Physics 101 Discussion Week 14 Explanation (2011)

D14-1 Colliding Rods

Q0. Obviously, this is about (linear) thermal expansion. What is the basic relation between the length increase ΔL of a rod of material with the linear thermal expansion coefficient α, when temperature is increased by ΔT?

$$\Delta L = \alpha \Delta T L.$$

1. gap

Q1. In this problem the gap is narrowed from both sides. What is the total increase ΔL_T of the length of both the materials (this is of course identical to the amount that the gap is narrowed), if temperature increase is ΔT (K)?

The total expansion ΔL_T is

$$\Delta L_T = \Delta L_{brass} + \Delta L_{Al} = (2\alpha_{brass} + \alpha_{Al})\Delta T.$$

Q2. What is the required ΔT to close the gap of size 1.3×10^{-3} m?

When ΔL_T is equal to the gap size, the gap is closed. Therefore, we need $\Delta L_T = 1.3 \times 10^{-3}$ m. This implies $\Delta T = 1.3 \times 10^{-3}/(2 \times 19 \times 10^{-6} + 23 \times 10^{-6}) = 1.3 \times 10^{-3}/61 \times 10^{-6} = 21.31$ (in K).

Q3. It is said that the initial temperature is 80°F. We must change the unit to °C. Perform this unit conversion and finish the problem.

80 °F$= T_c$ °C $= (5/9)(80 - 32) = 26.67$ °C. The final temperature must be $26.7 + 21.3 = 48.0$ °C.

2. longer rail

Q4. What is the relation between ΔL and L?

> They are proportional. Therefore, the answer to this question is obvious: Longer.

D14-2 Expanding block

Q0. What is the relation between the volume change due to thermal expansion of a hollow box and that of a solid block of the same shape (that is, you cannot see any difference in their appearances) made from the same material ?

> They are exactly the same, because the very thin surface of the block must accommodate the thermally expanded block without any wrinkle nor tear.

Q1. Let us work out the general relation between the linear thermal expansion coefficient α and the (volume) thermal expansion coefficient β: $\beta = 3\alpha$, where 3 is the spatial dimensionality.

> Consider a block of size $L_x \times L_y \times L_z$. The volume is initially $V(T) = L_x L_y L_z$. If the temperature is raised by ΔT, the linear dimension, say, in the x-direction becomes $L_x + \Delta L = L_x(1+\alpha\Delta T)$. Therefore, the volume at temperature $T+\Delta T$ is
>
> $$V(T+\Delta T) = L_x L_y L_z (1+\alpha\Delta T)^3 = V(T)(1+3\alpha\Delta T) + \text{ very small terms.}[1]$$
>
> Therefore, by definition, $\beta = 3\alpha$.

1. ΔV

Q2. Using this result, get ΔV.

> $T = 30$ °C and $\Delta T = 70$ °C. Therefore, $\Delta V = V(T) \times 3\alpha\Delta T = 1.5 \times 0.9 \times 0.7 \times (3 \times 16 \times 10^{-6} \times 70) = 0.945 \times 3.36 \times 10^{-3} = 3.175 \times 10^{-3}$ m^3.

2. slot

[1] As you know, $(1+x)^3 = 1+3x+3x^2+x^3$. Consider, for example, $x = 1/100$. $x^2 = 10^{-4}$, and $x^3 = 10^{-6}$. Thus, we may ignore higher order terms: $(1+x)^3 \simeq 1+3x$, and $1.01^3 \simeq 1.03$.

The similarity is maintained, because for this material thermal expansion is uniform and isotropic. No change.

D14-3 Neon tank

Q0. This Discussion concerns the ideal gas law and the concept of the atomic (molecular) mass.
(i) What is the significance of the ideal gas law?
(i') What is the ideal gas?
(ii) What does the atomic or molecular mass M_A mean?

(i) The ideal gas law implies $PV = Nk_BT = nRT$, where
k_B is the Boltzmann constant and R is the gas constant: $k_B = R/N_A$;
P is the pressure,
V is the volume,
$N = nN_A$ is the number of particles in the volume V (n is the amount of particles in moles), and
T is the absolute temperature.

(i') The 'ideal gas' implies a collection of non-interacting atoms (or molecules). As a mechanical system, it has only the kinetic energy, whose magnitude is indicated by T. The ideal gas law holds for every system consisting of N non-interacting particles.[2]

Thus, if we know P, V, and T of an ideal gas, we can count the number of particles in the volume.

(ii) The weight of $N_A = 6.02 \times 10^{23}$ (Avogadro's constant[3]) atoms (or molecules) in grams is called the atomic (molecular) mass of the pure substance.

If a substance's atomic or molecular mass is M_A, it implies that M_A g (NOT kg) of the substance contains exactly $N_A = 6.02 \times 10^{23}$ atoms or molecules, or 1 mole of the substance has a mass M_A (g).

1. n

In this case P, V and T are given, so we can immediately obtain n.

Since $R = 8.31$ J/mol·K, $n = PV/RT = 5 \times 10^5 \times 2/(8.31 \times 220) = 547$ moles.

[2] to TA: of course, we need a sufficiently high temperature.
[3] It is the number of atoms contained in 12 g of ^{12}C.

It is worth remembering that 1 mole of a gas occupies roughly 20 ℓ (liters) under 1 atm at around the room temperature (roughly 300 K). 1 atm $\simeq$ 100 kPa, so 1 mole roughly corresponds to 4 ℓ at 500 kPa. $2000/4 = 500$ moles is a reasonable answer. (1 $\mathrm{m}^3 = 1000$ ℓ.)

2. M

Q1. What is the mass of 1 mole of Ne?

Since its atomic mass is 20.2, the mass of 1 mole Ne = 20.2 g.

So, the answer to the question is $\cdots$

$20.2 \times 547 = 11,049$ g (NOT in kg). That is, about 11 kg.

3. K

Q2. What is the relation between the average kinetic energy $\langle K \rangle$ of (the translational motion of) a molecule and the gas temperature T?

$$\langle K \rangle = \frac{3}{2} k_B T.$$

We cannot exactly derive the above relation, but it is not hard to see $T \propto \langle v^2 \rangle \propto \langle K \rangle$.
(i) P is due to the impulse given by a particle bouncing back from the container wall.
(ii) A particle with speed v contributes the impulse proportional to v.
(iii) P also depends on how many particles bounce back from the wall per unit time, but
(iv) the number of particles hitting the wall per unit time is also proportional to v, so
(v) the contribution of particles with speed v to P must be proportional to $v^2 \propto K$.
Thus, $P \propto \langle K \rangle$.

So, you can calculate $\langle K \rangle$ easily as $\cdots$

$\langle K \rangle = (3/2) k_B T = (3/2) \times 1.38 \times 10^{-23} \times 220 = 4.554 \times 10^{-21}$ J.

4. average speed

Q3. What is the relation between the average kinetic energy $\langle K \rangle$ and the root-mean-square velocity v_{rms} of a molecule? What do you need to know?

$$\langle K \rangle = \frac{3}{2} k_B T = \frac{1}{2} m v_{rms}^2,$$

where m is the mass of a single Ne atom.

Q4. Thus, we need the mass m of a single atom/molecule. Assuming that you know m, get the formula for v_{ave} as a function of T.

$$v_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{2\langle K\rangle}{m}}.$$

Q5. What is the mass m of a single neon atom?

Since a collection of N_A atoms has mass 20.2 g, $m = 20.2 \times 10^{-3}/6.02 \times 10^{23}$ kg (notice that we have converted the unit from g to kg); the mass m of a neon atom is $0.020/6.022 \times 10^{23} = 3.32 \times 10^{-26}$ kg.

Q6. Use the formula obtained in Q4 and finish the question **4**.

$$v_{rms} = \sqrt{2 \times 4.554 \times 10^{-21}/3.32 \times 10^{-26}} \simeq 520\text{m/s}.$$

Q7 [extra]. Why is this speed rather close to the sound speed in the neon gas (at least you can use it to estimate the order of the sound speed in Ne)?

For a sound wave to propagate, energy must be transported actually, but it is carried by molecules (because the space between molecules is much larger than their diameter), so the sound propagation speed is not very different from the actual running speed of molecules.

5. heat

Q8. Suppose you wish to compute the energy needed to raise the temperature of a certain quantity of a substance by ΔT, what do you need?

You need the heat capacity C of the substance of the required quantity.

Q9. To obtain the heat capacity C of the substance of the given quantity, what materials constant (a quantity specific to each substance) do you need?

The specific heat (under a desired condition, say, under constant volume).

Q10. What is the specific heat of a monatomic gas under constant volume?

The constant volume specific heat is $3R/2$ per mole (as can be seen from the formula for $KE_{ave} = (3/2)RT$).

Q11. What is the heat capacity of the neon gas in the tank?

We have 547 moles, so the heat capacity of the gas is $547 \times (3/2) \times 8.31 = 6818$ J/K.

Q12. What is the question you have been considering? Give the answer.

You are asked to calculate the needed thermal energy to raise the temperature: $Q = C\Delta T = 6818 \times (660 - 220) = 3.0 \times 10^6$ J.

6. P_2

Q13. Write down the equation connecting P_2 and T_2.

$$P_2 = nRT_2/V.$$

Never calculate P_2 directly. Be (creatively) lazy.

Q14. Find P_2/P_1 in terms of the ratio of temperatures, and then finish the problem.

Obviously, $P_2/P_1 = T_2/T_1$, because the volume and the quantity of the neon gas do not change. Therefore, $P_2 = (T_2/T_1)P_1 = (660/220)5 \times 10^5 = 1.5 \times 10^6$ Pa (1.5 MPa).

7. energy ratio

Q15. What is the relation between $\langle K \rangle$ and T?

As we already discussed (derived) $\langle K \rangle \propto T$, which is worth remembering.

Q16. What is the ratio in terms of temperatures? Then, finish the solution.

$\langle K \rangle_2 / \langle K \rangle_1 = T_2/T_1$, so the ratio is 3.

8. speed ratio

Q17. What is the relation between the root-mean-square velocity v_{rms} and T?

Since $v_{rms} \propto \sqrt{\langle K \rangle}$, $v_{rms} \propto \sqrt{T}$.

Q18. What is the ratio in terms of temperatures? Then, finish the solution.

$v_{rms2}/v_{rms1} = \sqrt{T_2/T_1}$, so the ratio is $\sqrt{3}$.

D14-4 Heat Capacity

This is a problem to calculate the final temperature T_f (°C) reached by a compound system whose components have different temperatures initially.

Q0. In this case, thermal energy is not converted into any other form of energy, so you may think the total thermal energy is conserved. What do you need to relate the temperature changes and the required thermal energies for the changes?

Heat capacities. (You must be able to tell its definition for a given system.)

Q1. What does the conservation of thermal energy imply? Or, what is the consequence of the conservation of thermal energy?

If one part of the system loses energy, that energy must go to some other part of the system. Energy cannot vanish. The total energy of the system remains constant because it is isolated from the rest of the world.

Q2. We wish to calculate the gain of thermal energy by 18 cans of soda initially at $T = 1$ °C. What is their total heat capacity?

$18 \times 0.35 \times 3800 = 23940$ J/K.

Q3. What is the gain of thermal energy by the soda cans (notice that 'change' is always 'after' − 'before')? Pretend you know T_f (°C).

$23940(T_f - 1)$.

Q4. Following a logic parallel to the above, calculate the gain of thermal energy by the watermelon.

Its heat capacity is $6.5 \times 4186 = 27209$ J/K. Therefore, the gain of thermal energy by the watermelon is $27209(T_f - 30)$. As you will see (and expect) this is actually negative.

Q5. Now, apply the conservation of thermal energy to write down the formula for T_f, and finish the problem.

thermal energy gain of soda cans + thermal energy gain of water melon = 0.

That is,

$$23940(T_f - 1) + 27209(T_f - 30) = 0.$$

Therefore, $(23940 + 27209)T_f = 23940 \times 1 + 27290 \times 30$. You might be able to guess this relation immediately. Thus, $T_f = 16.5$ °C.

Before HE3 we will stop here.

D14-5 Conductivity

Q0. Suppose that heat flows through a long rod, and that the temperature at any point on the rod is constant with time (different points on the rod may have different temperatures). You choose a cross section at an arbitrary point along the rod, and study heat flow through it. What is the relation between the flow coming from the left and that leaving to the right (or vice versa)?

If, for example, more heat enters the point on the rod from the left than leaves that point on the rod to the right, then thermal energy will begin to build up at that point and the temperature there will have to go up. But if we know that the temperature at that point is constant, then thermal energy cannot build up at any point and the heat coming in must be equal to the heat going out.

1. T at P

Q1. This is a heat conduction problem. What is the general relation between the temperature difference ΔT and the heat flow (rate of heat transfer Q/t)?

The formula sheet tells us that $Q/t = kA\Delta T/L$, where A is the cross section and L the length of the heat conducting rod.

That is, rate of heat transfer is proportional to the slope (gradient) of the temperature.

Q2. This is a stationary heat conduction problem, so the observation we made in **Q0** must apply. To apply this, we must compute the heat flow coming from the A side and that leaving to the B side. To do this with the aid of the formula in **Q1** we need the temperature T_P °C at P, which is unknown. But our strategy is always to pretend that we know everything we need. What is the heat flow along A, assuming T_P?

$k = 120$ J/s·m·K, and $\Delta T = 100 - T_P$, so $Q/t = 120(100 - T_P)A/L$ (though A and L are given, we do not need them for (1), so let us not write their numerical values explicitly).

Q3. Do a similar thing on the B side, and then apply our conclusion at the beginning of this problem.

The flow through B is, since $k = 85$ J/s·m·K and $\Delta T = T_P$, $Q/t = 85T_P A/L$. The two obtained heat flows must be identical for T_P not to change:

$$120(100 - T_P)A/L = 85T_P A/L$$

or $120(100 - T_P) = 85T_P$. That is, $T_P = 12000/205 = 58.5$ °C.

2. melting ice

Q4. To melt ice at 0 °C, what do you need?

Latent heat.

It is required to loosen the water molecules firmly registered in a crystal lattice. This heat energy is consumed to increase the 'potential energy' of water molecules. Since the temperature is solely determined by the kinetic energy,[4] latent heat cannot change temperature.

Q5. What is the total energy required to melt 1 kg of ice to make 0 °C liquid water?

The latent heat of melting of water per 1 kg (under 1 atm at 0 °C) is on the formula sheet: $L_{f,water} = 33.5 \times 10^4$ J/kg. Thus, we need 33.5×10^4J.

Q6. Now, we need the absolute value of the heat flow Q/t along the rod. What is it?

[4] to TAs: Do not quibble. This is definitely OK for $T > 100$ K under the ordinary number density.

You can use rod A or rod B, because we know both have identical heat flow rates, but the B side is easier

$$Q/t = 85T_P A/L = 85 \times 58.5 \frac{0.05}{1.2} = 207 \text{ J/s}.$$

Find the needed time; finish the problem.

$33.5 \times 10^4/207 = 1618$ s (27 min).

3. moved P

B conducts heat poorly, so it would take longer.

D14-6 Radiation

Q0. What is the formula we need?

$e\sigma T^4 A$.
This must be the thermal energy lost per second (i.e., the power loss).

Q1. The temperature of the filament must be constant. Then, what can you conclude about the relation between the power supply and the power loss?

They must be identical.

Q3. Now, write down the power balance condition and finish the problem.

$e\sigma T^4 A$ must agree with 100 W.
Since $e = 1$, $5.67 \times 10^{-8} \times 3100^4 A = 100$ or $5.236 \times 10^6 A = 100$. That is, $A = 1.9 \times 10^{-5}\text{m}^2$.