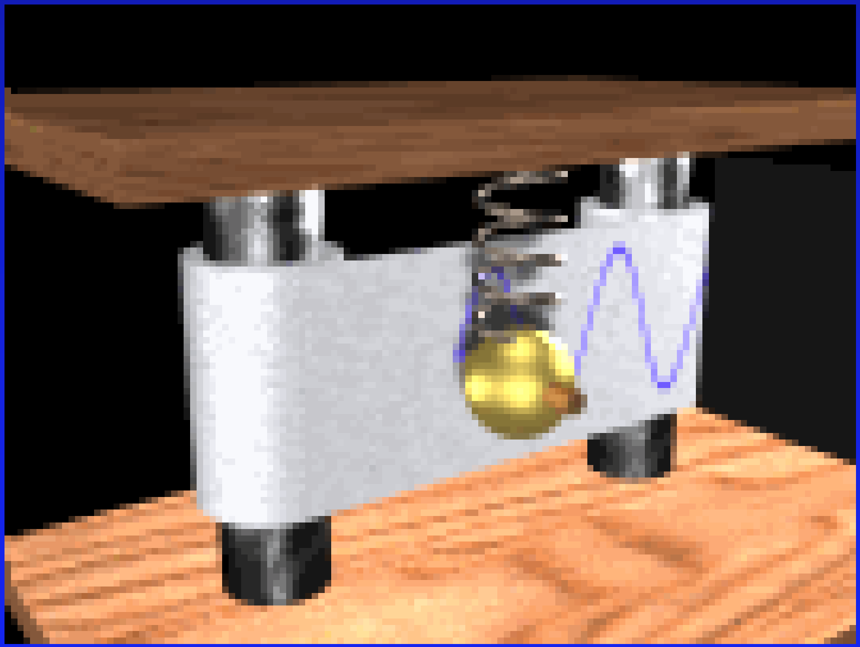
Exam III

Physics 101: Lecture 19
Elasticity and Oscillations



Overview

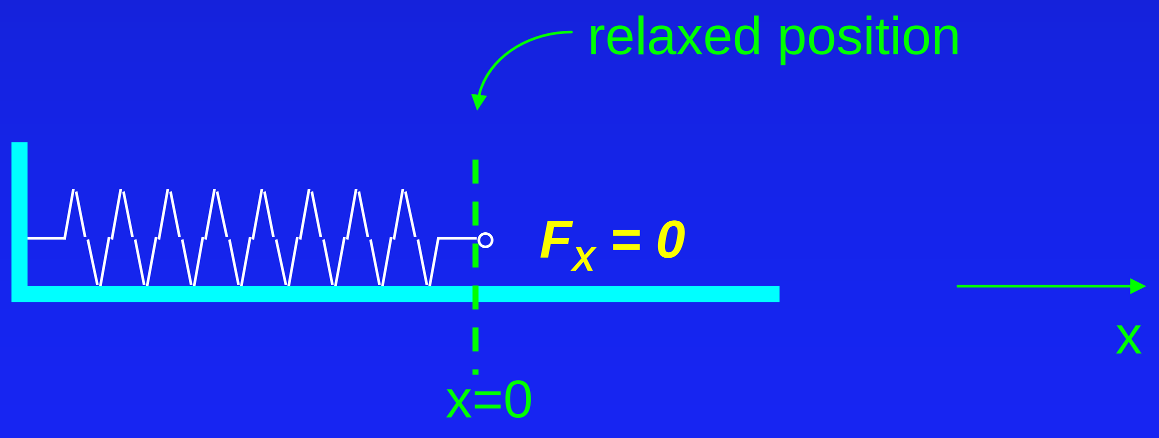
- Springs (review)
  - Restoring force proportional to displacement
  - $F = -k\,x$ (often a good approximation)
  - $U = \frac{1}{2} k\, x^2$
- Today
  - Young's Modulus (where does k come from?)
  - Simple Harmonic Motion
  - Springs Revisited

Springs

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

  - $F_X = -k\,x$

    Where x is the displacement from the relaxed position and k is the constant of proportionality.

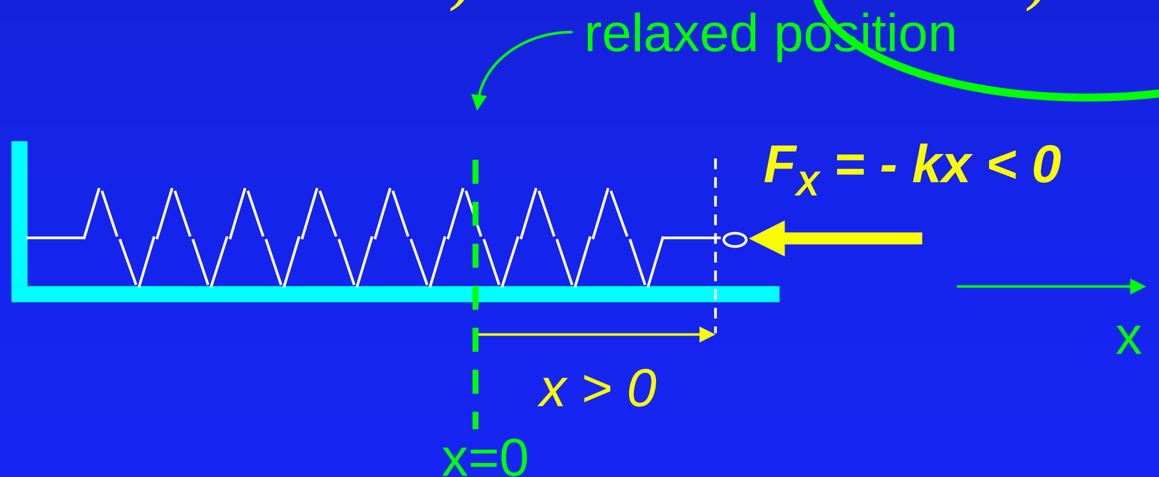


Springs ACT

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.
  - $F_X = -k\,x$ Where x is the displacement from the relaxed position and k is the constant of proportionality.

What is force of spring when it is stretched as shown below.

A) F > 0 B) F = 0 C) F < 0



Springs

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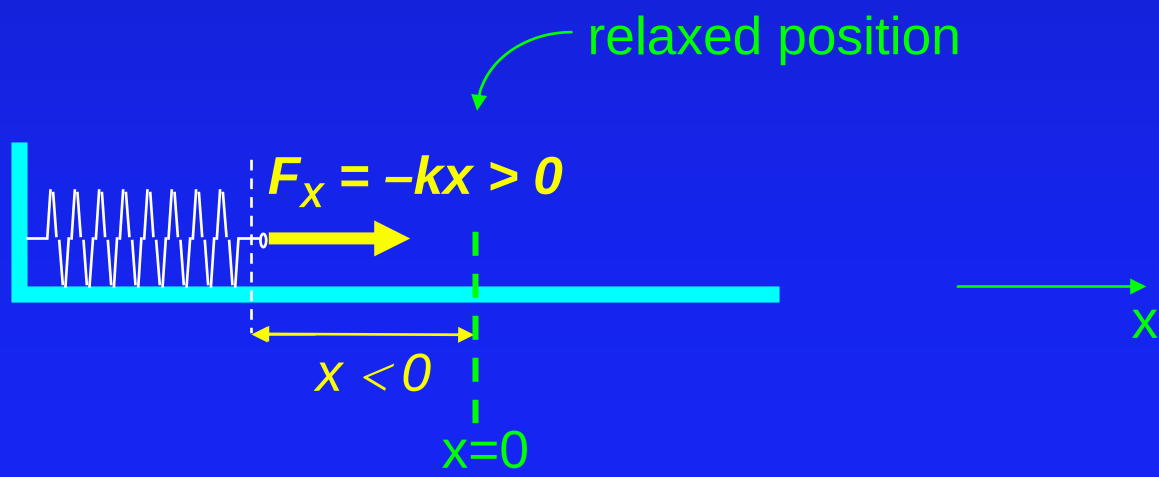
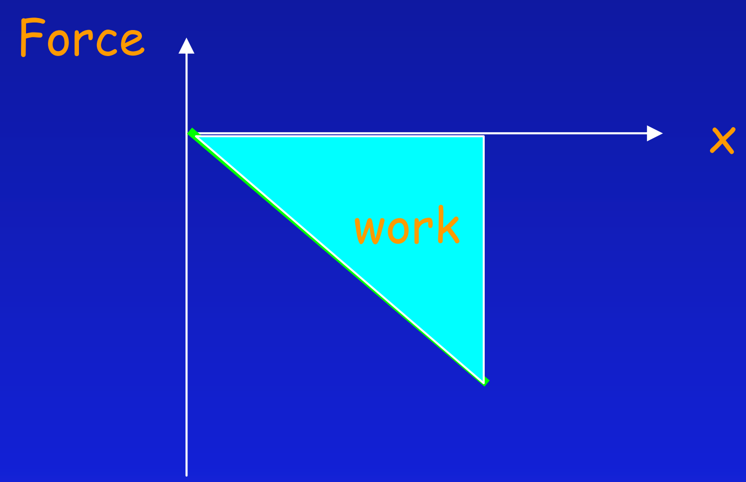


Potential Energy in Spring

- Hooke's Law force is Conservative
  - $F = -k\,x$
  - $W = -1/2\,k\,x^2$



  - Work done only depends on initial and final position
  - Define Potential Energy $U_{spring} = ½\,k\,x^2$

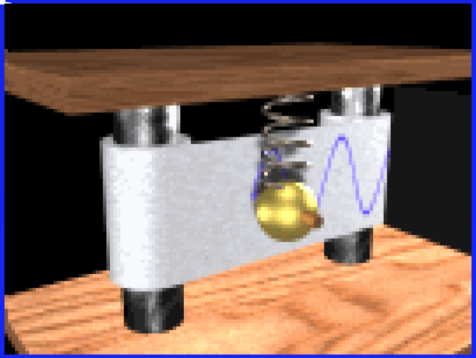
Young's Modulus

- Spring $F = -k\,x$ [demo]
  - What happens to "k" if cut spring in half?
  - A) decreases   B) same   C) increases
- k is inversely proportional to length!
- Define
  - Strain = $\Delta L / L$
  - Stress = F/A
- Now
  - Stress = Y Strain
  - $F/A = Y \; \Delta L/L$
  - $k = Y\,A/L$ from $|F| = k\,x$
- Y (Young's Modules) independent of L

Simple Harmonic Motion

- Vibrations
  - Vocal cords when singing/speaking
  - String/rubber band

- Simple Harmonic Motion
  - Restoring force proportional to displacement
  - Springs $F = -kx$

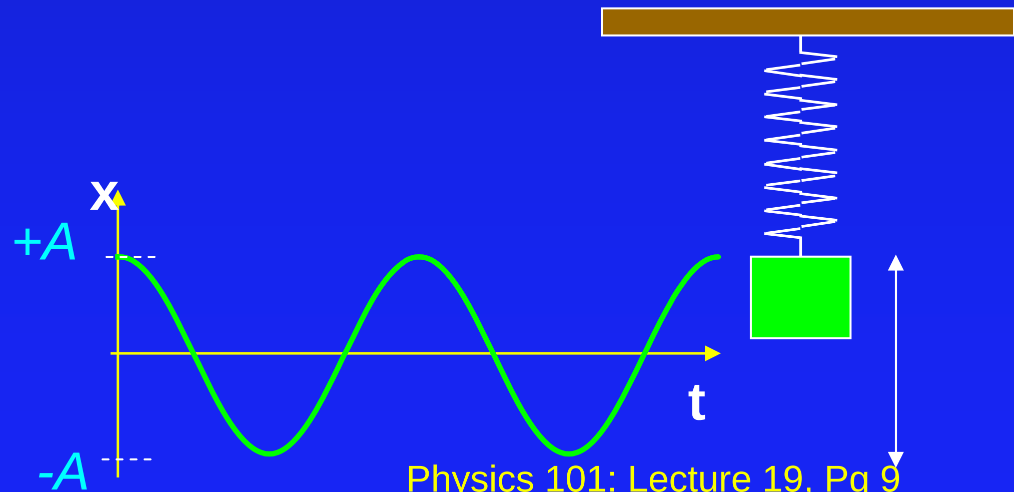


Spring ACT II

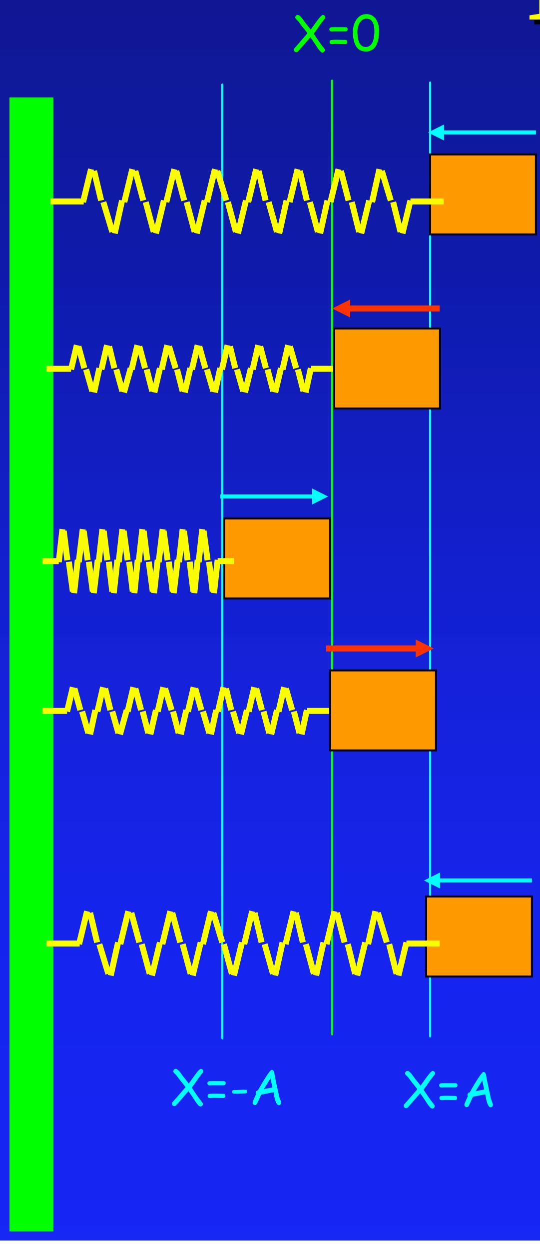
A mass on a spring oscillates back & forth with simple harmonic motion of amplitude A. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the magnitude of the acceleration of the block biggest?

1. When x = +A or -A (i.e. maximum displacement) ⟵ CORRECT
2. When x = 0 (i.e. zero displacement)
3. The acceleration of the mass is constant

F=ma



Springs and Simple Harmonic Motion



$X=A$; $v=0$; $a=-a_{max}$

$X=0$; $v=-v_{max}$; $a=0$

$X=-A$; $v=0$; $a=a_{max}$

$X=0$; $v=v_{max}$; $a=0$

$X=A$; $v=0$; $a=-a_{max}$

***Energy ***

- A mass is attached to a spring and set to motion. The maximum displacement is $x=A$
  - $\Sigma W_{nc} = \Delta K + \Delta U$
  - $0 = \Delta K + \Delta U$ or Energy U+K is constant!

    Energy $= \frac{1}{2} k x^2 + \frac{1}{2} m v^2$
  - At maximum displacement $x=A$, $v = 0$

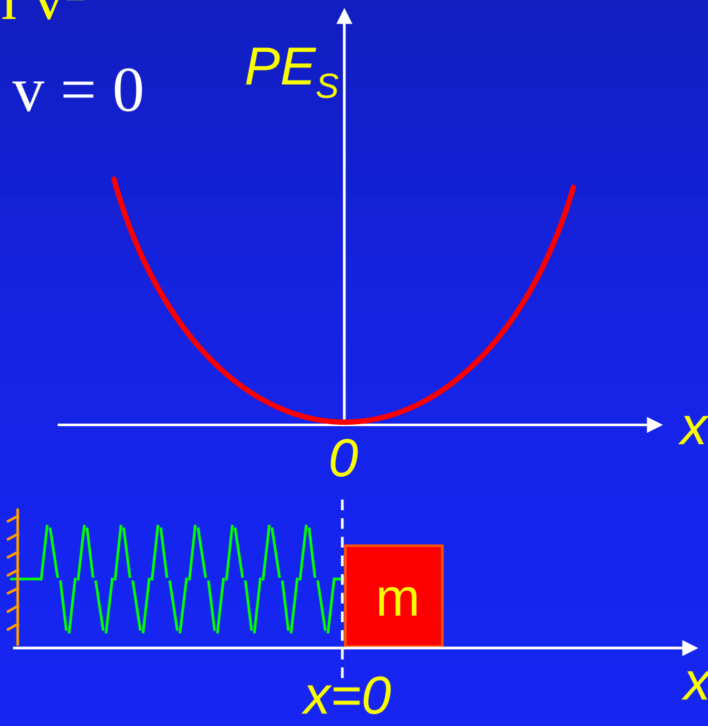
    Energy $= \frac{1}{2} k A^2 + 0$
  - At zero displacement $x = 0$

    Energy $= 0 + \frac{1}{2} m v_m^2$

  Since Total Energy is same

  $\frac{1}{2} k A^2 = \frac{1}{2} m v_m^2$

  $v_m = \text{sqrt}(k/m)\ A$

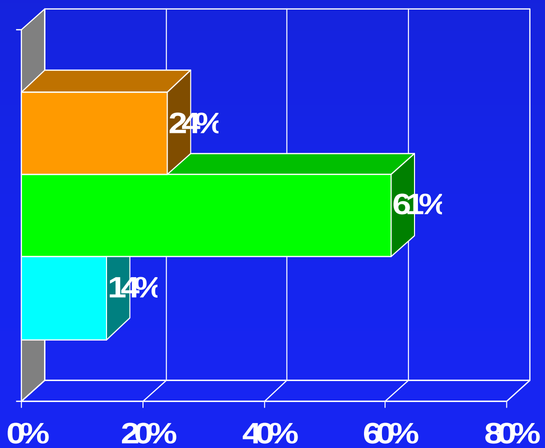


Preflight 1+2

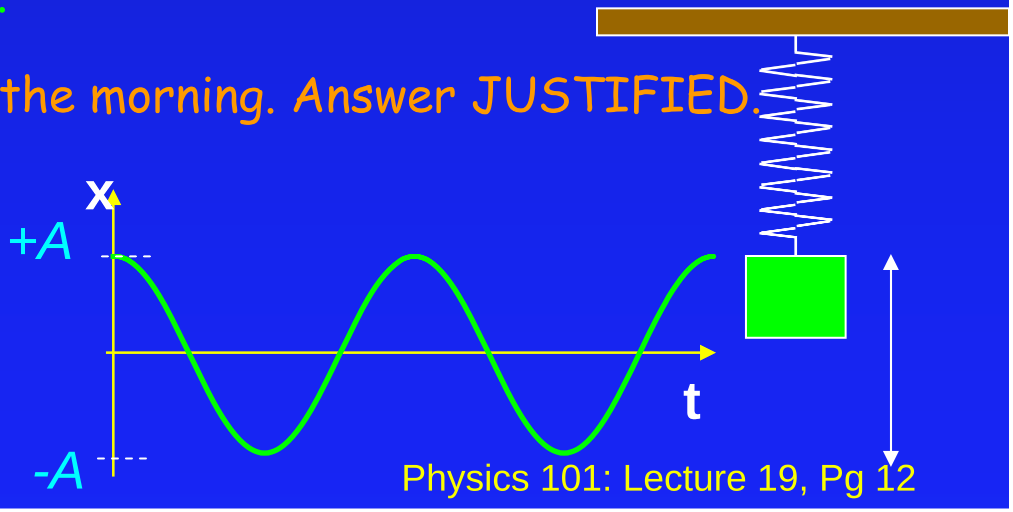
A mass on a spring oscillates back & forth with simple harmonic motion of amplitude A. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the speed of the block biggest?

1. When x = +A or -A (i.e. maximum displacement)
2. When x = 0 (i.e. zero displacement) ⟵ CORRECT
3. The speed of the mass is constant

"At x=0 all spring potential energy is converted into kinetic energy and so the velocity will be greatest at this point."



Its 5:34 in the morning. Answer JUSTIFIED.



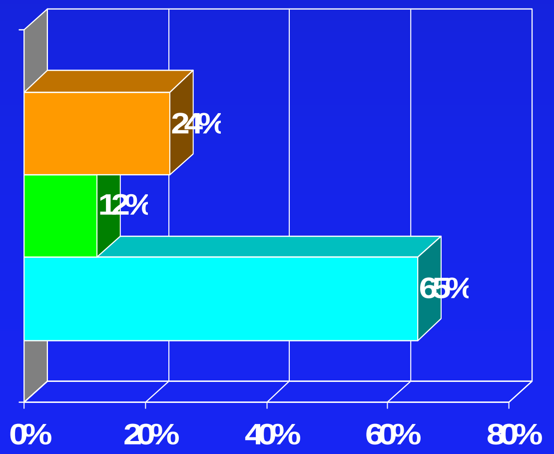
Preflight 3+4

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude *A*. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the total energy (K+U) of the mass and spring a maximum? (Ignore gravity).

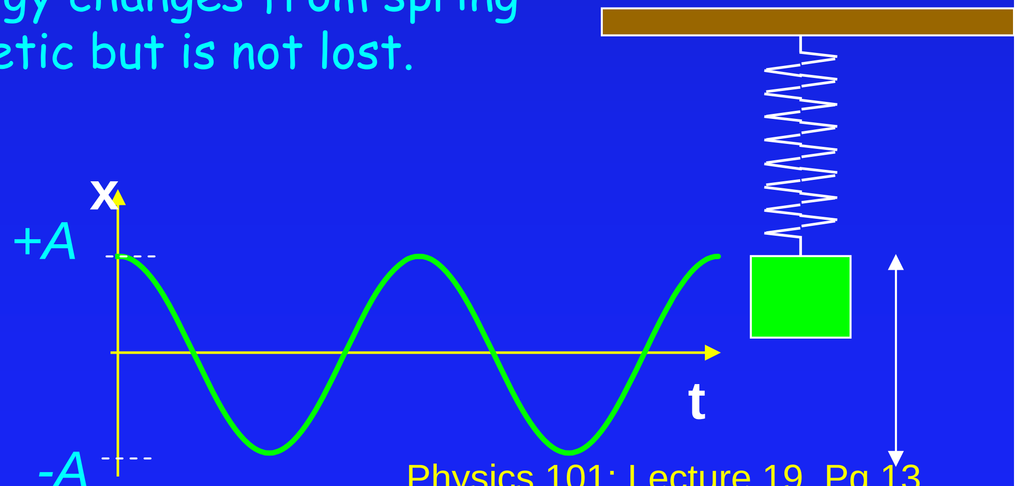
1. When x = +A or -A (i.e. maximum displacement)

2. When x = 0 (i.e. zero displacement)

3. The energy of the system is constant ← CORRECT



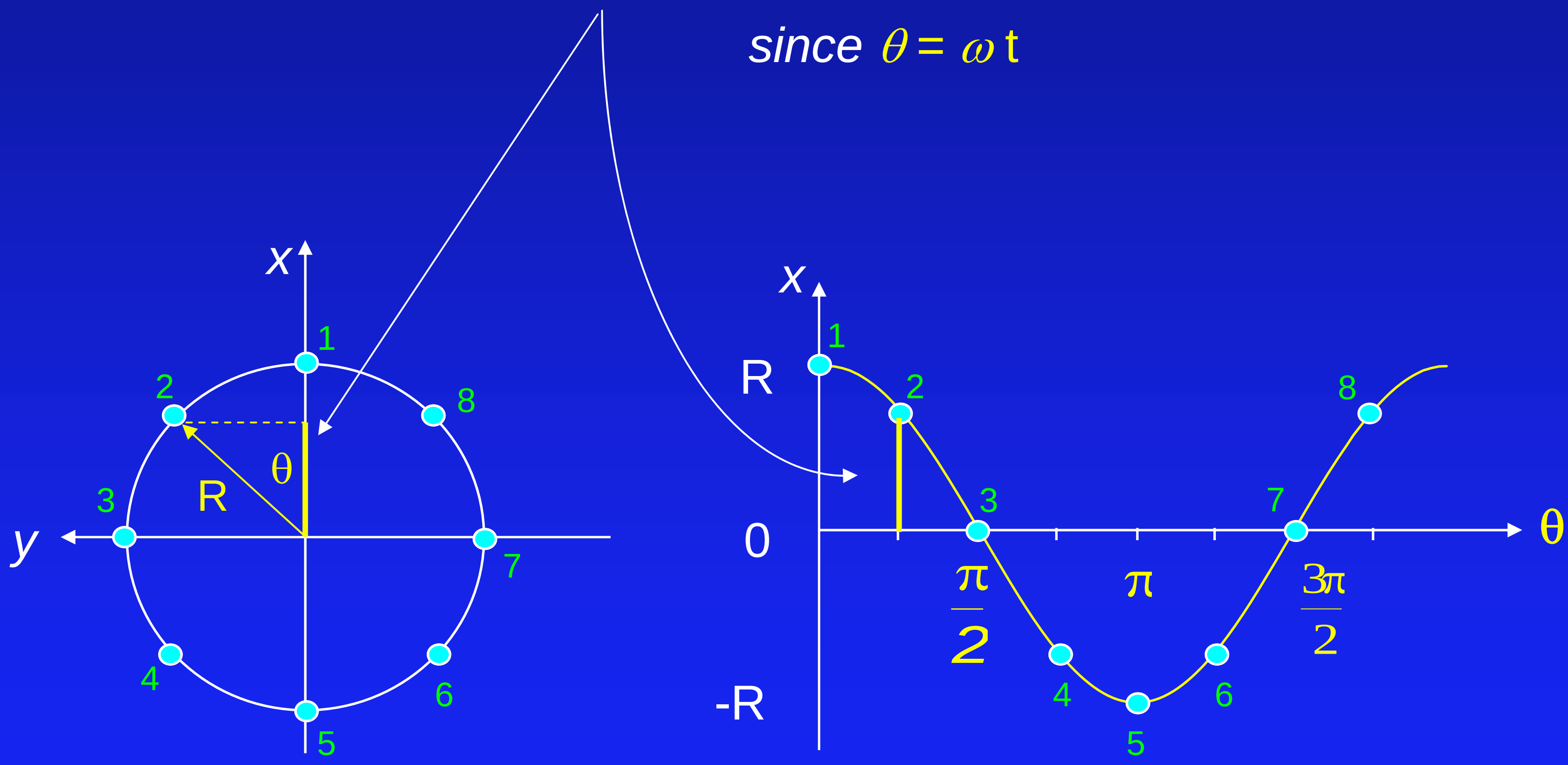
The energy changes from spring to kinetic but is not lost.



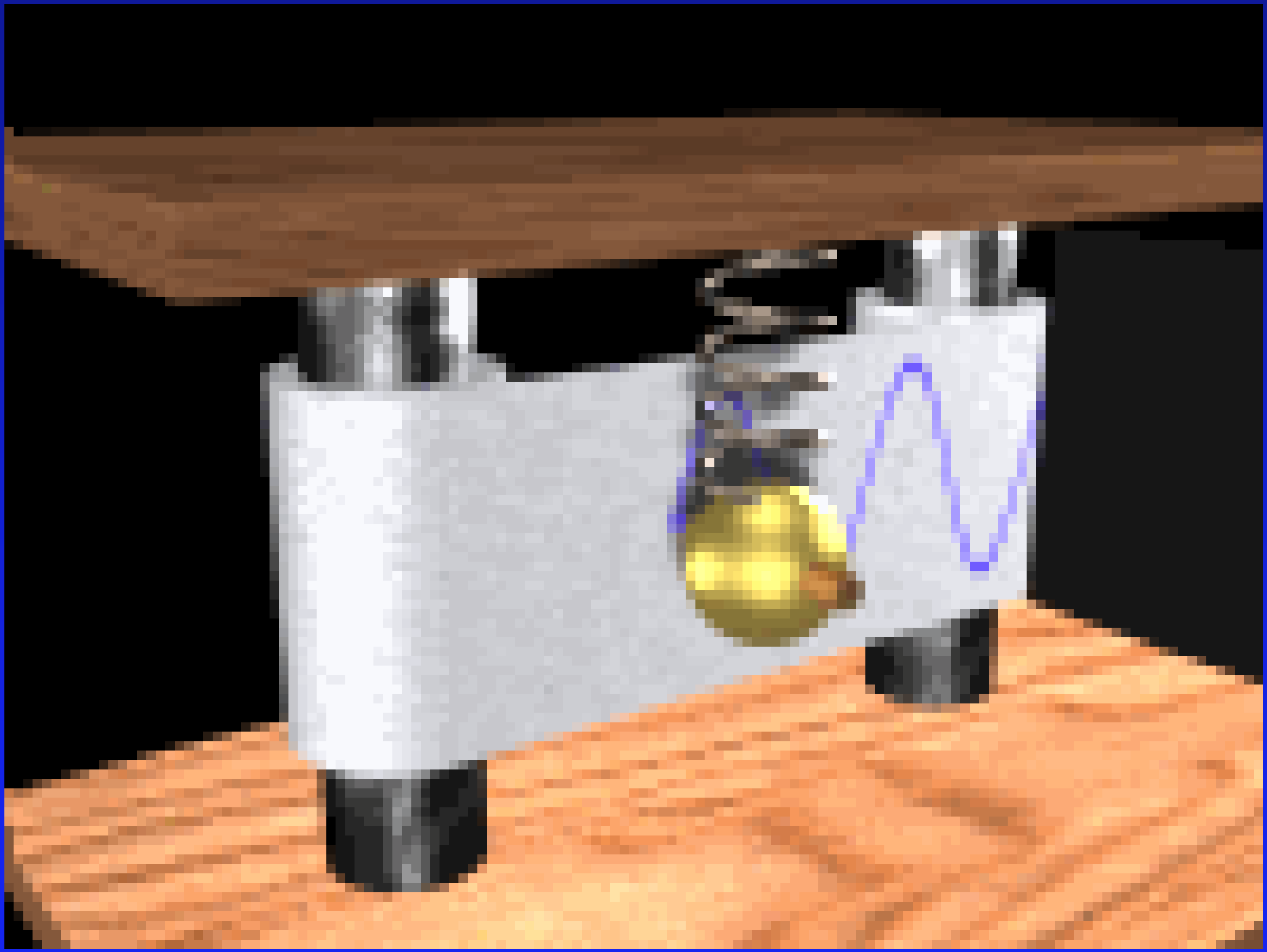
What does *moving in a circle* have to do with moving back & forth *in a straight line* ??

$$x = R\cos\theta = R\cos(\omega t)$$

since $\theta = \omega t$



SHM and Circles



Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$

$v(t) = -[A\omega]\sin(\omega t)$

$a(t) = -[A\omega^2]\cos(\omega t)$

OR

$x(t) = [A]\sin(\omega t)$

$v(t) = [A\omega]\cos(\omega t)$

$a(t) = -[A\omega^2]\sin(\omega t)$

$x_{max} = A$

$v_{max} = A\omega$

$a_{max} = A\omega^2$

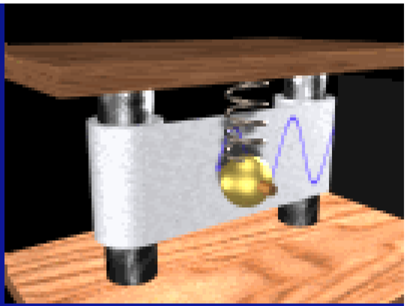
Period = T (seconds per cycle)

Frequency = f = 1/T (cycles per second)

Angular frequency = $\omega = 2\pi f = 2\pi/T$

For spring: $\omega^2 = k/m$

Example



A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

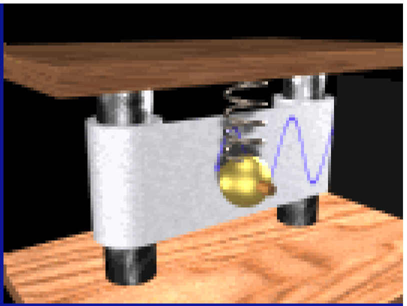
Which equation describes the position as a function of time x(t) =

A) 5 sin(ωt)   B) 5 cos(ωt)   C) 24 sin(ωt)

D) 24 cos(ωt)   E) -24 cos(ωt)

We are told at t=0, x = +5 cm. x(t) = 5 cos(ωt) only one that works.

Example



A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

What is the total energy of the block spring system?

A) 0.03 J   B) .05 J   C) .08 J

$E = U + K$

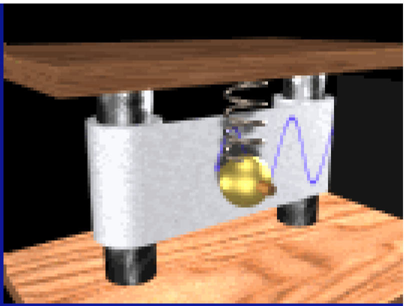
At t=0, x = 5 cm and v=0:

$E = \frac{1}{2} k x^2 + 0$

$= \frac{1}{2} (24 \text{ N/m}) (5 \text{ cm})^2$

$= 0.03 \text{ J}$

Example



A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

What is the maximum speed of the block?

A) .45 m/s B) .23 m/s C) .14 m/s

E = U + K

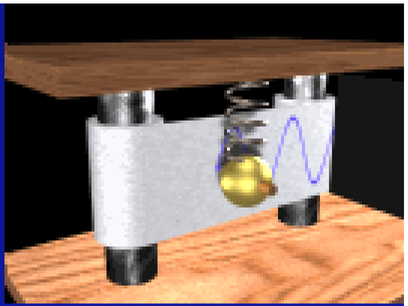
When x = 0, maximum speed:

$E = \frac{1}{2} m v^2 + 0$

$.03 = \frac{1}{2}\, 3\ \text{kg}\ v^2$

$v = .14$ m/s

Example



A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

How long does it take for the block to return to x=+5cm?

A) 1.4 s B) 2.2 s C) 3.5 s

ω = sqrt(k/m)

= sqrt(24/3)

= 2.83 radians/sec

Returns to original position after 2 π radians

T = 2 π / ω = 6.28 / 2.83 = 2.2 seconds

Summary

- Springs
  - $F = -kx$
  - $U = ½ k x^2$
  - $\omega = sqrt(k/m)$

- Simple Harmonic Motion
  - Occurs when have linear restoring force $F= -kx$
  - $x(t) = [A] \cos(\omega t)$ or $[A] \sin(\omega t)$
  - $v(t) = -[A\omega] \sin(\omega t)$ or $[A\omega] \cos(\omega t)$
  - $a(t) = -[A\omega^2] \cos(\omega t)$ or $-[A\omega^2] \sin(\omega t)$