Physics 101 Discussion Week 11 Explanation (2011)[1]

D11-1 Hydraulic lift

Q0. What is the relevant principle? Also you must know the definition of pressure.

Pascal's principle:
(1) The pressure of a (connected) confined fluid is everywhere the same if there is no gravity.
(2) Under gravity, the pressure of a (connected) fluid at the same depth is everywhere the same.

These imply that if the pressure of a confined fluid at a point is found to be increased by ΔP, then everywhere in the fluid the pressure is increased by the same amount ΔP.

Pressure = normal force/area.
The area A of a disk of diameter D is given by $A = \pi(D/2)^2 = (\pi/4)D^2$.

1: F

Q1. It is explicitly said that the heights of the surfaces of the liquid are the same. What can you say about the relation between the pressures at the surfaces?

Pascal says, "The same!"

Q2. Calculate the pressures (P_i and P_o) in terms of forces and areas of the input and output pistons. Then, apply Pascal's principle.

Pressure P_i at the input piston surface: $P_i = F/\pi(D_1/2)^2 + P_A = 4F/\pi D_1^2 + P_A$,
where P_A is the atmospheric pressure.
Pressure P_o at the output piston surface: $P_o = 4 \times 22,300/\pi D_2^2 + P_A$.
Following Pascal, we get

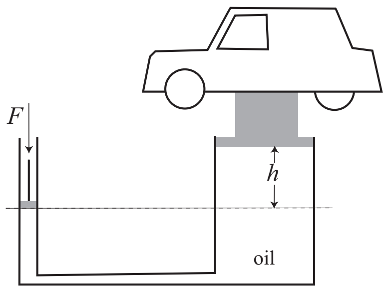
$$P_i = P_o \;\Rightarrow\; \frac{F}{D_1^2} = \frac{22,300}{D_2^2} \;\Rightarrow\; F = 22,300\left(\frac{D_1}{D_2}\right)^2 = 22,300\left(\frac{0.008}{0.140}\right)^2 = 72.8 \text{ N}.$$

2: 1.2 m higher

Q3. How can you apply Pascal's principle, when the piston surfaces are at different heights?

[1] Thanks to Lulu Li for checking this version.

The situation is depicted in the following figure:



Pascal's principle under the existence of gravity says that the pressures at the same heights (in the same fluid) are identical. Therefore, the pressure P_0 along the dotted line in the above figure is constant.

Q4. If you assume you know F, it is easy to obtain P_0. The pressure P_h just below the plunger is also easy to obtain: we have already calculated it in **Q2**: $P_h = 22,300/\pi(D_2/2)^2$. Remember the pressure acting on the area of the piston must support the weight of the car. What is the relation between P_0 and P_h? The needed formula is in the formula sheet.

$$P_0 = P_h + \rho g h,$$

where ρ is the density of the oil.

Now, you can compute P_0, so you can get F. Finish the question.

$$F = (P_h + \rho g h)\pi \left(\frac{D_1}{2}\right)^2 = F_0 + \rho g h \pi \left(\frac{D_1}{2}\right)^2,$$

where F_0 is the answer to **1**. $F - F_0 = 800 \times 9.8 \times 1.2\pi(4 \times 10^{-3})^2 = 0.473$ N. Consequently, $F = 73.3$ N.[2]

D11-2 U-tube

Q0. What is the relevant principle?

Again, Pascal's principle. Look at the answer to **Q0** of the preceding Discussion Problem.

However, you must not overlook 'a fluid' in the statement (2). (2) holds only if the points are at the same height of the same substance.

[2]Note for TA: In this case you could calculate the mass of the oil above the 'dotted line' in the figure to obtain the answer, but this method should be avoided, because only the height difference matters and the shape of the cylinder does not matter.

Q1. What is the most convenient points (or the height) to apply Pascal's Principle and what is your conclusion?

The same height, and in the same substance. Therefore for the points in the right and the left tubes below the dotted horizontal line at the same height must be the same (the same height AND in the same liquid 1).

Q2. The pressure differences between the liquid surfaces and the dotted horizontal lines must be the same in both the columns according to the answer to **Q1**. Calculate the pressure differences in the right and the left columns.

Left: $\Delta P_L = \rho_1 gh$,
Right: $\Delta P_R = \rho_2 g(h + H)$.

Now, you must be able to finish this problem.

The pressures at the dotted horizontal line are the same, and the pressure at the surfaces of the liquids must be the same (atmospheric pressure; because the density of air is much smaller the liquid densities, we may ignore it). Therefore,

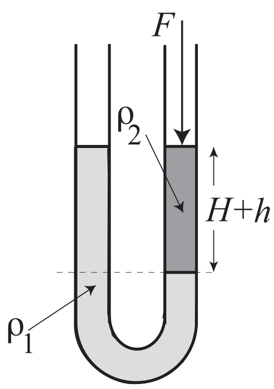
$$\Delta P_L = \Delta P_R \;\; \Rightarrow \rho_1 h = \rho_2 (h + H).$$

That is,

$$\frac{\rho_1}{\rho_2} = 1 + H/h.$$

2: F

Q3. Sketch the situation.



Q4. What is the pressures P_L in the left column and P_R in the right column at the horizontal broken line in the above figure? (You may ignore the atmospheric pressure, but if you do not wish to do so, call it P_A.)

$P_L = \rho_1 g(h+H)+P_A$ and $P_R = \rho_2(h+H)+F/A+P_A$, where A the cross section of the tube.

Now, you must be able to finish the problem.

Therefore, $P_L = P_R$ implies

$$F = A(\rho_1 - \rho_2)g(h + H).$$

D11-3 Floating raft

Q0. What is the key principle? Then, state its content. What other concepts do you need?

Archimedes' principle: the force exerted on the object immersed in a fluid by the fluid is exactly the force to support the displaced fluid by the object.

We must know what density means.

Q1. What is the minimum buoyancy force needed to support 7 persons?

$7 \times 80g = 560g$ N. Do not compute this number. DO NOT do numerical work until the last moment.

Q2. What is the buoyancy force acting on a log totally immersed in water?

To use Archimedes' principle, we need the volume of the displaced fluid = the volume of the log: $\pi r^2 l = \pi \times 0.2^2 \times 5 = 0.6283$ m^3. The mass of the displaced water is the volume × density = 628 kg. The buoyancy force must be able to support this mass, so $628g$ N per log.

Q3. Do not forget that each log has its own mass. What is the mass of a log?

The mass of the log is its volume × the density of the log $= 0.6283 \times 725 = 455.53$ kg.

Q4. How much extra mass can a log immersed in water support without sinking?

The buoyancy force − gravitational force $= 628g - 455g = 173g$ must be the excess upward force. Thus, each log can support up to 173 kg.

Finish the problem.

The mass that must be supported is 560 kg. $560/173 = 3.23$. That is, 3 logs are not enough, but 4 logs suffice.

D11-4 Buoyancy

In this case, you must clearly know the meaning of the reading of the spring balance/scale: the reading '25 kg' means that the tray is pushed down with a force of $25g$ N.

Q0. This is obviously a problem related to buoyancy. What is the key principle?

Archimedes' principle.
An object immersed in water feels less heavy than in the air, because water partially supports the object. The magnitude of this supporting force is exactly the force that can support the same volume of water as that of the object.

1: upper scale

Q1. Why has the reading of the scale changed to 25.4 kg from 25 kg? What causes the difference 0.4 kg?

Because the container pushes the tray with a force of $25.4g$ N. The increase $0.4g$ N must come from the ball. The ball is pushing down the water. Then, the action-reaction principle tells us that the water in the container must be pushing the ball upward. This force must be the buoyancy force.

Now, it should be easy to answer the question.

The ball is pushed up by the buoyancy force of $0.4g$ N, so the $Mg - 0.4g = (M - 0.4)g$ N must be the downward force acting on the ball. However, the ball is stationary, so this downward force must be counterbalanced by the upward force from the spring scale above. The action-reaction principle implies that this same force pulls the spring in the above scale down, causing its reading to be $(M - 0.4)$ (kg). Since the reading in A must be M, the reading must be reduced by 0.4 kg.

Something must oppose the gravitational force to prevent the ball from falling. If someone reduces the supporting force, then someone else must support more. The total force needed to support the same mass is always the same. In A the upper spring scale supported Mg. In B the water (supported by the lower spring scale) helps a bit ($0.4g$), so the upper scale needs only $Mg - 0.4g$ force.

2: volume

Q2. Why does the water surface rise?

Because the ball pushes water aside.

Q3. You can tell the volume of displaced water with the aid of Archimedes' principle. What is it?

The buoyancy on the ball is $0.4g$ N. Thus, 0.4 kg = 400 g of water must have been pushed aside by the ball. This is 400 cm^3, because the water density is 1 g/cm^3. The cross section of the container is $A = 0.01$ $m^2 = 100$ cm^2, so the height increase must be $400/100 = 4$ cm.

D11-5 Shower head

Q1. What is the (definition of the) volume flow rate?

The volume of fluid flowing through a cross section per unit time.

1: volume flow rate

The volume flow rate through the pipe is cross-sectional area × speed of fluid (in the numerical calculation the unit of length should be unified; e.g., use m everywhere):

$$\pi(0.65 \times 10^{-2})^2 \times 1.2 = 1.327 \times 10^{-4} \times 1.2 = 1.59 \times 10^{-4} \text{ m}^3/\text{s}.$$

2: shower hole

Q2. What principle do you need?

Continuity equation: this is just conservation of matter. A substance (without chemical reaction) cannot disappear or cannot be created from nothing. Therefore, if the flow is steady, what comes in must go out at the same rate (the same mass rate).

In our case, the substance is water and flow is not too fast, so we may assume that its density does not change (incompressible), so mass flow rate $\propto$ volume flow rate. That is, the volume flow rate must be constant everywhere.

Applying the volume conservation, finish the problem.

The total cross section of the shower holes is $A = 12 \times \pi[0.046 \times 10^{-2}]^2 = 7.98 \times 10^{-6}\text{m}^2$. The continuity equation implies

$$1.59 \times 10^{-4} = Av = 7.98 \times 10^{-6} v.$$

That is, $v = 1.59 \times 10^{-4}/7.98 \times 10^{-6} = 19.9$ m/s.

D11-6 Venturi meter

Q0. What is the fundamental equation we need? You must be able to explain each term in plain English.

The key is:
Mechanical energy and mass are conserved along a steady flow.
In more concrete forms:

$\ast$ Bernoulli's equation applicable to non-viscous fluid (i.e., fluid without mechanical energy dissipation):

$$P_1 + \frac{1}{2}\rho_1 v_1^2 + \rho_1 h_1 g = P_2 + \frac{1}{2}\rho_2 v_2^2 + \rho_2 h_2 g.$$

That is, along a steady flow:

$$P + \frac{1}{2}\rho v^2 + \rho h g = \text{constant}.$$

This is essentially conservation of total mechanical energy = energy stored in a compressed unit volume $+K + U$, where K is the kinetic energy, and U the potential energy.

$\ast$ Continuity equation for incompressible fluid: along a steady flow Av = constant.

1: v_2/v_1

This is a mere application of the continuity equation: $A_1v_1 = A_2v_2$. That is, $v_2/v_1 = A_1/A_2 = (r_1/r_2)^2 = (150/125)^2 = 1.44$.

2: v_1

Q1. We need one more relation. We have used conservation of mass already. The remaining conservation law must be about energy. Find the second relation and determine v_1.

We use Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho_1 v_1^2 = P_2 + \frac{1}{2}\rho_2 v_2^2,$$

or, using the ratio $v_2/v_1 = 1.44$ obtained above,

$$\Delta P = P_1 - P_2 = \rho(v_2^2 - v_1^2)/2, \;\Rightarrow\; 240 = 1.3(1.44^2 - 1)v_1^2 = 1.4v_1^2$$

That is, $v_1 = 13.1$ m/s.

3: mass flow rate

The total mass flow rate is: $A_1\rho v_1 = \pi 0.15^2 \times 1.3 \times 13.1 = 1.20$ kg/s.