Physics 101 Discussion Week 7 Explanation (2011)

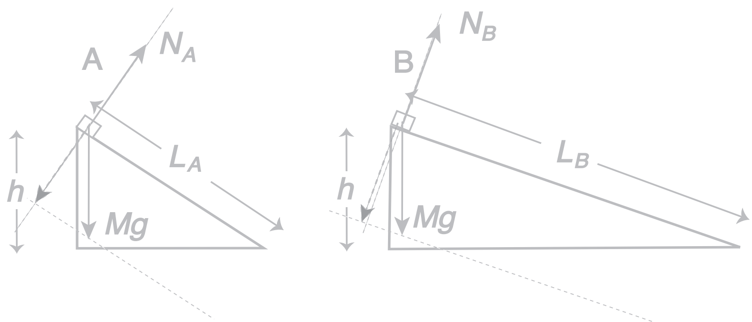
D7-1 Different Slopes

Q0. As is explicitly written in the problem, this is to check your understanding of work done by forces. What formula do you need on the formula sheet? Explain how to use it.

> $W_F = FS\cos\theta$. F is the magnitude of the force acting on an object which moves over distance S. θ is the angle between the force vector $\boldsymbol{F}$ and the vector $\boldsymbol{S}$ describing the displacement of the object.
> Look at the figures on p81 (there, S is written as Δx).

1: free-body diagram

You must not have any difficulty in drawing the diagrams. Quantitatively, pay due attention to the length of the vectors representing the normal force and the gravitational force.



The dotted arrows denote the vectors showing the negative of the normal forces.

2: work

Q1. Give a geometrical interpretation of the formula $W_F = FS\cos\theta$,[1] using the concept of projection. What is $F\cos\theta$, or what is $S\cos\theta$?

> $F\cos\theta$ is the projection of the force vector onto the direction of the displacement vector (notice that even the sign is correct) (Fig. A on the next page). It is a determination of how much the two vectors point in the same direction. Note that if $\boldsymbol{F}$ is perpendicular to $\boldsymbol{S}$, the work done by $\boldsymbol{F}$ would be zero.

[1] This is the scalar (or inner) product $\boldsymbol{F}\cdot\boldsymbol{S}$ of two vectors $\boldsymbol{F}$ and $\boldsymbol{S}$.

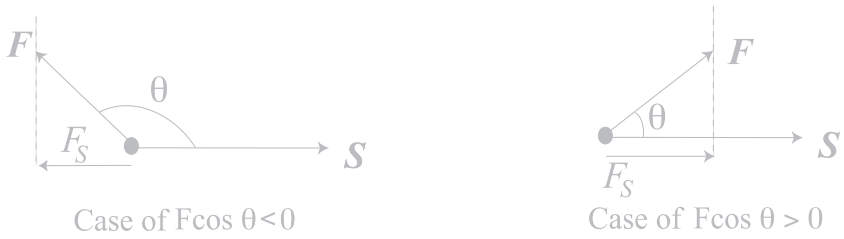


Fig. A: F_S is the projection of $\boldsymbol{F}$ onto $\boldsymbol{S}$.

As illustrate in Fig. A, to calculate the work done by $\boldsymbol{F}$, project it onto the displacement vector to obtain $F\cos\theta$ and then multiply S.

You can project the displacement vector $\boldsymbol{S}$ onto $\boldsymbol{F}$ as well (Fig. B). For the present problem this is wiser. The figure corresponding to the above is as follows. If the arrow of force $\boldsymbol{F}$ and the projected arrow of $\boldsymbol{S}$ point in the same direction, the work is positive; otherwise, negative.

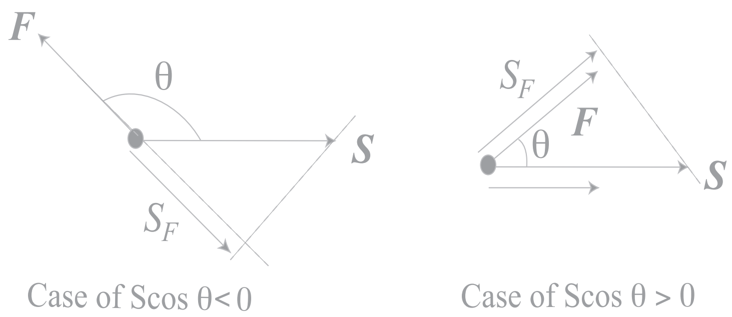


Fig. B: S_F is the projection of $\boldsymbol{S}$ onto $\boldsymbol{F}$.

Thus, we have learned the relation between the work and the force projected onto the displacement direction, and the relation between the work and the displacement projected onto the force direction.

Now, the answer to **2** should be obvious. Normal forces do not do any work. In contrast, gravitational forces Mg's do work, because they are not entirely orthogonal to the displacements; the works done by these gravitational forces are positive, because projections of these forces onto the displacement vectors have the same direction as the latter (as Fig. A Right).

Let us interpret W_F as the product of the magnitude F of the force × projection of displacement vector onto the force direction $S\cos\theta$.

For A and B forces are the same: their magnitudes are both Mg. The projections of the displacement onto the force direction are the same for A and B: both downward h. Hence, the works are identical for A and B: $W_F = Mgh$ for both A and B.

3: W_F

We have already answered this question: Mgh.

4: final speed

Q3. What is the fundamental law we need?

Work/kinetic energy theorem: $W_{NET} = \Delta K$.
ΔK is always the final value − the initial value of K.

Let us finish the calculation.

$W_{NET} = Mgh$, and $K = (1/2)Mv^2$, so

$$\frac{1}{2}Mv^2 = Mgh \quad \Rightarrow \quad v^2 = 2gh.$$

That is, $v = \sqrt{2gh}$.

Obviously, this has nothing to do with M, so you have already answered **5**.

How can you solve the same problem using the principle of conservation of mechanical energy?

Q4. First, explain what this principle implies.

If there is no friction (as in this problem), $E = K + U$ is constant. U is the potential energy = the energy associated with the height of the object = needed work to push it up to the height from some reference point. Here, the bottom of the incline is the reference point, so initially, $U = Mgh$ is the potential energy of the mass M at the top of the incline.

Q5. Give the initial and final U, K and the total (mechanical) energy E in terms of M, g, h and v.

Initial: $U = Mgh$; $K = 0$, because $v = 0$. Thus, $E = Mgh$.
Final: $U = 0$, because our height is measured from the bottom of the incline where the reference point lies;
$K = mv^2/2$.
Thus $E = mv^2/2$.

Q6. Apply the principle of conservation of mechanical energy and get v.

Initial E = final E. That is,

$$E = Mgh = \frac{1}{2}Mv^2.$$

We get the same formula we already saw above: $v = \sqrt{2gh}$.

D6-2 Blocks on pulley

Q0. What are the key principles?

Conservation of energy: The total mechanical energy, $K + U$ is preserved. On the formula sheet a more general formula $W_{nc} = \Delta E$ is given: the work done to the object is equal to the increase of the energy of the object.

Sometimes you might wonder whether you use conservation of energy or Newton's second law. Generally speaking, try to use conservation of energy first (especially when there is no friction). In this case, there is no friction, so definitely try conservation of energy.

1: v_M

Q1. To apply conservation of energy, write down the initial and final energies.

To this end you must be able to express U and K explicitly. For U you must choose the reference height. Any choice is OK, but usually it is wise to choose the height of the lowest point in the problem as the reference height. In our case, it is the position of the floor. Write the initial and the final U, K and total energy.

Initially:

There is no motion, so K is zero.

According to our choice of the height the block with mass m has no potential energy.

The block with mass M is at height h, so U is Mgh.

Thus, $E = Mgh$.

Finally, two blocks are moving at the same speed v_M.

$K = m \times v_M^2/2 + M \times v_M^2/2 = (m + M)v_M^2/2$.

The block with mass m is at height h from the floor, so it has U of mgh,

The block with mass M is almost on the floor, so it has no U.

Thus, $E = mgh + (m + M)v_M^2/2$

Q2. Apply conservation of mechanical energy and obtain v_M.

Since everything we need has been obtained, it is easy to write down initial $E =$ final E:

$$Mgh = mgh + (m+M)v_M^2/2 \quad \Rightarrow (M-m)gh = (m+M)v_M^2/2.$$

Thus,

$$v_M = \sqrt{2gh\frac{M-m}{M+m}}.$$

2: mass m going up

Now, only the smaller block with mass m is moving, and the other block is doing nothing, so we may forget about the latter.

Q3. You can apply Newton's second law, but the use of conservation of energy is much easier. Identify the initial and the final U, K and E and solve the problem.

Now, the initial state is at the time when the block with mass M crashes to the floor. We know the height h and the speed v_M of the lighter block.
Initial:
K $= mv_M^2/2.$
U $= mgh,$
$E = mv_M^2/2 + mgh.$
Final: this is the highest point, so the speed of the block must be zero.
K $= 0.$
U $= mgh_m,$
$E = mgh_m$
Now, the energy is conserved, so initial $E =$ final E implies

$$mv_M^2/2 + mgh = mgh_m \;\Rightarrow\; 2g(h_m - h) = v_M^2$$

That is,

$$h_m = h + \frac{v_M^2}{2g} = h + h\frac{M-m}{M+m} = \frac{2M}{M+m}h.$$

3: numerics

$v_M = \sqrt{6g(3/7)} = 5.02$ m/s.
$h_m = 30/7 = 4.286$ m.

4: changing m

If the smaller block is lighter, it should be easier to launch, so should be faster. Then, we can guess that the max speed is obtained when $m \simeq 0$, and the speed is solely determined by the larger block, so $v_M = \sqrt{2gh}$. This is indeed the case as you see from the explicit formula for v_M as a function of m.

D7-3 Block with friction

Read questions **1** and **2** first.

Q0. What is the most useful formula on the formula sheet? Perhaps you remember its name.

Work/energy theorem: $W_{NET} = \Delta K$
It is important that the work is the 'net' work. In our case, there are two forces; **1** and **2** ask their contributions separately.

1: W_F

We are interested in the work **done** to the block.

Q1. Is this work positive or negative? Can you explain your answer, using the formula $FS\cos\theta$?

Positive, that is, it increases the energy of the block.
Since the force and the displacement are in the same direction, $\theta = 0$. That is why $FS\cos\theta > 0$.

Q1'. [Incidental question] What is the work that the block does on the person pulling the block?

Because of the action-reaction law (Newton's third law), the force on the person is opposite to the force acting on the block. Thus, it is antiparallel to the moving direction of the person: $\theta = \pi$: exactly, the negative of the work done by the person.

Let us finish the calculation of the work.

It is just FD.

2: W_f

Q2. What is the kinetic friction force? Pay attention to its direction.

The normal force $= Mg$, so the kinetic friction force is $\mu_k Mg$. Its direction is opposite to the moving direction.

Q3. What is the work due to the friction force done on the block? Is it + or −?

It must be negative, because the force points in the opposite direction of the displacement ($\theta = \pi$). $W_f = -\mu_k MgD$.

3: final speed

Q4 We apply the work/energy theorem. What do you have to compute to obtain the final speed?

The net work $= W_T + W_F = (F - \mu_k Mg)D$ and the initial $K = (1/2)Mv_0^2$.

Q5 Let the final speed be v. Write down the work/energy theorem and finish the problem.

Do not forget that $\Delta X = X_{final} - X_{initial}$. The work/energy theorem tells us that

$$W_{NET} = \frac{1}{2}Mv^2 - \frac{1}{2}Mv_0^2 \quad \Rightarrow \quad v^2 = v_0^2 + 2W_{NET}/M.$$

That is,

$$v^2 = v_0^2 + 2\left(\frac{F}{M} - \mu_k g\right) D.$$

That is,

$$v^2 = 15^2 + 2(9.4 - 0.98) \times 35 = 814.4$$

Hence, $v = 28.5$ m/s.

D7-4 Slide

Q0. What principles do you need to solve this problem?

Conservation of energy to obtain the speeds for **1, 2** and **4** in the form of

$$mgy + \frac{1}{2}mv^2 = \text{ constant.}$$

Here, m is the mass of the particle, v is its speed, and y is its vertical position coordinate.

Newton's second law applied to a circular motion is needed to solve **3**.

1: h

Q1. This is a straightforward conservation of energy question. What is the total energy relative to the height of point A (or B)? What is the total energy at A in terms of v?

Initially at O there is no motion, so the kinetic energy is zero; the potential energy relative to position A is mgh, so the total energy must be mgh.

At A the height is obviously zero relative to A, but the block is moving at a speed v (= 1.3 m/s), so the total energy is $mv^2/2$.

Let us finish the question **1**.

$$mgh = \frac{1}{2}mv^2.$$

That is, $h = v^2/2g = 1.3^2/(2 \times 9.8) = 0.086$ m.

2: kinetic energy at B

Since energy is conserved, and A and B are at the same heights, the kinetic energies at A and B must be identical.

Consequently, the speeds at A and B must be identical and must be 1.3 m/s, so the kinetic energy is $mv^2/2 = 0.169$ J.

3: normal force

Q2. Assume that the block is at the highest point B with a speed $v = 1.3$ m/s. Itemize all the forces acting on it and specify their directions.

Recall **D6-3**.
Gravitational force mg (downward toward the center) and the normal force N (downward toward the center).

Q3. Write down Newton's second law in the radial direction. Use R (= 0.1 m) for the radius.

The direction of the acceleration is toward the center of the circle (centripetal).

$$m\frac{v^2}{R} = mg + N.$$

What is the normal force? Write it symbolically first and then give the numerical solution.

From the above formula, we obtain

$$N = m\left(\frac{v^2}{R} - g\right).$$

Thus, $N = 0.2(1.3^2/0.1 - 9.8) = 1.42 > 0$ N. The block can go safely without falling to C.

Since $mv^2 = 2mgh$, we could write $N = mg(2h/R - 1) = 0.2 \times 9.8(0.86 \times 2 - 1) = 1.411$ N (the difference is due to roundoff errors).

4 speed at C

You should have immediately realized that this is a conservation of energy problem. The initial total energy relative to the ground level must be the initial kinetic energy + the potential energy due to the height H.

This approach is OK, but before jumping into solving, look at the problem carefully. What do you know?

You know the speed at A, which is due to the drop by h from O. Now, you are actually asked the speed at C, which is due to the drop by $4h$ from O.

Q4. What is the speed of a block at the ground level, if it is released without any initial speed from the height y? If you know this formula, the answer to **4** should be immediate.

Conservation of energy tells us

$$mgy = \frac{1}{2}mv^2 \ \Rightarrow \ v = \sqrt{2gy}.$$

For $y = h$, $v = \sqrt{2gh}$. Therefore, if h in this formula is replaced by $4h$, v becomes $2v = 2.6$ m/s.