Physics 101 Discussion Week 4 Explanation (2011)

D4-1 Pushing the shopping carts

1: force

Q1 What is the key concept/law?

Newton's second law: $\sum \boldsymbol{F} = m\boldsymbol{a}$. This is a one dimensional problem, so you can consider only the x-component. (Look at the formula sheet.)

Q2. Finish the problem.

The total mass is $10m$, so $F = 10ma$.

2: net force/per cart

Q3 This is almost the same as 1. The only difference is the mass. $\sum \boldsymbol{F}$ should be obtained immediately.

Each cart experiences the acceleration a, so ma must be the net force on each cart.

3 force on 8th cart by 7th

The fundamental law we need is Newton's second law. We must apply Newton's law beyond the 7th cart.

Q4 How many carts the 7th cart must push to maintain the acceleration a? What is their total mass?

3 carts $= 3m$.

Q5 We know the acceleration. Complete the solution.

It may help to draw the free body diagrams for the 10th, 9th, and 8th carts before you draw the diagram for the 7th. F due to the 7th cart on the 8th move three carts (the 8th, 9th and 10th carts): $F = 3ma$.

4: numerics

$a = 0.05$, $m = 30$, so numerically, the above answers read:
1: $F = 10ma = 15$ N.
2: $F = ma = 1.5$ N.
3: $F = 3ma = 4.5$ N.

D4-2 Block and Elevator

Q0. What do you think are the key points of the problem?

(i) The maximum static friction force F is given by $F = \mu_s N$, where N is the normal force.
(ii) Since the elevator may be moving with acceleration, we need Newton's second law in the vertical direction to determine N.

Let us solve the questions in a unified fashion, i.e., let us follow the instruction in the last paragraph on p46. Let us take the y-coordinate to be vertical upward (as illustrated).

Q1. What forces do you have to take into account? Draw the free body diagram in the y-direction.

Gravitational force F_G (downward) and the normal force N (upward) due to the elevator floor.

Q2. Write down Newton's second law for the block in the y direction, writing the y-component of its acceleration to be a_y and its mass M. Write the magnitude of the gravitational force as Mg.

$$Ma_y = N - Mg. \tag{1}$$

Q3. From (1) you can obtain N, so you can compute the maximum static friction force F. Find N in terms of a_y, g and M. Then, find F.

$$N = M(a_y + g).$$

In this formula you must understand the meaning of the sign of a_y. $a_y > 0$ implies that the acceleration is in the positive y-direction; according to our convention this means an upward acceleration.

From N we can find the max static friction force as

$$F = \mu_s M(a_y + g). \tag{2}$$

Q4. Suppose the acceleration in the y-direction of the block is $a_y = +2$ m/s^2. Can you tell in which direction (upward or downward) the block is moving (i.e., the direction the elevator is moving)?

No, never.
∗ If the velocity is upward, and if its speed is increasing, $a_y > 0$.
∗ If the velocity is downward, and if its speed is decreasing, $a_y > 0$, because $v_y < 0$ and its magnitude is diminishing, so a_y must have the opposite sign of v_y.

If the velocity is downward and its speed is increasing, what is the sign of a_y?

(i) constant speed upward

Q5. What is a_y? Then, answer the question.

a_y is the rate of change of the (y-component of the) velocity, so if the elevator speed is constant, then $a_y = 0$. Therefore, $F = \mu_s Mg$, the same as the stationary case.

(ii) constant speed downward

You should know the answer immediately. No difference!

Lesson: we can never feel the absolute velocity.

(iii) accelerating upward

Q6. What is the sign of a_y? Then, answer the question.

Our y-coordinate is so chosen that upward is the positive direction. Therefore, $a_y > 0$; $a_y + g > g$, so F is larger. [You feel heavier.]

(iv) accelerating downward

Q7. What is the sign of a_y? Then, answer the question.

Our y-coordinate is so chosen that upward is the positive direction. Therefore, $a_y < 0$; $a_y + g < g$, so F is less. [If the elevator falls freely, $a_y = -g$! You are weightless!]

(v) upward; slowing down

Q8. What is the sign of the y-component of the velocity? Is it increasing or decreasing? If you can answer this question, you must know the sign of a_y. Then, answer the question.

The y-component of the velocity is positive, and its magnitude is diminishing, so $a_y < 0$. Therefore, F is less.

(vi) downward; slowing down

Q9. What is the sign of the y-component of the velocity? Is it increasing or decreasing? If you can answer this question, you must know the sign of a_y. Then, answer the question.

The y-component of the velocity is negative, and its magnitude is diminishing, so something positive must be being added. Hence, $a_y > 0$. Therefore, F is larger.

Respect your experience:
If you imagine that you are in the elevator, probably you can guess the answers above.

Thinking about extreme cases may also be helpful.
∗ What happens if the descending elevator hits the ground?
∗ Would the box feel heavier or lighter if the elevator accelerated downward at 9.8 m/s?

D4-3 Ball Toss upward

Q0 What is the main theme of this problem? What are the key points?

1D kinematics of a point mass (small mass) under constant acceleration:
∗ The basic equations for the x components of the displacement vector and the velocity are given on p43.
∗ In short, you have only to understand these three equations. Be able to explain to you friends how to use them!

1 max height

Let us choose the y-coordinate to be upward. There are different methods to answer the question.

Q1. Let us use the v_0, Δy, and g relation. How do you proceed?

We use

$$v^2 = v_0^2 + 2a\Delta x.$$

We identify the symbols as: $v = 0$ (at the highest point), $v_0 = 39.2$ (the initial speed), and $\Delta y = h_{max} - h$. That is,

$$0 = v_0^2 - 2g(h_{max} - h).$$

This implies that

$$h_{max} = h + v_0^2/2g = 15 + 39.2^2/(2 \times 3.7) = 15 + 207.65 = 222.65 \text{ m}$$

We may study the motion in more detail.

Q2. When does the ball reach the highest point?

We use $v = v_0 + at$ with $v_0 = 39.2$ m/s and $a = -3.7$ m/s^2 (Notice the $-$ in front of 3.7).
At the highest point, the y velocity vanishes: $v(t) = 0$ (v at time t vanishes). Writing $v(t)$, we can determine the time required:

$$v(t) = v_0 - gt = 0,$$

that is, $t = v_0/g = 39.2/3.7 = 10.59$ s.

Q3. What is the height of the highest point?

We use the formula $y = y_0 + v_0 t + (1/2)at^2$ with $y_0 = h = 15$ m, $y = h_{max}$, $v_0 = 39.2$ m/s, and $a = -3.7$ m/s^2 at time $t = 10.59$ s. Now,

$$h_{max} = 15 + 39.2 \times 10.59 - (1/2) \times 3.7 \times 10.59^2 = 15 + 415.1 - 207.52 = 222.6 \text{ (m)}.$$

Needless to say, we get (almost) the same answer.

2: time in the air

According to our calculation above, it takes 10.59 s to reach the highest point. To answer the question, we need the time to fall from the highest point to the ground freely with zero initial velocity.

Q4. How long does it take the ball to fall from 222.6 m to the ground?

We use the formula

$$y = y_0 + v_0 t + (1/2)at^2 \tag{3}$$

with $y = 0$ (ground), $y_0 = 222.6$ m , $v_0 = 0$ (initially without motion at the highest point), and $a = -3.7$ m/s^2 ($-$ is because our y coordinate uses the positive direction upward). Thus,

$$0 = 222.6 = (1/2) \times 3.7t^2,$$

so $t = \sqrt{2y_0/g} = \sqrt{2 \times 222.6/3.7} = 10.97$ s.

Thus, $10.59 + 10.97 = 21.56$ s is the total time in atmosphere.

We should be able to obtain this from (3) with the following interpretation: $y = 0$, $y_0 = h = 15$ m, $v_0 = 39.2$ m/s, and $a = 3.7$ m/s^2.

$$0 = 15 + 39.2t - (3.7/2)t^2.$$

Solving this for t, we get $t = (39.2 + \sqrt{39.2^2 + 3.7 \times 30})/3.7 = 21.6$ s, which is consistent with the previous answer.

3: v_G

Q5. From **Q3**, we know v_G is the final speed of the ball falling from the height 222.6 m.

It takes 10.97 s to fall from the highest point with zero initial velocity, so the final speed is

$$v = v_0 - 3.7t = 0 - 3.7 \times 10.97 = -40.59.$$

That is, the $v_G = 40.59$ m/s ($-$ in the above formula is because the velocity is downward). This should be slightly larger than the initial speed 39.2 m/s at the tower top.

We should be able to get the same result, starting from the tower top at $t = 0$. We know it takes 21.56 s from the tower top to the ground with the initial velocity $v_0 = +39.2$ m/s. We use $v = v_0 + at$:

$$v_G = v_0 - gt = 39.2 - 3.7 \times 21.56 = -40.47 \text{ m/s}$$

Although there is a slight numerical error, this is consistent with the previous answer.

4: 5 s.

We have already solved a similar question. The y coordinate at time t is obtained as $y = y_0 + v_0 t + (1/2)at^2$. Therefore,

$$y(5) = 15 + 39.2 \times 5 - (1/2)3.7 \times 5^2 = 15 + 196 - 46.25 = 164.75 \text{ m}.$$

5: v at 7 m

We are discussing 1D kinematics of motion with constant acceleration. Then, we have only three equations to take into account. We should use

$$v^2 = v_0^2 + 2g\Delta y,$$

where $v_0 = 39.2$ m/s, $g = -3.7$ m/s^2, and $\Delta y = 7 - 15 = -8$. Therefore, $v = \sqrt{39.2^2 + 2 \times 3.7 \times 8} = 39.95$ m/s.

D4-3 Block on Incline

Q0 What is the point of this problem? What is the relevant formula (principle)?

∗ Newton's second law: $\sum \boldsymbol{F} = m\boldsymbol{a}$.
∗ x and y components can be considered totally separately.

1

2: Normal force

Q1 Let us consider x and y directions separately. The y-direction is perpendicular to the slope. Write the equation describing the second law in the y-direction.

$$0 = F_N - Mg\cos 40^\circ, \tag{4}$$

where F_N is the normal force. From (4) we get $N = Mg\cos 40^\circ = 10g\cos 40^\circ = 75.1$ N.

3: *a*

Q2 Now, the x-direction. As already noted in **1** itemize all the forces in the x-direction and write down the second law. Use a for the acceleration.

Since there is no friction, there is only one force: gravitational force:

$$Ma = Mg\sin 40^\circ. \tag{5}$$

That is, $a = g\sin 40^\circ = 6.3$ m/s^2.

4: doubling mass.

No change, obviously from (5)

5. 5m traveling

Q3 Let us choose the origin of our coordinates to be the starting point of the motion. Write down the formula for the x coordinate $x(t)$ at time t. $x(t) = 5$ should give you the answer.

The initial velocity (x-component) is zero. Now we know $a = 6.3$ m/s^2.

$$x(t) = 0 + 0 \times t + \frac{1}{2}6.3t^2 = 3.15t^2$$

Therefore, t $(= \sqrt{2\Delta x/a}\ ) = \sqrt{10/6.3} = 1.26$ s.