Physics 101 Discussion Week 5 Explanation (2011)

D5-1 Ball Toss

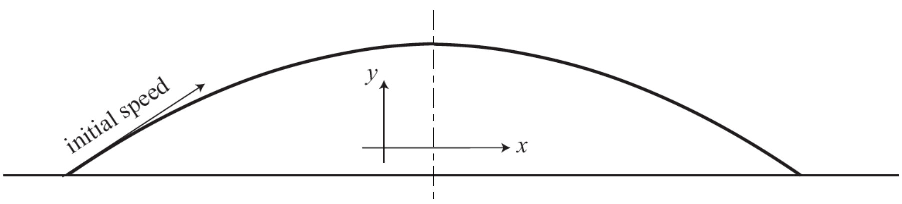
Q0 What is the main theme of this problem? What are the key points?

2D motion in space.
∗ It is important to recognize that the motion in the x-direction (horizontal) and that in the y-direction (vertical) can be separately considered (no interference between them!).
∗ The basic equations for the x and y components of the displacement vector and the velocity are given on p57.
∗ Clearly recognize that these equations are essentially two copies of the formulas for 1D motions in the formula sheet.

1: sketching motion

This is a parabola. **See the figure on p48** (various features in this figure are worth memorizing).

You must clearly recognize that the trajectory of the ball is symmetric around its highest point.



Let us be a bit quantitative now.

Q1. Let us choose the x-coordinate to be horizontal (left to right) and the y-coordinate to be vertical (upward) as in the above figure.

We choose the initial position to be the origin: $(x_0, y_0) = (0, 0)$. To use the equations mentioned above (i.e., $x = x_0 + v_0 t + (1/2)at^2$, etc.), we need the initial velocity and the acceleration vectors. Identify them.

The initial speed is 39.2 m/s, and its direction is 30° (upward) from the horizontal ground. Therefore,

$$\boldsymbol{v}_0 = (39.2\cos 30°, 39.2\sin 30°) = (33.9, 19.6) \text{ m/s}.$$

The acceleration $\boldsymbol{a}$ is given by $\boldsymbol{a} = (0, -g) = (0, -9.8)$ m/s^2. Watch out for the signs. You must be able to explain why the y-component is negative.

Q2. What is the important characteristic of the motion in the x-direction?

The speed in the x-direction is constant.
Then, if you know the duration of the motion, you can easily calculate the distance the ball moves in the horizontal (x) direction.

Q3. You must clearly recognize that the trajectory of the ball is symmetric around its highest point. Therefore, if we can know the position of the highest point, our sketch would be fairly accurate.

We already know well how to determine the highest point (recall **D4-3**). How long does it take for the ball to reach the highest point?

At the highest point $v_y = 0$, so $v_{0y} - gt = 0$. Thus, after $t = v_{0y}/g = 19.6/9.8 = 2$ s the ball reaches the highest point. Now, you can answer **2** (because the trajectory is symmetric around the highest point) and **3**.

2: time

Q4. How long does it take for the ball to fall from the highest point and hit the ground? You can easily answer the problem by symmetry.

2 s. Therefore, $2 \times 2 = 4$ s is the time during which the ball is in the air.

3: h_{max}

Q5. Write down the y-coordinate of the ball $y(t)$ at time t. Then, find the maximum height.

$$y(t) = v_{0y}t - \frac{1}{2}gt^2 = 19.6t - \frac{1}{2}9.8t^2. \tag{1}$$

The highest point is reached at $t = 2$, so $y(2) = 19.6 \times 2 - (1/2)9.8 \times 2^2 = 39.2 - 19.6 = 19.6$ m.

4: D

Q6. We know that the motion along the x-axis is at a constant velocity, but let us write an explicit formula for the x-coordinate of the ball $x(t)$ at time t. Then, answer the question.

$$x(t) = v_{0x}t = 33.9t.$$

The duration of the motion is 4 seconds, so $x(4) = 33.9 \times 4 = 135.6$ m.

5: height when $\boldsymbol{x} = 100$

Q7. What do you know already? Everything! $y(t)$ would give the height; $x(t)$ gives the traveled distance. How long does it take for the ball to travel 100 m horizontally?

We must solve

$$x(t) = 33.9t = 100.$$

Therefore, $t = 100/33.9 = 2.95$ s.

Q8. What is the height at this time $t = 2.95$ s?

We have only to compute $y(2.95)$. From (1)

$$y(2.95) = 19.6(2.95) - \frac{1}{2}9.8(2.95)^2 = 15.18.$$

15.2 m is the height. You can calculate this height going backward in time (using a time machine!) for $4 - 2.95 = 1.05$ s: $y(1.05) = 15.18$!

6: landing speed

Answer without any calculation.

By symmetry, [or imagine you take a movie of the motion and play it backward in time (and make its mirror image around the center), you will see exactly the same motion] $|v_G| = 39.2$ m.

Q9. You should also be able to calculate the final velocity explicitly using $\boldsymbol{v}(t)$. [But even in this case you should be able to get immediately the answer without any calculation.]

You could honestly calculate as

$$\boldsymbol{v}(4) = (v_x(4), v_y(4)) = (33.9, 19.6 - 4.9 \times 4^2) = (33.9, -19.6) \quad \text{m/s}.$$

From this we get $\sqrt{33.9^2 + 19.6^2} = 39.2$ m/s. Note that the y-component changes the sign. That is the only change!

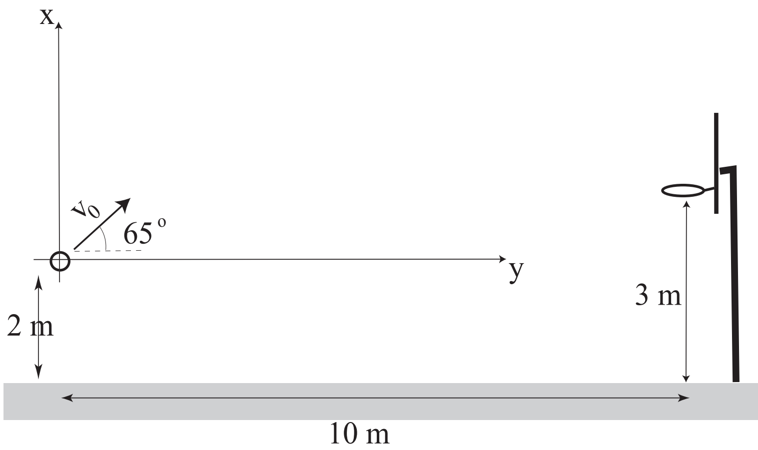
7: velocity at the highest point

We already know the answer: $\boldsymbol{v}$ at the highest point is horizontal. That is, (c). Explicitly, $(v_x(2), v_y(2)) = (33.9, 0)$ m/s.

D5-2 Basketball

Q0. What is the point of this problem?

Truly 2D motion; actually, you have only to understand a constant acceleration motion (2D kinematics). x and y directions can be considered separately (as x and y components of vectors). Probably, we need not quote the relevant formulas anymore.



1. 10 m travel

Q1. As you see we must know the position of the ball (not only the x but also the y coordinate) of the ball at time t. To this end, we need initial data. Choose the xy-coordinates as above. Give the initial velocity vector $\boldsymbol{v}_0$, the initial position $\boldsymbol{x}_0$ and the acceleration $\boldsymbol{a}$. Give them in terms of v_0, etc.

$\boldsymbol{v}_0 = (v_0 \cos 65°, v_0 \sin 65°)$,
$\boldsymbol{x}_0 = (0, 0)$ (due to our choice of the coordinates),
$\boldsymbol{a} = (0, -g) = (0, -9.8)$ m/s^2.

Q2. Write down $x(t)$ and $y(t)$.

$$x(t) = v_0 t \cos 65°, \tag{2}$$
$$y(t) = v_0 t \sin 65° - 4.9t^2. \tag{3}$$

The rest must be a matter of calculation.

Q3. We wish to hit the position $(10, 1)$ m (do not forget our origin is 2 m above the ground). Write down the simultaneous equations you must solve based on (2) and (3).

$$\begin{aligned} 10 &= v_0 t \cos 65^\circ, & (4) \\ 1 &= v_0 t \sin 65^\circ - 4.9t^2. & (5) \end{aligned}$$

Q4. Solve (4) and (5).
Hint: $\sin 65^\circ \times$ (4) $- \cos 65^\circ \times$ (5) can get rid of v_0 (m).

Following the hint, we get

$$10 \sin 65^\circ - \cos 65^\circ = 4.9t^2 \cos 65^\circ$$

or

$$8.64 = 2.07t^2.$$

That is, $t = \sqrt{8.64/2.07} = 2.04$ s. Now, using (4), we get

$$v_0 = 10/2.04 \cos 65^\circ = 11.6 \quad \text{m/s}.$$

D5-3 Dragging blocks

Q0. What are the key points of this problem?

1) Kinetic friction force $f = \mu_k N$.
2) Newton's second law in 2D.
   * The x and the y directions (orthogonal directions) can be totally decoupled when you consider the second law.
   * Whenever you apply the second law, itemize all the forces acting on the object beforehand.

Let us perform numerical calculation only at the very end, so let us use symbols: m_1, m_2, etc.

2: y-direction

Since there is no (accelerated) motion in the y direction, the normal forces from the surface must cancel gravity:

block 1: $m_1 a_{y1} = N_1 - m_1 g = 0$;

block 2: $m_2 a_{y2} = N_2 - m_2 g = 0$,

where N_i is the normal force acting on the block i with mass m_i kg.

3: x-direction

Q1. [Itemize the forces] Since you must have done **1**, you must have itemized all the forces acting on the blocks. What are they for block 1 and for block 2 in the x direction?

Block 1: kinetic friction $\mu_k m_1 g$ and tension T.
Block 2: kinetic friction $\mu_k m_2 g$, tension T, and F.

Q2. What is the relation between a_{x1} and a_{x2}?

The distance between the two blocks never changes, so they move together. Then, their velocities and their rates of change must be identical. $a_{x1} = a_{x2}$.

Q3. [Setting up equations] Write down the second law for these two blocks in terms of symbols. The point of the symbolic approach is that you need not worry about what quantity you know and what not. Just pretend you know everything. Let us write $a_{x1} = a_{x2} = a$,

$$m_1 a = -\mu_k m_1 g + T, \tag{6}$$

$$m_2 a = -\mu_k m_2 g - T + F \tag{7}$$

Q4. Solve the above simultaneous equations for a. You should have realized that adding these two equations can get rid of T.

If we add the left hand sides of (6) and (7), we get $(m_1 + m_2)a$. If we add their right hand sides, we get $-\mu_k g(m_1 + m_2) + F$. That is, we obtain

$$(m_1 + m_2)a = -\mu_k g(m_1 + m_2) + F,$$

or

$$a = -\mu_k g + \frac{F}{m_1 + m_2}.$$

4: T

Q5. Get T.

Put a into (6) or

$$T = m_1(a + \mu_k g) = m_1 \frac{F}{m_1 + m_2}.$$

Numerically, $a = -0.25 \times 9.8 + 40/12 = 0.88\ \mathrm{m/s^2}$, and $T = 4 \times (40/12) = 13.3$ N.

D5-4 Blocks and Pulley

Q0. What are the key points?

Newton's second law.
In this case, we immediately know that we have only to consider the second law along the string; that is, only in the x direction for block 1, and only y for block 2.

The x-component of the acceleration of block 1 and negative of the y-component of the acceleration of block 2 must be identical (the same logic we used in **D5-3**): $a_{1x} = -a_{2y}$. We have already answered **3**.

First, let us finish **2** and **3**.

2

$T_1 = T_2$, because the string is massless.

3

We already answered this.

4: second law

Writing $a_{1x} = -a_{2y} = a_1$, write down the equation of motion.

Q1. Itemize the forces acting on the blocks.

Block 1: tension T and friction $f = \mu_k Mg$.
Block 2: tension T and gravity Mg.

Q2. Write down the equation of motion for these blocks.

$$
\begin{aligned}
Ma &= -\mu_k Mg + T, && (8)\\
-Ma &= -Mg + T. && (9)
\end{aligned}
$$

Here, the negative sign in front of the frictional force is because the friction opposes the motion, which is in the positive x direction in our case.

Q3. Solve the simultaneous equation to get a.

Subtracting (9) from (8), we obtain

$$2Ma = (1 - \mu_k)Mg.$$

Therefore, $a = (1 - \mu_k)g/2$.

5: numerical

General lesson: Especially when the equations are clean, do not put numbers until you get the answers symbolically.

It is very clear that M does not matter in this case.

$$a = 9.8(1 - 0.2)/2 = 0.4g = 3.92 \text{ m/s}^2.$$