Physics 101 Discussion Week 6 Explanation (2011)

D 6-1 Friction

Q0. Read **1** and **2**. What are the key points of the problem?

Although you need Newton's second law to answer **2**, the problem is essentially about static and kinetic friction. Look at the graph on p43. A technical point is how to handle F (how to decompose it into two convenient directions).

1: normal force

Q1. We wish to write down the force balance conditions (or Newton's second law) in the directions perpendicular and parallel to the slope surface. To this end, find the parallel and perpendicular components of F. We must also do the same thing for the gravitational force whose magnitude is mg.

Let us choose the coordinates as in **D2-5**. For the force $\boldsymbol{F}$

x-component (parallel component): $-F\cos 40°$.

y-component (perpendicular component): $-F\sin 40°$.

For the gravitational force

x-component: $mg\sin 40°$.

y-component: $-mg\cos 40°$.

Here, pay due attention to the signs.

Q2. Write down the force balance condition in x and y directions.

First, you should itemize the forces: normal force N, $\boldsymbol{F}$, Mg, and the (static) friction force f (which must be in the positive x direction).

The force balance conditions read:

x-direction: $f - F\cos 40° + mg\sin 40° = 0$,

y-direction: $N - F\sin 40° - mg\cos 40° = 0$.

Q3. Find the normal force N and the friction force f from the above equations.

Notice that $f = \mu_s N$ is applicable ONLY when the block is on the verge of sliding.

$$\begin{aligned} N &= F\sin 40^\circ + mg\cos 40^\circ = 360\sin 40^\circ + 20\times 9.8\cos 40^\circ = 381.5\ \mathrm{N}, \\ f &= F\cos 40^\circ - mg\sin 40^\circ = 360\cos 40^\circ - 20\times 9.8\sin 40^\circ = 150\ \mathrm{N}. \end{aligned}$$

2: with oil

Q4. Write down Newton's second law in the x direction, and obtain a.

First, itemize forces: F and mg, no friction. We know their x components, so

$$ma = -F\cos 40^\circ + mg\sin 40^\circ.$$

Thus, $a = -(F/m)\cos 40^\circ + g\sin 40^\circ = -18\cos 40^\circ + 9.8\sin 40^\circ = -7.5\ \mathrm{m/s^2}$. That is, the block is pushed up along the slope.

D6-2 Carnival Ride

Q0. What are the key points of the problem?

Kinematics of circular motion:
∗ Definitions: $\Delta\theta = \Delta x/r$ (radian); $v = \omega r = \Delta\theta/\Delta t$; $\alpha = \Delta\omega/\Delta t$.
∗ Understand the correspondence to the linear motion on p69.
Dynamics of circular motion:
∗ The most important observation is that circular motion means there is acceleration. Acceleration requires force = **centripetal force**, which always points at the the center of the circle.
∗ $a = v^2/r$ gives the acceleration due to centripetal force toward the center.

1: normal force

Q1. What happens if there is no normal force N due to the retaining wall? (Or what is N doing to you?) Notice that the floor is frictionless, so there is no force parallel to the platform acting on your feet.

If no N (or no retaining wall), you will fall off, because your trajectory will be a straight line tangential to the circle. N makes you go circularly around the center. N is the centripetal force that provides the needed acceleration toward the center.

Q2. What is the acceleration toward the center due to N?

The acceleration toward the center is

$$a = V^2/R.$$

Therefore, Newton's second law tells us that $N = Ma = MV^2/R$.

2: gravity

This is easy: $F_G = Mg$.

3: friction

Q3. Find the inequality satisfied by μ_s.

The static friction on your back must keep you from falling, so the frictional force must not be smaller than F_G just obtained: $Mg \le N\mu_s = Ma\mu_s$. That is,

$$\mu_s \ge g/a = Rg/V^2.$$

4: numerics

The inequality just above implies $\mu_s \ge 5 \times 9.8/10^2 = 0.49$.

5: altering M

You must regard this a stupid question: the answer is too obvious. No change! But this is a triumph of algebraic approach. If you put numbers from the start, a clean conclusion is hardly obtained.

D6-3 Ball along hoop

Q0. What is the point of the problem?

While the ball moves on the vertical ring, both the gravitational and normal forces have effects.

You must know how to compute the centripetal force. Needless to say, we need Newton's second law applied to the vertical direction.

1: minimum speed

Q1. What is the condition that the ball does not fall off the hoop? [Hint: what happens to the normal force on the ball from the hoop, if the ball falls off?]

Since the ball ceases to touch the hoop, the normal force would vanish. Remember that the normal force is inward toward the center of the track. The ball must still be in contact with the track, if the normal force is greater than zero. Therefore, the condition is that the normal force to be in the radial direction (toward the center of the hoop) must be positive.

Q2. Let us compute the normal force N (this must be downward) on the ball with a tangential speed V at the top of the hoop. Apply Newton's second law to the ball in the vertical direction. As usual, itemize the forces first.

There are two forces acting on the ball, the gravitational force Mg downward, and the normal force N downward. The sum of these forces must be equal to the centripetal force:

$$m\frac{V^2}{R} = N + mg.$$

Therefore, $N = m(V^2/R - g)$

Q3. Combining the obtained N and the observation in **Q1**, you must know the answer.

$$N = m\left(\frac{V^2}{R} - g\right) \geq 0,$$

so $v_{min}^2/R = g$ or $v_{min} = \sqrt{gR}$.

3: numerics

$v_{min} = \sqrt{gR} = \sqrt{4.9} = 2.2$ m/s.

4: $m = 10$ g

No change, obviously.

D6-4 Centripetal acceleration

Q0 This Discussion is obviously about rotational kinematics and dynamics. What are the key formulas on the formula sheet? What is the most important feature of the circular motion?

$a_c = v^2/R = \omega^2 R$ (This is under **Dynamics**.)
Definitions of ω, etc., and rotational kinematic formulas (notice their close similarity to linear motion).

The most important feature of the circular motion is that it requires continual acceleration to happen, so it requires continual application of force.
The needed force is perpendicular to the instantaneous velocity (that is tangential to the circle) and points to the center. It is called the centripetal force.

1: tension

Q1 In this case the tension provides the needed centripetal force. Write down Newton's second law in the radial direction.

$$m\frac{v^2}{\ell} = T \tag{1}$$

2: $\ell \to 2\ell$

(1) immediately gives the answer: $T_2 = T/2$.

3: centrifuge

The only complication, if any, is to obtain the angular speed ω. One revolution is 2π.

$r\omega^2 = 0.2(2\pi \times 1000/60)^2 = 2193\text{m/s}^2$. This is $224g$.