Physics 101 Discussion Week 1 Explanation

D1-1 Unit conversion

2: baseball.

We should use an algebraic approach. The most important idea of the algebraic or symbolic approach is to pretend that you know everything you wish to know; if you do not know some quantity, simply call it x and pretend that you know it.

Q0 Let us apply this basic idea to our problem. Then the statement in the problem reads that the ball thrown by a good pitcher can traverse the home plate in x seconds. Write the required time x (seconds) in terms of the quantities in the problem.

Since the needed time is equal to the distance/the speed with the distance = 50 cm, and the speed = 95 miles/hour,

$$x \text{ s} = \frac{50 \text{ cm}}{95 \text{ miles/hour}}.$$

Q1 The rest is pure algebra. We need the ratios of the units with the same dimensions: hour/sec and mile/cm. Write x in terms of these ratios.

You may treat the units just as ordinary numbers:

$$x = \frac{1}{\text{s}} \frac{50 \text{ cm}}{95 \text{ miles/hour}} = \frac{50}{95} \frac{\text{hour}}{\text{s}} \frac{\text{cm}}{\text{mile}}.$$

Q2 Compute the needed unit ratios: hour/sec and cm/mile.

$$\frac{\text{cm}}{\text{mile}} = \frac{\text{cm}}{\text{km}} \frac{\text{km}}{\text{mile}} = \frac{0.01\text{m}}{1000 \text{ m}} \frac{\text{km}}{1.6 \text{ km}} = 10^{-5}/1.6.$$

Also

$$\frac{\text{hour}}{\text{s}} = \frac{3600 \text{ s}}{\text{s}} = 3600.$$

Q3 Combining all the results, finish the calculation.

$$x = \frac{50}{95} \times 3600 \times 10^{-5}/1.6 = 0.0118.$$

That is, the ball zips through the home plate in 0.0118 seconds.

3: heart.

In this case, let us first compute the amount of blood pumped out from the heart in a single day: x gallons/day.

Q4 Set up the algebraic relation determining x in terms of the data in the problem.

Since we have 70 beats per minute, in one minute, the heart pumps out 70×76 cm^3. Therefore (let us avoid/postpone numerical calculation as much as possible),

$$x \frac{\text{gallon}}{\text{day}} = 76 \times 70 \frac{\text{cm}^3}{\text{minute}}.$$

Q5 The rest is pure algebra. We need the ratios of the units with the same dimensions: cm^3/gallon and min/day. Write x in terms of these ratios.

$$x = 76 \times 70 \frac{\text{cm}^3}{\text{gallon}} \frac{\text{day}}{\text{minute}}.$$

The remaining task is to compute the unit ratios and finish the numerical calculation of x.

$$\frac{\text{cm}^3}{\text{gallon}} = \frac{\text{cm}^3}{\text{liter}} \frac{\text{liter}}{\text{gallon}} = \frac{\text{cm}^3}{1000 \text{ cm}^3} \frac{\text{liter}}{3.785 \text{liter}} = 1/3785,$$

and

$$\frac{\text{day}}{\text{minute}} = \frac{24 \times 60 \text{ minute}}{\text{minute}} = 24 \times 60.$$

Therefore,

$$x = 76 \times 70 \frac{1}{3785} \times 24 \times 60 = 2023.$$

That is, 2023 gallons per day.

The question asks the average amount of blood output per life of the heart. The average life span is 85×365 days, so the answer we wish to have is

$$2023 \times 85 \times 365 = 6.28 \times 10^7 \text{ gallons}.$$

D1-2 Dimension

1: dimensional homogeneity.

The key point is

$$[X^\alpha Y^\beta \cdots] = [X]^\alpha [Y]^\beta [\cdots].$$

Looking at one or two examples, you should not have any difficulty.

(a) $F = ma$

Q1 Convert the equation into the one for dimensions.

$$[F] = [ma] = [m][a].$$

Q2 Write all the dimensions in terms of the fundamental dimensions M, L, and T.

$$[F] = ML/T^2,$$
$$[m][a] = M \times L/T^2.$$

Therefore, the equation is dimensionally homogeneous.

Let us look at another example.

(b) $x = at^3$

Q3 Convert the equation into the one for dimensions.

$$[x] = [at^3] = [a][t]^3.$$

Q4 Write all the dimensions in terms of the fundamental dimensions M, L, and T.

$$[x] = L,$$
$$[a][t]^3 = (L/T^2) \times T^3 = LT.$$

Therefore, the equation is NOT dimensionally homogeneous.

The answers for the remaining questions are given below:

(c) $[E] = ML^2/T^2$, $[mv/2] = [m][v] = M(L/T)$. NO.
(d) $[max] = [m][a][x] = M(L/T^2)L = M(L/T)^2$. OK.
(e) $[v] = L/T$, $[\sqrt{Fx/m}] = ([F]L/M)^{1/2} = ((ML/T^2)L/M)^{1/2} = L/T$.
Therefore, the equation is dimensionally homogeneous.
(f) $[F] = ML/T^2$, $[E/vt] = [E]/[v][t] = M(L/T)^2/L = ML/T^2$. OK.

2 Let us simply compute $[q]$.

$$[q] = [mv^2/xa] = [m][v]^2/[x][a] = M(L/T)^2/[L(L/T^2)] = M.$$

D1-5 Simple manipulations

For these questions supposedly the best strategies are given.

1.

$$570 = f\frac{330 - v}{288}, \quad (1)$$
$$520 = f\frac{330 + v}{372}. \quad (2)$$

(1) and (2) are rewritten as

$$288 \times 570 = 330f - vf, \quad (3)$$
$$372 \times 520 = 330f + vf. \quad (4)$$

If we add these two equations, we obtain

$$288 \times 570 + 372 \times 520 = 660f. \quad (5)$$

This allows us to obtain f: $f = 541.82$.

(4) tells us that $fv = 372 \times 520 - 330f$. Therefore, $v = 372 \times 520/f - 330 = 372 \times 520/541.82 - 330 = 27.0$.

2.

$(m + M)/9 = m$. What is M/m?

$$m + M = 9m, \;\Rightarrow\; 1 + (M/m) = 9. \quad (6)$$

Thus, $M/m = 8$.

3.

$E = MV^2/2$ and $E = F^2/2k$. Obtain V in terms of F, M, and k. [You may assume $V > 0$]

$$MV^2/2 = F^2/2k, \;\Rightarrow\; V^2 = F^2/kM \tag{7}$$

Thus, $V = F/\sqrt{kM}$.

4.
You must clearly recognize that (i) and (ii) are the same questions; only different symbols are used.

(i) $K = I\omega^2/2$ and $L = I\omega$. Write K in terms of L and I.

$$K = I(L/I)^2/2 = L^2/2I.$$

(ii) should give $U = F^2/2k$.

5.
Let $\beta_1 = 10\log_{10}(I_1/I_0)$ and $I_D = I_1/D^2$. Obtain $\beta_D = 10\log_{10}(I_D/I_0)$ in terms of β_1 and D.

$$\beta_D = 10\log_{10}(I_D/I_0) = 10\log_{10}[(I_1/D^2)/I_0] = 10\log_{10}(I_1/I_0) - 20\log_{10} D.$$

That is, $\beta_D = \beta_1 - 20\log_{10} D$.

If you have any trouble in understanding the above calculation, you must review the logarithm.