Physics 101 Discussion Week 2 Explanation (2011)

D2-2 Displacement

Before answering each question, let us describe the displacements of the professor.

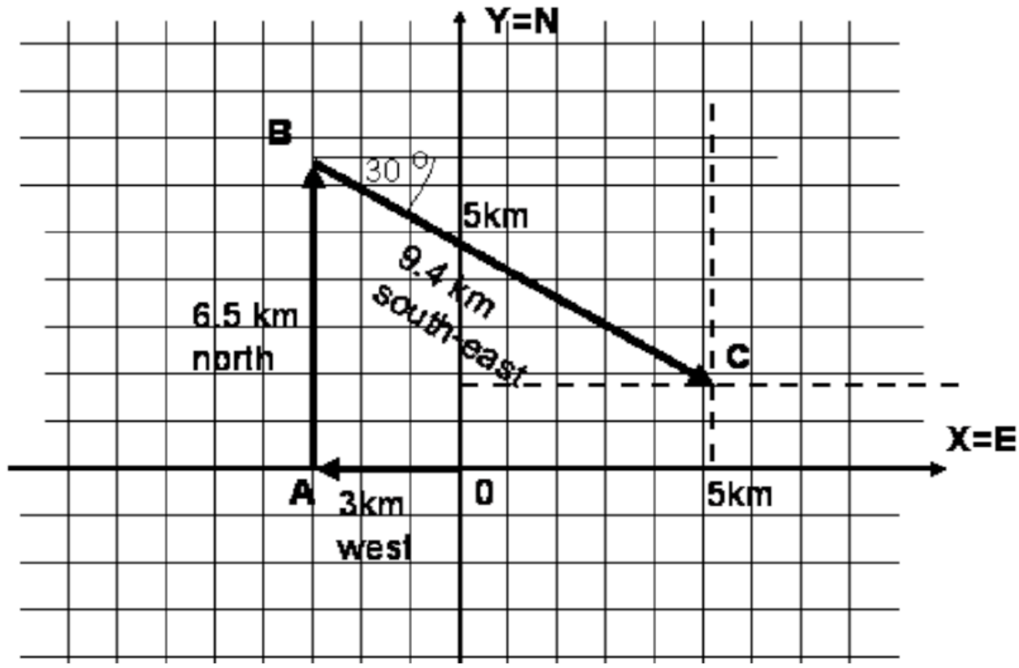
Q0. What do you think are the key points of the problem?

(1) Displacements are vectors. Thus, on a plane they are 2-dimensional vectors.
(2) 2D vectors can be expressed in coordinates, if we choose a 2D coordinate system as $\boldsymbol{r} = (x, y)$.

Q1. It is advantageous to visualize the trajectory of the professor on the xy-plane, assuming the positive y direction is North. Find such expressions for three displacement vectors of the professor.

The displacement vectors are as follows (the length unit is km):
3.0 km due west: $\boldsymbol{d}_1 = (-3, 0)$,
due north for 6.5 km: $\boldsymbol{d}_2 = (0, 6.5)$,
30° south of east for 9.4 km: $\boldsymbol{d}_3 = 9.4 \times (\cos(-30°), \sin(-30°)) = (8.14, -4.7)$.

Q2. Assuming that the origin of our coordinate system is at the Union, sketch the trajectory.



Q3. What is the coordinates of the final destination of the professor?

Since we have chosen the origin to be the starting point, the final position vector $\boldsymbol{r}$ must be identical to the total displacement: $\boldsymbol{r} = \boldsymbol{d}_1 + \boldsymbol{d}_2 + \boldsymbol{d}_3$. Thus, in coordinates

$$\boldsymbol{r} = (-3, 0) + (0, 6.5) + (8.14, -4.7) = (5.14, 1.8)$$

in km.

Now, we can read off the required answers to the questions.

1: how far north

This can be obtained by the y-component of $\boldsymbol{r}$: 1.8 km.

2: how far east

This can be obtained by the x-component of $\boldsymbol{r}$: 5.14 km.

3: total distance

Q4. For a 2D-vector $\boldsymbol{v} = (a, b)$, what is its magnitude (length) $|\boldsymbol{v}|$?

By definition

$$|\boldsymbol{v}| = \sqrt{a^2 + b^2}.$$

Recall Pythagorus' theorem.

Since the total displacement vector is $\boldsymbol{r} = (5.14, 1.8)$, the distance from the origin $|\boldsymbol{r}| = \sqrt{5.14^2 + 1.8^2} = \sqrt{29.66} = 5.45$ (km).

4: direction

Q5. For a 2D-vector $\boldsymbol{v} = (a, b)$, what is its angle θ from the positive x-direction?

From the definition of θ, we obtain (note that $\tan\theta = b/a$)

$$\theta = \arctan(b/a)$$

Since the total displacement vector is $(5.14, 1.8)$, $\theta = \arctan(1.8/5.14) = 19.3°$.

You must check whether your answers above are reasonable; look at the illustration in **Q2**.

D2-3 Rescue operation

Q0. What do you think are the crucial points of this problem?

(1) Velocities are vectors.
(2) If an object is moving on a board at a velocity $\boldsymbol{v}$, and if the board is moving at a velocity $\boldsymbol{V}$ relative to you the observer, the object moves at a velocity $\boldsymbol{v}+\boldsymbol{V}$ relative to you; that is, you observe that the object is moving at a velocity $\boldsymbol{v}+\boldsymbol{V}$.
If you jump on the board and sit on it, the object is moving at a velocity $\boldsymbol{v}$ relative to you; that is, you observe that the object is moving at a velocity $\boldsymbol{v}$.

You must recognize that the river surface is just the board in the above illustration.

1: direction of the boat

If you get on the boat that passively flows with the river, you jump on the surface of the river that is moving at a velocity 4 km/h in the direction of the arrow in the figure.

Q1. What is the velocity of the drowning child observed by you (i.e., relative to you) on the boat flowing passively with the river ?

The child is flowing with the river, so the child has a zero velocity relative to the river just as you have. That is, you observe that the child is not moving at all!

Now, the answer to **1** must be obvious

The boat should directly head to the child. The flow of the river is just the motion of the board on which you and the object are on in the above illustration.

2: rescue time

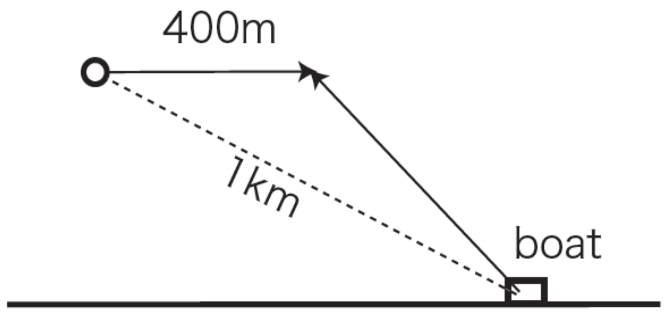
This must also be obvious, because you can totally ignore the flow of the river. Therefore, $1/10 = 0.1$ (hr) $= 6$ (min) is the needed time.

3: displacements

Now, the question is relative to the earth, or relative to the observer sitting on the river bank. If we know the position of the child at the moment he is rescued, you know the displacement vector of the boat as well.

Q2. What is the displacement of the drowning child by the time when he is rescued? Then, sketch the displacement vectors of the child and the boat.

It is equivalent to the question: how much distance is the child flowed in 0.1 hours? The river speed is 4 km/h, so the child flows 400 m downstream.



D2-4 Block and wires

D2-1 Problem **4** is directly related.

Q0. What do you think are the key points of the problem?

(1) Forces are vectors.
(2) The total force $= \sum \boldsymbol{F}_i = 0$ is the force balance condition (that is, under this condition, an object initially with a velocity zero keeps its zero velocity).

1-3. Before drawing the forces, answer Q1 and Q2.

Q1. There are three forces in the problem. Identify them.

Two tensions $\boldsymbol{T}_1$, $\boldsymbol{T}_2$ and the gravitational force $\boldsymbol{F}$ acting on the block.

Q2. What is the force balance condition in terms of these force vectors?

$$\boldsymbol{T}_1 + \boldsymbol{T}_2 + \boldsymbol{F} = 0.$$

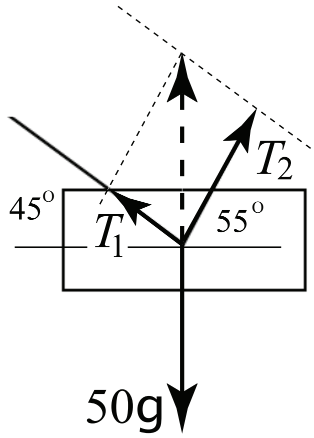
Its geometrical expression is that these three vectors make a closed triangle.

Q3. Draw the forces in the figure in the booklet. When you draw the force vectors in the figure, BE SURE that the force balance condition is explicitly correct.

The figure is on the next page.

Pay attention to the parallel auxiliary broken line segments in the following illustration to construct the 'force parallelogram.'

Q4. Using the coordinates given in the figure, write the tensions in components. This answers **2** and **3**. Also give $\boldsymbol{F}$ in components.



$$
\begin{aligned}
\boldsymbol{T}_1 &= (-T_1\cos 45^\circ, T_1\sin 45^\circ),\\
\boldsymbol{T}_2 &= (T_2\cos 55^\circ, T_2\sin 55^\circ),\\
\boldsymbol{F} &= (0, -50g).
\end{aligned}
$$

Q5. Write down the force balance condition in components.

In symbols this was answered in **Q2**. Now, putting the results in **Q4** into it, we get

$$
\begin{aligned}
&(-T_1\cos 45^\circ, T_1\sin 45^\circ) + (T_2\cos 55^\circ, T_2\sin 55^\circ) + (0, -50g)\\
&\qquad = (-T_1\cos 45^\circ + T_2\cos 55^\circ, T_1\sin 45^\circ + T_2\sin 55^\circ - 50g) = 0.
\end{aligned}
$$

4: balance condition in components.

This is now trivial; simply read off the component relations.

$$
\begin{aligned}
-T_1\cos 45^\circ + T_2\cos 55^\circ &= 0, && (1)\\
T_1\sin 45^\circ + T_2\sin 55^\circ - 50g &= 0. && (2)
\end{aligned}
$$

5: obtain tensions.

Q6. Write the simultaneous equations, calculating the coefficients numerically.

$$
\begin{aligned}
-0.71T_1 + 0.57T_2 &= 0, && (3)\\
0.71T_1 + 0.82T_2 &= 490. && (4)
\end{aligned}
$$

Q7. Solve the above simultaneous equation for T_1 and T_2.

In this case, the easiest way to solve the equations is to add (3) and (4) to obtain

$$1.39T_2 = 490.$$

That is, $T_2 = 352.5$ (N). Then, (3) tells us that $T_1 = (0.57/0.71)T_2 = 283.0$ (N).

D2-5. Incline and friction

Q0. What do you think are the key points of the problem?

(1) Force as vectors/force balance condition as in D2-4.
(2) Static friction: its magnitude is generally determined by the stationary condition.

(3) HOWEVER, the largest possible magnitude F_{max} of the static friction force is related to the magnitude N of the normal force with the coefficient of static friction as $F_{max} = \mu N$.

1: free-body diagram

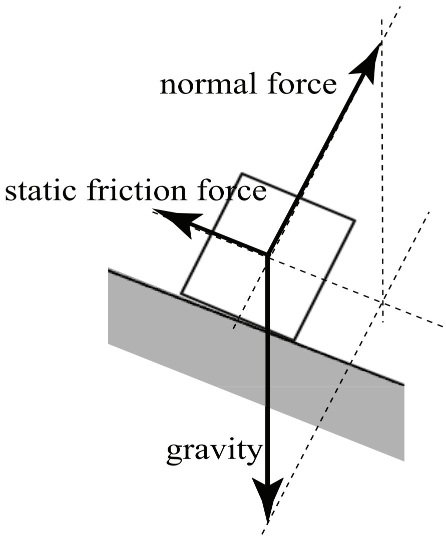
Q1. Before drawing any force diagram, identify all the forces. There are three forces in the problem.

These three forces are: the normal force $\boldsymbol{N}$, the friction force $\boldsymbol{f}$, and the gravitational force $\boldsymbol{F}$ acting on the block.

$\boldsymbol{N} + \boldsymbol{f} + \boldsymbol{F} = 0$ is the force balance condition.

Q2. Draw the forces in the figure in the booklet. When you draw the force vectors into the figure, BE SURE that the force balance condition is explicitly correct.

Again, pay attention to the dotted lines that describe the parallelogram indicating the vector addition rule.



2: normal force.

Q3. If you draw the force balance parallelogram as in the figure above, you can immediately read off the answer. Try to do so, and also write down all the forces in components using the coordinates given in the booklet. Assume that the slope angle is a general θ (in the figure it is 30°).

From the diagram we see that the y-component of the gravitational force $\boldsymbol{F}$ must cancel the normal force:

$$F\cos\theta = N.$$

The magnitude of $\boldsymbol{F}$ is $5g = 49$ N, so the normal force must have the magnitude $49 \times \cos 30° = 42.4$ N. Do not mix up sin with cos.

Forces in components read (DO NOT FORGET SIGNS)

$$\boldsymbol{N} = (0, N), \quad \boldsymbol{f} = (-f, 0), \quad \boldsymbol{F} = (F\sin\theta, -F\cos\theta).$$

Q4. Explicitly write the force balance condition in components.

$$(0, N) + (-f, 0) + (F\sin\theta, -F\cos\theta) = (F\sin\theta - f, N - F\cos\theta) = 0$$

3: μ_s.

Q5. Obtain the friction force.

The x-component of the force is on the verge of becoming positive when $\theta = 40°$, but still the box does not move. Therefore, the net force on the box must still be zero. That is, using the result of **Q2** above $F\sin\theta - f = 0$, or $f = F\sin\theta = 49\sin 40° = 31.5$ N.

Q6. What is the relation between the frictional force, the normal force, and the coefficient μ_s of static friction?

This is actually a tricky question (our favorite question to ask in exams!). (2) and (3) in the answer to **Q0** say:

The magnitude of the friction force f is generally just identical to the 'pulling force' (the component parallel to the friction surface; in our example it is $F \sin\theta$) UNTIL it starts to slide. Only the friction force on the verge of sliding, that is, only the MAXIMUM possible friction force is given in terms of the coefficient of static friction μ_s as $f = \mu_s N$.

This is the situation we have for $\theta = 40°$.

Q7. What is the normal force in terms of F and θ?

From the force balance condition, there must not be any net y-component: $N - F\cos\theta = 0$, or $N = F\cos\theta$.

Q8. Now, you know f and N in terms of F and $\theta = 40°$. Find μ_s.

$f = F\sin\theta$ and $N = F\cos\theta$, so $\mu_s = f/N = F\sin\theta/F\cos\theta = \tan\theta$. That is, $\mu_s = \tan 40° = 0.84$. Notice that the mass of the box does not matter at all.

D2-6. Blocks and pulley

Q0. What do you think are the key points of the problem?

(1) Force balance conditions.
(2) Static friction: when you encounter static friction, always recall (2) and (3) in the answer to **Q0** of the preceding problem.

1: free-body diagram

Q1. For any force balance problem, you must itemize all the forces first. Do so.

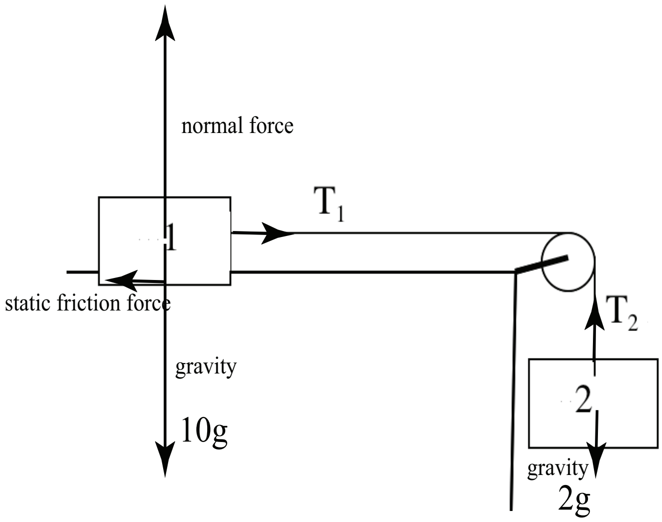
Tensions, normal forces, and gravity are usually relevant.

2: tensions

They must be identical. Trivial force balance.

3: T_2

They must be identical, since there is NO motion.



4: μ

Q2. What is the magnitude of the friction force acting on block 1?

Read the answer to **Q0** of **D2-5. The friction force must balance T_1, so it is $2g =$ 19.6 N.**

Q3. What is the maximum possible friction force? Is it larger or smaller than $T_1 = 2g$ N?

The maximum static friction force that can act on block 1 is $0.3 \times 6g = 1.8g$. This is smaller than $T_1 = 2g$. Don't use $g = 9.8$; **avoid unnecessary numerical calculation**.

Therefore, the static friction cannot withstand pulling, and the block must start moving.