Physics 101 Discussion Week 10 Explanation (2011)

D10-1 Disk and incline

First, scan the whole set of questions.

Q0. What are the key points of the problem? You must be able to explain the words you use.

This is a problem about rotational kinetic energy, conservation of total energy, and associated work/energy theorem.

(1) If something is rolling on a floor or on an incline, it is experiencing not only translational motion but also rotational motion. Thus, the kinetic energy K consists of two parts: translational kinetic energy due to translational speed v and that due to rotational angular speed ω:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

Identify this formula in the formula sheet. What is m? What is I? Where does I show up? Look at the formula sheet and review the related equations such as $I\alpha = \sum \tau$.

(2) You must find the relation between the translational speed v of the rotational center and the angular speed ω around it; if there is no slippage, we have $v = R\omega$, where R is the radius of the rotating object.

(3) If there is no slippage, energy is not lost, so you may use conservation of energy.

(4) The total energy can change due to work: work/energy theorem: $\Delta E = W$. Here, W is the work done to the object (in our problem, a disk).

1: I

Q1. Clearly write down the definition of the moment of inertia of a collection of masses. What is the intuitive meaning of moment of inertia I?

The moment of inertia around a given point O is defined as

$$I = \sum m_i R_i^2,$$

where R_i is the distance of mass m_i from O.

Roughly speaking, I measures how much mass is moved over how long of a distance (during a slight rotation). Thus, even if the total mass is maintained, I can be made as large as you wish; cats know this as we learn in **D10-5**.

Calculate the moment of inertia of the given disk.

If you do not know integration, there is no honest way to compute it; simply look at the formula sheet:

$$I = \frac{1}{2}mR^2 = 4 \times 0.25^2/2 = 1/8 = 0.125 \text{ kg}\cdot\text{m}^2.$$

2: total kinetic energy

Q2. We know the total kinetic energy consists of two parts as discussed in **Q0**, but this is useless, since we know neither v nor ω. We have already realized that the point of **D10-1** is conservation of energy. What is the key observation?

The initial potential energy (relative to the bottom) is totally converted into the final kinetic energy of the disk.

Calculate the initial potential energy U and finish the question.

$$U = mgh = 4 \times 9.8 \times 2 = 78.4 \text{ J}.$$

Therefore, the total kinetic energy must be 78.4 J according to the conservation law.

3: K ratio

Q3. Write down the translational and the rotational parts of the kinetic energy. We know $v = R\omega$ without slip.

$$K_{trans} = \frac{1}{2}mv^2, \quad (1)$$

$$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{4}mR^2\omega^2 = \frac{1}{4}mv^2. \quad (2)$$

In the second line do not forget that there are two 1/2 factors.

Thus, the ratio is:

$$\frac{K_T}{K_R} = \frac{(1/2)mv^2}{(1/4)mv^2} = 2.$$

That is, the ratio is constant as long as the disk is rolling without slip.

4: ω

Now, we already have everything we need.

With the aid of (1), (2) and $I = mR^2/2$ $(= 1/8)$

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}m(R\omega)^2 + \frac{1}{2}I\omega^2 = \frac{3}{2}I\omega^2. \tag{3}$$

This must be equal to the total energy 78.4 J. Therefore, we have $(3/2)I\omega^2 = 78.4$ or $\omega^2 = 78.4(2/3)/0.125 = 418$. Hence, $\omega = 20.45$ rad/s.

5: frictionless case

Q4. If there is no friction, what happens to the rotational angular speed ω? Is there any torque acting on the disk around its center?

There is no torque, so the angular speed ω around the center must be constant; if it is not rotating initially, it will not start rotating.

Q5. What can you say about the rotational kinetic energy?

It does not change; since it is initially zero, it is zero. That is, all the potential energy must be converted into translational kinetic energy.

Finish the problem. Is the translational speed faster or slower without friction than with friction?

The sliding speed must be faster if there is no friction, so it takes less time to slide down.

If you wish to use formulas, look at (3). The second term does not exist in the present case, so inevitably v becomes larger.

D10-2 M vs. I

Q0. What is the key issue of the problem?

The kinetic energy K of rotating body can consist of two parts, translational K_T and the rotational kinetic energy K_R.

(0) Energy conservation, since there is no friction. Key points worth remembering are:

(1) If there is no dissipation, the total mechanical energy $K + U$ is conserved.
(2) Dissipation does not occur under two conditions:
(2-1) No friction between the body and the floor: K_R is preserved (cannot change).
(2-2) No slip between the body and the floor: in this case the rotational speed and the translational speed are related rigidly as $V = R\omega$. That is, K_T/K_R is a fixed number that depends on the geometry of the system. (The force between the body and the floor is a static friction force that is determined by the equation of motion for the rotating body: $I\alpha = \tau$. The static friction force does not do any work, because it is static, so there is no displacement of the point upon which the force acts.)

1: frictionless

As is noted in **Q0**. if the cylinder is not rotating initially, without friction, it never starts rotating. Therefore, the problem without friction is just the same as that of a sliding block of a given mass.

Q1. Suppose the cylinder slides down for the height difference h. What is the kinetic energy of the cylinder?

Since the mass of the cylinder is not mentioned, let us pretend that we know it as M.

The total mechanical energy is conserved.
Initially: $K_T = 0$, $U = Mgh$ (relative to the final position), so the total mechanical energy is $E_{init} = Mgh$.

Finally: $U = 0$.
Therefore, conservation of E implies the final $K_T = Mgh$. If we write the final speed as V, then $K_T = MV^2/2 = Mgh$. Thus, $V = \sqrt{2gh}$. Notice that the result is independent of the mass M.

This answers the question. Both A and B have the same speed, if they slide down the frictionless slope for 1 m.

2: no slip.

Again, no mechanical energy is lost.

Q2. If there is no slip and if the center of mass has a translational speed V, what is the rotational angular speed ω of the cylinder of radius R?

$V = R\omega$.

Q3. Suppose the cylinder has mass M and moment of inertia I, what is the total kinetic energy K of the cylinder moving on a non-slippery floor with a translational speed of V?

$$K_T = \frac{1}{2}MV^2 = \frac{1}{2}MR^2\omega^2, \quad K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{I}{R^2}V^2,$$

so

$$K = \frac{1}{2}\left(M + \frac{I}{R^2}\right)V^2 = \frac{1}{2}(MR^2 + I)\omega^2.$$

Q4. Suppose the cylinder rolls down for the height difference h. What is the kinetic energy of the cylinder?

The total energy (relative to the final position) is $E = U_{init} = Mgh$. After rolling down, U is totally converted into kinetic energy, so

$$K = \frac{1}{2}\left(M + \frac{I}{R^2}\right)V^2 = Mgh$$

Therefore,

$$V^2 = \frac{gh}{2(1 + I/MR^2)}.$$

This implies that, if I is constant, larger M implies larger V.

We have answered the problem.

A and B both have the same I, but the mass of B is larger than that of A, so B must be faster.

3: friction force

This is the case that is neither frictionless nor without slip.

Q5. What is the relation between the angular acceleration α and the torque τ?

$$I\alpha = \tau$$

The equation is around the center of the cylinder.

Q6. What is the torque on the cylinder?

$I\alpha = 1.17$ N·m must be the torque due to the frictional force (no other force can produce any torque around the center of the cylinder).

Q7. Since there is no slip, the force between the cylinder and the slope is static friction. What is it?

$\tau = fR$, because the radius and the friction is perpendicular. Therefore, $f = 5.85$ N.

D10-3 Massive Pulley

Q0. What are the key points of the problem?

You must clearly recognize that M_1 and M_2 have the same speed v, which is related to the angular speed ω of the massive pulley as $v = R\omega$, because there is no slip (unfortunately, it is not stated clearly in the problem).

The key principle is conservation of mechanical energy.

1: h

Q1. This is of course a very standard conservation of energy question. What do we know (or are we given)?

The final velocity of the blocks $v = 4$ m/s. This means that we know ω as well. The displacement of 3 m is useless.

Q2. As is always with this kind of problems, clearly state the initial and final energies: write down initial and final kinetic energy, potential energy and total energy, pretending you know all the needed quantities (in terms of v and ω above).

Initial:

K: Nothing is moving, so $K_i = 0$,

U: Only M_2 changes its height, so we have only to pay attention to the potential energy of M_2 relative to the final height. Therefore, $U_i = M_2 gh$,

The total $E_i = K_i + U_i = M_2 gh$.

Final:

The kinetic energies of the blocks are $(1/2)M_1 v^2$, and $(1/2)M_2 v^2$, respectively. The kinetic energy of the pulley is

$$\frac{1}{2}I\omega^2 = \frac{1}{4}MR^2\omega^2 = \frac{1}{4}Mv^2.$$

Do not forget there are two 1/2 factors, one for the definition of kinetic energy, and the other from the moment of inertia for the disk.

Combining all the results, the total final kinetic energy is

$$K_f = \frac{1}{2}\left(M_1 + M_2 + \frac{M}{2}\right)v^2.$$

$U_f = 0$, because we define it relative to the final position.
$E_f = K_f + U_f = (M_1 + M_2 + M/2)v^2/2$.

Finish the problem, applying conservation of total mechanical energy.

$E_i = E_f$:

$\frac{1}{2}\left(M_1 + M_2 + \frac{M}{2}\right)v^2 = \frac{1}{2}(15{+}25{+}10){\times}4^2 = 25{\times}4^2 = 400 = M_2gh = 25{\times}9.8h$,

or $245h = 400$. $h = 1.63$ m.

2: disk → hoop

You must be able to answer this question immediately: Less than 4 m/s. Answer the following question:

Q3. Comparing a disk and a hoop with the same masses and radii, for which object do you have to rotate more portions of its mass farther when you rotate the whole object?

The mass of the hoop is concentrated on the outer edge of the corresponding disk, so you must move much more mass in the case of the hoop than that of the disk. The hoop is harder to rotate.

Quantitatively, this difference shows up in the formula of the moments of inertia around the center MR^2 vs $MR^2/2$, or $I_{hoop} > I_{disk}$.

D10-4 Merry-go-round

Q0. What are the key concepts/principles for this Discussion? Quote the relevant formulas from the formula sheet and read them in plain English.

Angular momentum L and its conservation when there is no net external torque. Relevant formulas are:

$$L = I\omega.$$

"Angular momentum around a given rotational center (axel) is equal to the moment of inertia around the same center times the angular speed around the same center."

$$\sum \tau_{ext} \Delta t = \Delta L.$$

"The change of angular momentum during a short time is equal to the time span times the net external torque."

Clearly recognize that this is a disguised version of Newton's second law:

$$\tau = I\alpha,$$

as can be seen from $\Delta L = \Delta(I\omega) = I\Delta\omega = I\alpha\Delta t = \sum \tau \Delta t$.

1: kinetic energy

Q1. You know $K = (1/2)I\omega^2$, so you have only to calculate I of the whole system. Do so, and then finish the problem.

Go back to the definition of I: $I = \sum m_i R_i^2$ = the sum for the merry-go-round + mR^2. Thus,

$$I_{total} = \{(1/2)M + m\}R^2 = (150/2 + 30) \times 3^2 = 105 \times 3^2 = 945 \text{ kg}\cdot\text{m}^2.$$

Therefore, $K = (1/2)I_{total}\omega^2 = 4252.5$ J. (The moment of inertia of the merry-go-round is $I_{mg} = 675$ kg·m^2.)

2: child moves

Q2. Are you sure that the angular momentum of the system is conserved?

The merry-go-round + boy system is rotating horizontally. The gravitational forces on them are parallel to the axis of rotation, so no torque is caused by them. Thus, there is no external torque at all. Therefore, the total angular momentum must be conserved.

Q3. This is a typical problem of conservation of angular momentum. Just as with any other problem that uses conservation laws, write the initial and final angular momenta explicitly, pretending that you know everything needed.

Initial angular momentum: (I_{mg} is the moment of inertia of the merry-go-round)

$$L_i = I_{total}\omega_i = (I_{mg} + mR^2)\omega_i = 945\omega_i.$$

Final angular momentum: since the boy is at the center (and we may ignore his 'thickness'), he does not contribute to the moment of inertia of the system:

$$L_f = I_{total}\omega_f = I_{mg}\omega_f = 675\omega_f.$$

Now, applying conservation of angular momentum, finish the problem.

$$945\omega_i = 675\omega_f \quad \text{or } \omega_f = \frac{675}{945}\omega_i = 4.2 \text{ rad/s.}$$

3: work

Q4. What do you utilize to answer the question?

Work/energy theorem: $\Delta E = E_f - E_i = W$.

Q5. Calculate ΔE. That is, recognize E_i and E_f clearly.

Potential energy is not needed.
Initial energy, which is obtained in **1** (but copied here)
$E_i = K_i = \frac{1}{2}I_i\omega_i^2 = (1/2)945 \times 3^2 = 4252.5$ J.
Final energy:
$E_f = K_f = \frac{1}{2}I_f\omega_f^2 = (1/2)I_{disk}\omega_f^2 = (1/2)675 \times 4.2^2 = 5953.5$ J.
Thus, the kinetic energy (= total energy) is increased by $\Delta E = 5953.5 - 4252.5 = 1701$ J.

Q6. Discussion ends here, but how does the boy do this much of work? [Hint: when the boy moves inward on the rotating disk, is he faster or slower than the tangential speed of the disk at his new position?]

D10-5 falling cat

Q0. What do you think is the key point of this Discussion?

The moment of inertia I can be changed without changing the mass; if the mass is displaced farther away from the rotational center, I can be made bigger. Actually, it can be made as large as you wish.

[after you learn angular momentum] Even if the angular momentum is zero, the angular displacement can be made nonzero.

Q1. It is said that an average cat can do this, even when she is dropped from $h = 30$ cm height. What is the time t allowed for the cat to rotate to land correctly?

From $(1/2)gt^2 = h$, $t = \sqrt{2h/g} = \sqrt{0.6/9.8} = 0.24$ s.
You might say perhaps we must subtract the length of the legs; if $30 \rightarrow 15$ cm, $t = 0.17$ s, which is not much different from the original estimate.