# Physics 101 Discussion Week 8 Explanation (2011)

## D8-1 Impulse

**Q0**. What are the key points of the problem?

If two objects collide, one object gives an impulse $\boldsymbol{I}$ to the other (and vice versa), whose momentum $\boldsymbol{P}$ is altered by the impulse. Impulse and momentum are vectors; here, for simplicity, we study only one directions, so we understand these quantities as the $x$-components of vectors.
∗ You must be able to explain what impulse is.
∗ You must be able to give the definition of momentum.

**Q1**. Explain the relation between the impulse $I$ and the change $\Delta P$ in momentum with the aid of the equation of motion (= Newton's second law).

$$I = \Delta P. \tag{1}$$

The 'original form' of the equation of motion Newton wrote was

$$\frac{\Delta P}{\Delta t} = F.$$

Impulse is $I = F\Delta t$, so this implies (1). That is, (1) is a disguised second law.

## 1: impulse

**Q2**. To obtain the impulse, we should know the force $F$ and its duration $\Delta t$ or the change of the momentum $\Delta P$. Since we know the change of the velocity of the ball, we should be able to calculate the change $\Delta P$ of the momentum of the ball. Find it.

$\Delta P$ = the final momentum − the initial momentum. Respect the sign.

$$\Delta P = 0.15 \times (-45) - 0.15 \times 40 = -6.75 - 6.0 = -12.75 \text{ N·s}$$

Therefore, $I = -12.75$ N·s (N·s = kg·m/s). Since it is negative, it is in the negative $x$-direction.

## 2: $F$

**Q3**. Write down the relation between the impulse and the force $F$ that lasts for $\Delta t$? Then, answer the question.

$$I = F\Delta t,$$

so $F = -12.75/0.6 = -21.25$ N, i.e., 21.25 N in the negative $x$-direction. (You need not put $-$ as long as you clearly recognize its direction in this calculation.)

## 3: $v_{bat}$

**Q4**. What principle can you use to obtain the velocity (equivalently, the momentum) of the bat? Notice that you know the initial momentum of the bat.

You could think in two ways:
1) You could think this as a collision problem. Then, recall conservation of momentum. You must be able to tell what it means.
2) The force acting on the bat must be obtainable with the aid of the action-reaction principle. This implies the impulse equal to $-I$ should have worked on the bat.

Method 1)
First, let us use conservation of total momentum to obtain the final momentum of the bat.

**Q5**. What is the total initial momentum of the ball + the bat? What is the final total momentum (pretend you know the answer $v_{bat}$)?

Initial: $P_i = 0.15 \times 40 + 0.9 \times (-30) = -21$ kg·m/s (or N·s).
Final: $P_f = 0.15 \times (-45) + 0.9v_{bat} = -6.75 + 0.9v_{bat}$ kg·m/s (or N·s).

**Q6**. Apply conservation law and finish the problem **3**.

There is (assumed to be) no external force in the $x$-direction, so we may apply conservation of the $x$-component of the total momentum:

$$P_i = -21 = -6.75 + 0.9v_{bat} = P_f,$$

so $0.9v_{bat} = -14.25$ or $v_{bat} = -15.8$ m/s $\Rightarrow$ 15.8 m/s still in the negative $x$-direction.

Method 2)
Let us use the impulse acting on the bat. We know that the change of the momentum $\Delta P$ of the ball is due to the impulse $-12.75$ N·s.

**Q7**. Write $\Delta P$ for the bat and finish the problem.

$\Delta P$ = the final momentum − the initial momentum:

$$\Delta P = 0.9v_{bat} - 0.9 \times (-30) = 0.9v_{bat} + 27.$$

This is equal to +12.75 (= − the impulse given to the ball): $0.9v_{bat} = -14.25$. The answer is the same as before.

## 4: kinetic energy loss

**Q8**. Is the total kinetic energy conserved when the total momentum is conserved?

Generally speaking, there is no reason for conservation of kinetic energy even if the total momentum is conserved.

**Q9**. There is only one way to answer the question: let us calculate the kinetic energy before and after batting.

The initial kinetic energy:
$(1/2) \times 0.15 \times (40)^2 + (1/2) \times 0.9 \times (30)^2 = 120 + 405 = 525$ J.
The final kinetic energy:
$(1/2) \times 0.15 \times (45)^2 + (1/2) \times 0.9 \times (15.8)^2 = 151.9 + 112.3 = 264.2$ J.
Therefore, $\Delta KE = 264.2 - 525 = -260.8$ J, i.e., 260.8 J was lost.

# D8-2 Exploding ball

$m_1 = 5$ kg and $m_2 = 15$ kg are assumed in the solution below.

**Q0**. What is the key principle for the problem?

Conservation of momentum.

## 1: momentum ratio

**Q1**. What is the initial total momentum?

0.

**Q2**. Suppose $\boldsymbol{p}_2$ is the momentum (vector) of mass $m_2 = 15$ kg piece. What is the momentum (vector) $\boldsymbol{p}_1$ of mass $m_1 = 5$ kg piece?

Since the momentum is conserved,

$$\boldsymbol{p}_1 + \boldsymbol{p}_2 = \boldsymbol{0}.$$

That is, $\boldsymbol{p}_1 = -\boldsymbol{p}_2$.

**Q3**. Now the answer should be easy.

$$p_1/p_2 = 1.$$

## 2: speed ratio

**Q4**. What is the relation between $\boldsymbol{p}_1$ and $\boldsymbol{v}_1$?

$$\boldsymbol{v}_1 = \boldsymbol{p}_1/m_1 = \boldsymbol{p}_1/5.$$

Analogously, $\boldsymbol{v}_2 = \boldsymbol{p}_2/15$.

Finish the question, using the answer to **1**.

$$v_1/v_2 = (p_1/5)/(p_2/15) = 15/5 = 3.$$

## 3: kinetic energy ratio

**Q5**. What is the most useful relation for this purpose between $K$ and $p$ and $v$?

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m} = \frac{1}{2}mv \times v = \frac{1}{2}pv.$$

Finish the problem.

$$\frac{K_1}{K_2} = \frac{v_1 p_1}{v_2 p_2} = \frac{v_1}{v_2} = 3.$$

or

$$\frac{K_1}{K_2} = \frac{p_1^2/m_1}{p_2^2/m_2} = \frac{m_2}{m_1} = 3.$$

Notice that the lighter piece can have much more kinetic energy than the heavier piece.

## 4: speeds

**Q6**. Write $v_1$ in terms of $K_1$ and $m_1$ in symbols. Then, get the numerical answer.

$$K_1 = \frac{1}{2}m_1 v_1^2 \;\Rightarrow\; v_1 = \sqrt{2K_1/m_1}$$

**Q7**. [Extra question] Write $p_1$ in terms of $K_1$ and $m_1$ in symbols.

$$K_1 = \frac{1}{2m}p_1^2 \;\Rightarrow\; p_1 = \sqrt{2m_1 K_1}.$$

## D8-3 Sticking blocks

**Q0**. What is the key principle for the problem?

Conservation of momentum.

## 1: before collision

**Q1**. Describe the velocity vectors in components with respect to the $xy$-coordinates given in the problem.

For $M_1$: $\boldsymbol{v}_{1i} = (15, 0)$ m/s.
For $M_2$: $\boldsymbol{v}_{2i} = (0, 20)$ m/s.

## 2: total momentum

**Q2**. Find the initial total momentum vector $\boldsymbol{P}$.

$$\boldsymbol{P} = M_1\boldsymbol{v}_{1i} + M_2\boldsymbol{v}_{2i} = 10(15, 0) + 5(0, 20) = (150, 100) \text{ kg·m/s}.$$

## 3: before collision

Identical to $\boldsymbol{P}$ due to conservation of momentum.

## 4: after collision

$P$ is identical to that of before collision due to conservation of momentum.

## 5: $|\boldsymbol{P}|$ etc.

$$|\boldsymbol{P}| = \sqrt{150^2 + 100^2} = 180 \text{ kg·m/s}, \quad \theta = \tan^{-1}\frac{100}{150} = 33.7°.$$

The speed is $|\boldsymbol{P}|/M = 180/15 = 12.02$ m/s, where $M = 10 + 5 = 15$ kg is the total mass.

# D8-4 Colliding pucks

**Q0**. What is the key issue?

This is a problem about conservation of momentum. However, perhaps the key issue is to write down the info in the problem clearly in formulas. To express everything in vectors is the easiest approach; Minimize the use of your own CPU (brain).

Let formulas think.

## 1, 2: vector representations

**Q1**. Expressing all the velocity vectors (before and after the collision) in components, write down all the important information in the problem to understand the collision.

Initial velocities

Puck A: $\boldsymbol{v}_{Ai} = (2\cos 30°, 2\sin 30°) = (\sqrt{3}, 1)$ m/s.

Puck B: $\boldsymbol{v}_{Bi} = (0, -6)$ m/s.

Final velocities. Notice that Puck A has only the $y$ component (i.e., its speed is equal to the absolute value of its $y$ component) $u_A$, and that Puck B has only the $x$ component $u_B$. We do not know $u_A$ and $u_B$, but the key to algebraic approach is to pretend you know everything! Now, write down the velocities after the collision.

Puck A: $\boldsymbol{v}_{Af} = (0, u_A)$,

Puck B: $\boldsymbol{v}_{Bf} = (u_B, 0)$.

We know the masses, so we can know momenta.

## 3, 4: final speeds.

**Q2**. Calculate the total momentum before and after the collision (first express them in symbols), and obtain $u_A$ and $u_B$.

Conservation of momentum reads

$$M_A \boldsymbol{v}_{Ai} + M_B \boldsymbol{v}_{Bi} = M_A \boldsymbol{v}_{Af} + M_B \boldsymbol{v}_{Bf}.$$

That is,

$$2(\sqrt{3}, 1) + 1(0, -6) = (2\sqrt{3}, -4) = 2(0, u_A) + 1(u_B, 0) = (u_B, 2u_A)$$

Therefore, $u_B = 2\sqrt{3}$ and $u_A = -2$. That is, Puck B moves at speed $2\sqrt{3}$ m/s, and Puck A at 2 m/s.

## 5: energy loss

The initial total kinetic energy is

$K_i = \frac{1}{2} M_A v_{Ai}^2 + \frac{1}{2} M_B v_{Bi}^2 = 4 + 18 = 22$ J.

The final total kinetic energy is

$K_f = \frac{1}{2} M_A v_{Af}^2 + \frac{1}{2} M_B v_{Bf}^2 = 4 + 6 = 10$ J.

Therefore, 12 J is lost.

# D8-5 Personal spaceships

This is an excellent (= nontrivial) problem. If you understand all the arguments here, your understanding of momentum is excellent.

**Q0**. What are the key concepts/principles for this problem?

Conservation of momentum $\boldsymbol{P}$ and the concept of impulse $\boldsymbol{I}$.

You must clearly understand relative velocities:
The (relative) velocity of A relative to B is $\boldsymbol{v}_A - \boldsymbol{v}_B$.
You must understand that Newton's second law is the basic principle:

$$\frac{\Delta \boldsymbol{P}}{\Delta t} = \boldsymbol{F} \quad \Rightarrow \quad \Delta \boldsymbol{P} = \boldsymbol{F} \Delta t = \boldsymbol{I}.$$

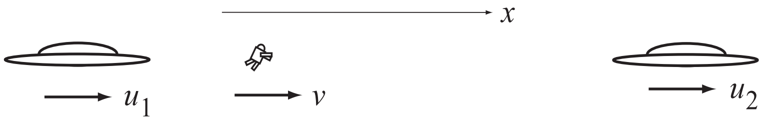
Here, $\boldsymbol{P}$ is the total momentum and $\boldsymbol{F}$ is the total external force.
If there is no net external force, $\Delta \boldsymbol{P} = 0$, i.e., conservation of momentum.

**1**

**Q1**. Is the first spaceship moving relative to distant stars after the astronaut leaves?

Yes.

Let us choose the coordinate direction of the relative motion to be $x$. Let $v$ be the $x$ component of the velocity relative to distant stars of the astronaut, $u_1$ that of the first spaceship and $u_2$ that of the second spaceship henceforth as is shown in the figure below.




**Q2**. Write down what you know (what you are given) about $v$, $u_1$ and $u_2$. Then, identify what you must know.

The second spaceship is still stationary with respect to distant stars: $u_2 = 0$.
The astronaut's velocity relative to that of the first spaceship is 20 m/s: $v - u_1 = 20$ (m/s).

You are asked to obtain the astronaut's speed relative to the second spaceship: $v - u_2$. Thus, you must know $v$, but you know only $v - u_1$; $u_1$ is not given.

**Q3**. What is the second relation between $v$ and $u_1$ you can obtain with the aid of the principle we have already mentioned in **Q0**?

The total momentum of the first spaceship + the astronaut must be conserved. That is,

$$Mu_1 + mv = 0 \quad \Rightarrow 450u_1 + 80v = 0.$$

**Q4**. Solve the two relevant equations obtained above for $v$ (and $u_1$).

$$\begin{aligned} v - u_1 &= 20, \\ 8v + 45u_1 &= 0. \end{aligned}$$

Multiply 45 to the first equation to obtain $45v - 45u_1 = 900$, and then add this to the second equation (add the right-hand sides and left-hand sides, respectively), to obtain

$$53v = 900$$

Therefore, $v = 16.98 \simeq 17.0$ m/s. Since $u_2 = 0$, this must be the answer. By the way, $u_1 = -3$ m/s. Is this sign reasonable?

## 2

**Q5**. Write down what you know (before landing) and what you want (after landing).

Before landing: $v = 17$ m/s, $u_2 = 0$.
After landing, we know nothing, so let us pretend we know the velocity $V$ of the astronaut + the second space ship.

**Q6**. What is the relation between the velocities before and after the astronaut's landing? (What is conserved?)

The total momentum of the astronaut + the second spaceship is conserved. What is it? It is $mv$. Write down the formula for $V$.

$$mv + Mu_2 = mv = (m + M)V,$$

where $u_2 = 0$ has been used.

Now, it should be easy to finish the problem.

$$V = \frac{m}{m + M}v = \frac{8}{53} \times 16.98 = 2.563 \simeq 2.6 \text{ m/s}.$$

**Q7**. [Extra question] Can you confirm that the total momentum of the whole system (that of the two spaceships + the astronaut) is conserved?

The velocities after the landing is over: $u_1 = -3$ m/s, $u_2 = v = V = 2.6$ m/s. Therefore, the final total momentum of the system is

$$Mu_1 + (m + M)V = -450 \times 3.02 + 530 \times 2.56 \simeq 0.$$

This is not exactly zero due to the rounding errors. In symbols, $u_1 = -20m/(m+M)$, $V = 20[M/(m + M)] \times [m/(m + M)]$, so the above sum is exactly zero.

## 3: impulse

**Q8**. What is the relation between the impact $I$ given by the astronaut to the second spaceship and the momentum $\Delta P$ imparted by him to the spaceship?

Identical! $\Delta P = I$.

Calculate $\Delta P$ of the second spaceship.

$$\Delta P = MV - 0 = 450 \times 2.56 = 1150 \text{ N}\cdot\text{s}.$$

Thus, the impulse must be about 1150 N·s.

## 4: impulse to ship I

Since only a portion of the momentum of the astronaut is imparted to the second ship, the magnitude of the impulse to the ship I must be larger than that given to the ship II.