\omega_f^2 - \omega_0^2}{2\alpha} = \Delta\theta$$

$$\frac{20^2 - 0^2}{2 \times 15} = \Delta\theta$$

$\Delta\theta = 13.3$ radians

1 Revolutions = 2 π radians

$\Delta\theta = 13.3$ radians

= 2.12 revolutions

Summary of Concepts

- Uniform Circular Motion
  - ➔ Speed is constant
  - ➔ Direction is changing
  - ➔ Acceleration toward center $a = v^2 / r$
  - ➔ Newton's Second Law $F = ma$

- Circular Motion
  - ➔ θ = angular position radians
  - ➔ ω = angular velocity radians/second
  - ➔ α = angular acceleration radians/second^2
  - ➔ Linear to Circular conversions $s = r\,\theta$

- Uniform Circular Acceleration Kinematics
  - ➔ Similar to linear!