Physics 101 Discussion Week 13 Explanation (2011)[1]

D13-1 Electromagnetic Waves

Q0. What is the fundamental relation among the frequency f, wavelength λ, and the speed v of the wave?

$f\lambda = v$.

Q1. **1-3** are only trivial applications of the basic formula you just answered, so simply finish them.

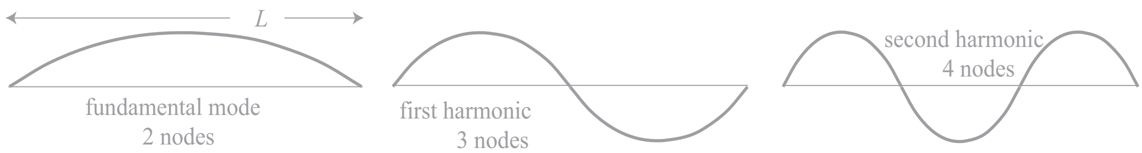
1. $f = 2.4 \times 10^9$. Therefore, $\lambda = 3 \times 10^8/(2.4 \times 10^9) = 0.125$ m, or 12.5 cm. (microwave ovens us 12.24 cm wave).
2. $f_{FM}/f_{phone} = \lambda_{phone}/\lambda_{FM} = 0.125/2.967 = 0.0421$ (i.e., 101.1 MHz).
3. $f = c/\lambda = 3 \times 10^8/700 \times 10^{-9} = 4.29 \times 10^{14}$ Hz $= 430$ THz.

D13-2 Cello

1. fundamental mode

Q0. What is the fundamental mode of a chord of length L? Sketch it, and the second and the third longest wavelength modes as well.

The mode without any internal node is the fundamental mode.



Both the ends are fixed, so the ends are always the nodes. How many 'half waves' we can neatly place on the chord characterizes the standing wave modes.

Q1. For the fundamental mode, what is the relation between the chord length L and the wavelength λ?

[1] Checked by Lulu Li

Obviously from the figure, $\lambda/2 = L$, or $\lambda = 2L$.

What is, then, L for the cello string?

Of course, $L = 0.7$ m.

2. F

Q2. What property of the wave is influenced by the tension F in the chord?

The wave speed v is dependent on F.

Q3. What is the relevant formula on the formula sheet?

$$v^2 = F/(m/L).$$

You must be able to tell the meaning of the symbols in this formula.

Q4. Formula Sheet tells us that the wave speed is given by $v = \sqrt{F/(m/L)}$. Since m/L is given, we need v to find F. What is v?

The frequency is given, so in order to use $v = f\lambda$, we need λ, but we have already computed it in **(1)**. $f\lambda = v = 330 \times 1.4 = 462$ m/s.

Q5. Find the string tension F in terms of v and m/L. Then, compute it numerically.

$F = (m/L)v^2$. $m/L = 8 \times 10^{-6}$ kg/m, so $F = (m/L)v^2 = 8 \times 10^{-6} \times 462^2 = 1.71$ N.

3. m

Q6. We know the relation among F, mass density m/L, and the wave speed $v = f\lambda$. Solve the relation for f (Write f in terms of m, L, λ and F).

Since $v = \sqrt{F/(m/L)} = f\lambda$,

$$f = \frac{\sqrt{FL}}{\lambda\sqrt{m}}. \tag{1}$$

Q7. What are you allowed to change in the above formula?

Now, F and L are required to be kept constant. The wave must be the lowest frequency, so $\lambda = 2L$ cannot be changed. Thus, we can change only m.

Q8. Under the required condition can you see $f \propto 1/\sqrt{m}$?

This should be clear from eq.(1), since $\lambda = 2L$, and F and L are kept constant.

Q9. What is the required mass m?

The fundamental frequency f is to be halved, so the mass density must be changed as $m/L \to 4m/L$. That is, the new mass density is 3.2×10^{-5} kg/m. Therefore, the total mass m is $3.2 \times 10^{-5} \times 0.7 = 0.021$ g.

D13-3 Piano

1. m/L

Q0. Since the tension F and the mass density m/L are discussed in the problem, the question must be related to the wave speed v. The wavelength $\lambda = \lambda_1$ is given, so to find the wave speed, we need the frequency f, since $v = f\lambda$. What is the relation between the period T and the frequency f?

$f = 1/T$

Q1. We are asked to calculate m/L. What is the relation between m/L and wave speed v?

$v = \sqrt{F/(m/L)}$.

Q2. The wavelength λ_1 is given. Find v in terms of T and λ_1.

Formula sheet tells us $v = \lambda_1 f$, so $v = \lambda_1/T$.

Q3. Find m/L in terms of T, λ_1 and F.

m/L and F, v are related as in the answer to **Q1**, so we can solve as

$$m/L = F/v^2.$$

Introducing the formula for v obtained in **Q2** to the above formula, we have

$$m/L = FT^2/\lambda_1^2 = 944/330^2 = 8.67 \times 10^{-3} \text{ kg/m}.$$

$(v = \lambda_1/T = 1.26/3.82 \times 10^{-3} = 330 \text{ m/s})$.

2. $F \to F/9$

Q4. Write down the equation for the wavelength λ in terms of F, T and m/L.

$$\lambda = \sqrt{F/(m/L)}/f = T\sqrt{F/(m/L)} \tag{2}$$

Q5. Can you see $\lambda \propto \sqrt{F}$, if the string is the same and T is the same?

In eq.(2) T and m/L are constant, so $\lambda \propto \sqrt{F}$.

Q6. Obtain λ_b.

Since $F \to F/9$, $\lambda \to \lambda/3$. Therefore, $\lambda_b = 1.26/3 = 0.42$ m. This is equal to $\lambda_1/3$.

3. harmonics

Since $\lambda_b = \lambda_1/3$, obviously, $n = 3$. The rightmost figure in **D13-1** is its sketch.

D13-4 Tornado warning

Q0. Since you see 'dB (decibel)', this is about loudness β of sound. Write down the definition of the loudness. Why is the loudness is related to the logarithm of the intensity I of the sound?

The loudness β of sound is defined by

$$\beta = 10 \log_{10} \frac{I}{I_0} \text{ (dB)}.$$

When the intensity changes from I to $I + \Delta I$, we notice it only when $\Delta I/I$ is sufficiently large; that is, only the ratio matters. Thus, we feel the loudness increase due to $I \to 2I$ and that due to $2I \to 4I$ are identical; we feel these changes as due to the same increments of loudness. Therefore, taking the logarithm of intensity is a convenient way to define the loudness that matches our sensation.

More formally, ...
Loudness defined as above is a measure of perceived intensity of the sound. It uses a logarithmic scale, reflecting that the general physiological reaction (or perceived intensity) R and the physical intensity of the stimulus S are related as (essentially, it is the so-called Weber-Fechner law[2])

$$\Delta R \propto \Delta S/S.$$

This is a reasonable relation, because, as already mentioned, we can detect the change ΔS in physical intensity only relative to the absolute intensity S. This relation implies $R \propto \log_{10} S$, because $\log_{10}(1+x) \propto x$ for small x and $\log_{10}(S + \Delta S) - \log_{10} S = \log_{10}(1 + \Delta S/S) \propto \Delta S/S$.

Q1. What is the intensity I of a sound (wave)?

The intensity I of sound is the energy carried by the sound wave per unit area per unit time. Therefore, you may say I is the power carried by the sound wave through unit area.

Thus, the unit of I is $\mathrm{J/m^2{\cdot}s} = \mathrm{W/m^2}$.

It is additive. That is, if you hear two sounds of intensity I_1 and I_2 simultaneously, the total intensity of the sound you hear is given by $I_1 + I_2$.

Those who need a review of logarithms, try LQ1-LQ7.

LQ1. Suppose $y = 10^x$. How do you get x from y $(\neq 0)$?

$x = \log_{10} y$. This is the definition of $\log_{10}$. That is,

$$10^{\log_{10} x} = x, \quad \log_{10} 10^x = x.$$

[2] According to Wikipedia Ernst Heinrich Weber (1795-1878) found this empirically and Gustav Theodor Fechner (1801-1887) later offered a theoretical interpretation of Weber's findings, which he called simply Weber's law.

LQ2. Give $\log_{10} 10$ and $\log_{10} 1$ (no calculator, please).

$\log_{10} 10 = 1$ and $\log_{10} 1 = 0$.

Why is $10^0 = 1$ reasonable?

LQ3 Find $\log_{10} 2$, $\log_{10} 3$ (with a calculator).

$\log_{10} 2 = 0.30103\cdots$, and $\log_{10} 3 = 0.4771\cdots$.

This implies that $10^{0.30103\cdots} = 2$, and $10^{0.4771\cdots} = 3$.

LQ4. Suppose $x_1 = \log_{10} y_1$ and $x_2 = \log_{10} y_2$. What is $\log_{10}(y_1 \times y_2)$?

$$\log_{10}(y_1 \times y_2) = \log_{10} y_1 + \log_{10} y_2 = x_1 + x_2.$$

Demonstration:
We know $y_1 = 10^{x_1}$ and $y_2 = 10^{x_2}$, so we have

$$y_1 y_2 = 10^{x_1} \times 10^{x_2} = 10^{x_1+x_2}.$$

This implies, by definition (remember $y = 10^x \iff x = \log_{10} y$), $\log_{10}(y_1 y_2) = x_1 + x_2$.

LQ5. Suppose $y \neq 0$. What is $\log_{10}(1/y)$? What is $\log_{10} y^4$?

$\log_{10}(1/y) = -\log_{10} y$, because $y = 10^x$ implies $1/y = 10^{-x}$. $\log_{10} y^4 = 4 \log_{10} y$. Generally, $\log_{10} x^a = a \log_{10} x$ for any a. If we set $a = -1$, we get $\log_{10}(1/x)$.

LQ6. Suppose $x_1 = \log_{10} y_1$ and $x_2 = \log_{10} y_2$. What is $\log_{10}(y_1/y_2)$?

We know $y_1/y_2 = y_1 \times (1/y_2)$, so

$$\log_{10}(y_1/y_2) = \log_{10} y_1 + \log_{10}(1/y_2) = \log_{10} y_1 - \log_{10} y_2 = x_1 - x_2.$$

LQ7. To close this review, calculate $\log_{10} 4$, $\log_{10} 5$, $\log_{10} 6$, $\log_{10} 7$, $\log_{10} 8$ and $\log_{10} 9$ without using a calculator, but assuming $\log_{10} = 0.3010$ and $\log_{10} 3 = 0.4771$. [You may use the fact that $49 = 7^2 \simeq 50$.]

$$\log_{10} 4 = \log_{10} 2^2 = 2 \times \log_{10} 2 = 0.6020$$
$$\log_{10} 5 = \log_{10}(10/2) = \log_{10} 10 - \log_{10} 2 = 1 - 0.3010 = 0.6990$$
$$\log_{10} 6 = \log_{10}(2 \times 3) = \log_{10} 2 + \log_{10} 3 = 0.3010 + 0.4771 = 0.7781$$
$$\log_{10} 7 = \log_{10} \sqrt{49} = (1/2) \log_{10} 49 \simeq (1/2) \log_{10} 50 = (1/2)(\log_{10} 5 + 1) = 1.6990/2 = 0.8595.$$
[How accurate is this?]
$$\log_{10} 8 = \log_{10} 2^3 = 3 \times \log_{10} 2 = 0.9030$$
$$\log_{10} 9 = \log_{10} 3^2 = 2 \times \log_{10} 3 = 2 \times 0.4771 = 0.9542.$$

Q2. Let I_3 be the sound intensity of the siren at 3m from the source. What is its loudness β_3 (= 120 dB) at 3 m in terms of I_3?

By definition
$$\beta_3 = 10 \log_{10}(I_3/I_0). \quad (3)$$

1. at 12m

Q3. What is the intensity I_D of the sound, if we know I_1 (the intensity at $D = 1$ m)?

$I_D = I_1/D^2$.
Notice that $I_D \times 4\pi D^2$ (intensity $\times$ the area of the sphere of radius D = the total energy going out from the sphere of radius D) is constant (conservation of energy).

Imagine a pulse originating from a central source. If the pulse expands uniformly to a sphere one meter in radius, it's energy must be divided uniformly across the the surface area of that one meter sphere. After it has expanded to a sphere five meters in radius, it's energy will have been divided down even further, but it's total energy must go somewhere, the energy per unit area times the area of the sphere at any stage in expansion must be constant.

Q4. What is the intensity I_{12} of the sound of the siren at a distance $D = 12$ m in terms of the intensity I_3 of the sound of the siren at a distance $D = 3$ m?

We know $I_{12} = I_1/12^2$ and $I_3 = I_1/3^2$, so we have $I_{12}/I_3 = (3/12)^2$ (= 1/16). That is, $I_{12} = I_3/16$.

Q5. Using the definition of loudness, calculate $\beta_{12} - \beta_3$.

$$\beta_{12} - \beta_3 = 10 \log_{10}(I_{12}/I_0) - 10 \log_{10}(I_3/I_0) = 10 \log_{10} \frac{(I_{12}/I_0)}{(I_3/I_0)} = 10 \log_{10}(I_{12}/I_3).$$

Q6. Now, you should be able to calculate β_{12}, since you know the ratio I_{12}/I_3.

$$\beta_{12} = \beta_3 + 10\log_{10}(1/16) = 120 - 10\log_{10} 16 = 120 - 40 \times 0.3010 = 108 \text{ dB}.$$

2. two sirens

Q7. Suppose you hear two sounds from independent sources. Their intensities at your location are I_A and I_B, respectively. Then, what is the intensity of the total sound actually you hear when both sources are on?

$I_A + I_B$.

Q8. In this problem you have two sources with an identical intensity I_3, so the intensity you hear is $2I_3$. Calculate the loudness β.

Simply, we follow the definition: $\beta = 10\log_{10}(2I_3/I_0) = \beta_3 + 10\log_{10} 2 = 123$ dB. (Or you can proceed as follows. The intensity is doubled, so $\beta_{new} - \beta_{old} = 10\log_{10} 2 = 3.01$.)

Q9 [Extra question] Suppose the identical sirens are placed at 3 m and at 12 m from you, and are on. What is the loudness of the sound due to these sirens you actually hear?

$\beta = 10\log_{10}[(I_3 + I_{12})/I_0] = 10\log_{10}[(I_3/I_0) \times (1 + 1/16)] = \beta_3 + 10\log_{10}(17/16) = 120 + 0.26 = 120.3$ dB. Is this number intuitively acceptable?

D13-5 Fire truck

Q0. This is obviously a Doppler effect problem. What is it qualitatively and quantitatively?

Qualitatively:
Roughly speaking, if the distance between you and the sound source is shrinking, you hear a higher pitch sound; otherwise, you hear a lower pitch sound.

Quantitatively:
The relevant formula is found on the formula sheet:

$$f_o = \frac{v_w - v_0}{v_w - v_s} f_s.$$

Here, v_w is the speed of the wave (the sound wave in air in our case).

Clearly remember the sign convention.

Check your result against your daily experiences (ambulance siren, etc). The signs of v_o and v_s are $+$ (resp., $-$), if the direction of the motion is the same as (resp., opposite to) the propagation direction of the wave being observed.

For example, if the source is running toward you, the observer, the sound wave and the source are both running toward you, so $v_s > 0$. If you are running toward the sound source, you, the observer, are running against the wave, so $v_o < 0$, etc.

Q1. Since the observer is not moving, the above formula simplifies. Write down the simplified version we can use for this problem.

Since the observer is not moving, $v_o = 0$, so

$$f_o = \frac{v_w}{v_w - v_s} f_s.$$

Here, $v_w = 330$ m/s is the sound speed.

We do not know f_s nor v_s.

Q2 We need two pieces of information; they are actually given. Express the two conditions in formulas, writing the speed (absolute value) of the truck as $|v_s|$.

When the truck is running toward you, $f_o = 460$, and $v_s = +|v_s|$, because both the truck and the sound are traveling toward you (are running in the same direction),

$$460 = f_s \frac{330}{330 - |v_s|}.$$

When the truck is receding from you, $f_o = 410$ and $v_s = -|v_s|$, because the sound is coming toward you, but the truck is going away from you (the truck is running against the sound wave you observe). Therefore,

$$410 = f_s \frac{330}{330 + |v_s|}.$$

Q3. Solve the simultaneous equation obtained in Q2, and finish the problem.

Solving for f_s, we get

$$460(330 - |v_s|) = 410(330 + |v_s|)$$

or

$$330(460 - 410) = (460 + 410)|v_s|$$

This gives $|v_s| = 19.0$ m/s. That is, it requires 264 seconds (4.4 min) to cover 5 km.