Physics 101 Discussion Week 9 Explanation (2011)

D9-1 Balancing book

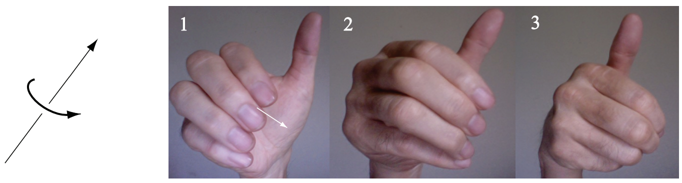
Q0. What are the key points of the problem? You must be able to explain the words you use.

The condition for a rigid object to be in equilibrium is:
(i) The sum of all the forces (the net force) acting on the body is zero: $\sum \boldsymbol{F} = \mathbf{0}$.
This is a vector equation: Remember that X and Y components of the forces must balance separately.
(ii) The sum of the torques around a point (the net torque around a point) is zero: $\sum \boldsymbol{\tau} = \mathbf{0}$.[1]

What is torque?
Look at the figures on p103 (or on the next page of Explanation).

SIGN convention

Any rotation around an axis has a sign. + rotation is illustrated in the following figure. This will be very important in Phys 102.



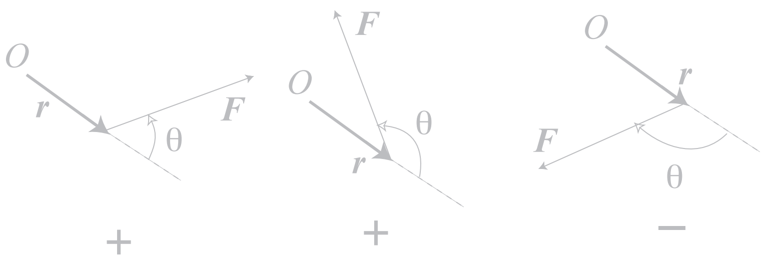
+ rotation around the thumb (right-hand rule)

When all forces and lever arms (the distance to the center of rotation) are confined to the plane of the page, rotations are considered around the normal direction to the sheet (the normal direction upward toward you). Thus, counterclockwise rotation is positive.

Notice that under the condition (i), if (ii) is confirmed at a point, it holds around any point in space.

The sign convention of the torque: counter-clockwise rotation = +. See the figure on the next page (the magnitude of the torque is $Fr \sin\theta$):

[1]Here, 'a point' is any point in the world (as noted below in the main text). Under the condition (i), if (ii) holds around a particular point, then (ii) holds around any point in the universe. Thus, you must choose the point wisely.



1: torque about the left end

The book: $0.2g$ (in N, downward); the arm length $= L/2$. The torque is $-0.1gL$ (clockwise).
Fingers: F_1 (upward); the arm length $= L/4$. The torque is $+F_1L/4$ (counter-clockwise).
Thumb: F_2 (downward); the arm length $= 0$. The torque $= 0$.

2: F_1

Q1. Calculate the total torque around the left end of the book. Then, obtain F_1 from the equillibrium conditon.

The total torque around the left end in equilibrium must satisfy

$$F_1L/4 - 0.1gL = 0,$$

so $F_1 = 0.4g = 3.92$ N.

3: F_2

Q2. This is the condition on the total force. Calculate the net force on the book in equilibrium, and find F_2.

The balance of the forces in the vertical direction is ($F_1 = 0.4g$ is used) (upward is + in the sheet of paper)

$$-F_2 + 0.4g - 0.2g = 0.$$

Therefore, $F_2 = 0.2g = 1.96$ N. 1.96 N downward.

4: additional mass

Q3. [Review] What are the equilibrium conditions for a rigid object to be in equilibrium?

(i) Total force is zero (in our case the y-component of the total force vanishes).
(ii) Total torque around a point vanishes.

Q4. The torque condition may be applied around any point. However, you should choose the point cleverly to have simple equations. What is a convenient point around which we consider the torque balance?

F_1 and F_2 are unknown, so we should choose a point where the arm length to one of the unknown forces is zero. Also we should choose a point around which all the needed arm lengths are easy to calculate.

In our case, without doubt it is the left end. The second best choice is the point where F_1 acts on.

Q5. Calculate the total torque around the left end to obtain F_1.

The total toques around the left end is

$$-0.1gL + F_1L/4 - 0.4gL = 0.$$

Therefore, $F_1 = 2g = 19.6$ N (upward).

Q6. As an exercise let us apply the torque balance condition around the point $L/4$ from the left end. What can you get?

$$(L/4)F_2 - (L/4) \times 0.2g - (3L/4) \times 0.4g = 0,$$

where the middle term is due to the book itself. This gives

$$F_2 - 0.2g - 3 \times 0.4g = 0$$

or $F_2 = 0.2g + 1.2g = 1.4g = 13.7$ N. We have finished the problem. However, applying the force balance condition in the y direction may be easier as you see just below.

Q7. Write down the force balance condition and obtain F_2 (we already know $F_1 = 2g$).

The y component force balance condition is

$$-F_2 + F_1 - 0.6g = -F_2 + 1.4g = 0.$$

Therefore, $F_2 = 1.4g = 13.7$ N. 13.7 N downward.

D9-2 Ladder

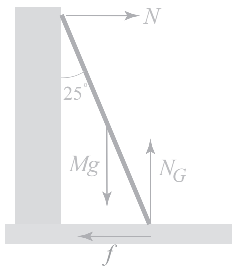
Q0. This is a ladder balance problem. What are the key principles for the problem?

The equilibrium condition for a rigid body consists of two conditions:
(i) The total force (2D vector!) vanishes (i.e., x and y components sum to zero individually).
(ii) The total torque around a point vanishes.
For (ii) you must choose the point wisely.

1: normal force

Q1. In any case, what you should do first is to itemize all the forces. Do this.

At the top end of the ladder: no friction, so the force N is perpendicular to the wall.
At the bottom of the ladder: there is a normal force N_G from the ground and a frictional force f (in the direction toward the wall).
There must be a gravitational force Mg acting at the center of gravity of the ladder.
See the figure below.



Q2. Since we are asked to obtain N, you may have realized that the force balance condition (i) above is not needed, but, in any case, let us write down force balance conditions.

Horizontal direction: $N = f$.
Vertical direction: $Mg = N_G$.
Since we know M, we know N_G, but we can know neither N nor f. We need one more condition. Of course, you know what it is.

Q3. Now, we must use the torque balance condition. Choose the best point around which the condition should be considered, and find N.

There are three candidate points, the top end, the center of gravity of the ladder, and the bottom end. The top end is not good, because we wish to know N. Since we know Mg the center of gravity is not good, either. The bottom end is the only wise choice.

Q4. Around the bottom end, what is the torque τ_N due to the normal force N? Is it positive (counterclockwise) or negative (clockwise)?

Obviously, it is clockwise (negative). The magnitude of the torque is the arm length (L) $\times$ force $N \times$ sin(the angle between the force and the arm $= 90° - 25° = 65°$). Therefore, $\tau_N = -NL \sin 65°$.

Notice that if we measure the angle from the arm to the force direction, it is $-115°$, so the sign of the torque τ_N can be obtained automatically: $\tau_N = NL \sin 115° = -NL \sin 65° = -NL \cos 25°$.

Q5. Around the bottom end, what is the torque τ_G due to the gravitational force Mg? Is it positive (counterclockwise) or negative (clockwise)?

Obviously, it is counterclockwise (positive). The magnitude of the torque is the arm length ($L/2$) $\times$ force $Mg \times$ sin(the angle between the force and the arm $= 25°$). Therefore, $\tau_G = Mg(L/2) \sin 25°$.

Notice that if we measure the angle from the arm to the force direction, it is $155°$, so the sign of the torque τ_G can be obtained automatically: $\tau_G = Mg(L/2) \sin 155° = Mg(L/2) \sin 25°$.

Finish the problem.

The torque balance implies $\tau_N + \tau_G = 0$ or

$$-NL \cos 25° + Mg(L/2) \sin 25° = 0.$$

That is, $N = (Mg/2) \tan 25° = 49 \tan 25° = 22.8$ N.

2: extra weight

The answer should be intuitively obvious. More force is needed at the top end to support the ladder, so N must increase.

Let us do it quantitatively. Let m be the mass of the person and assume that he is x (m) from the bottom of the ladder along the ladder. How does the torque balance equation read now? The following should be obvious:

$$-NL\cos 25^\circ + (Mg(L/2) + mx)\sin 25^\circ = 0.$$

Thus, N must increase with x.

D9-3 Block on pulley

Q0. What is the key principle for the problem?

The problem can be solved in different ways, but using conservation of energy is the simplest. You must be able to write down the kinetic energy K of the rotating object:

$$K = \frac{1}{2}I\omega^2,$$

where I is the moment of inertia. What is it? How to compute it? See p79 and look at the formula sheet.

1: *K*

Q1. Is the mechanical energy conserved?

Yes.

Q2. Then, we have only to compute the total energy at the moment when the calculation of energy is the easiest. When is it? After answering this, finish the problem.

Just before the block starts to fall.

At this moment $K = 0$, and the potential energy $U = mgh$. This is subsequently converted to the kinetic energy of the block and the pulley.

2: speed

Q3. We must write K in terms of the speed v of the block and the rotational speed ω of the pulley. What is the relation between v and ω?

Since there is no slip of the string, v must be equal to the peripheral speed of the pulley that is equal to $R\omega$, so $v = R\omega$.

Q4. You must know the kinetic energy of the block: it is $mv^2/2$. How about that of the pulley?

It must be

$$\frac{1}{2}I\omega^2,$$

where I is the moment of inertia.

What is I for a hoop of radius R and mass M? $I = MR^2$.

Q5. Write down the total kinetic energy in terms of v, M and m. Then, finish the problem.

Since we know $v = R\omega$,

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}MR^2\omega^2 = \frac{1}{2}(m + M)v^2.$$

That is, $v = \sqrt{2K/(m + M)}$

D9-4 Winding Down

Q0. For this problem it is obvious that you need both rotational kinematics and dynamics. Write down key formulas.

Rotational kinematics:

One complete revolution $= 2\pi$ rad.

$\omega = \omega_0 + \alpha t$: you must be able to tell what ω_0, ω and α means.

Or, you must be able to read this formula in English.[2]

$\theta = \theta_0 + \omega_0 t + (1/2)\alpha t^2$. Read the formula in plain English.

Rotational dynamics:

Newton's second law: $I\boldsymbol{\alpha} = \sum \boldsymbol{\tau}$. Read this in English.[3]

You must clearly recognize the one-to-one correspondence to the linear dynamics: $M\boldsymbol{a} = \sum \boldsymbol{F}$,

that is, $M \leftrightarrow I$, $\boldsymbol{a} \leftrightarrow \boldsymbol{\alpha}$, $\boldsymbol{F} \leftrightarrow \boldsymbol{\tau}$.

Kinetic energy: $(1/2)I\omega^2$ (pay attention to the correspondence to $(1/2)Mv^2$).

1: ω

[2] If the angular speed is ω_0 at time $t = 0$ (i.e., initially) and if the rotation is accelerated with a constant angular acceleration α, then at time t the angular speed is ω.

[3] Angular acceleration times moment of inertia is equal to the sum of all the torques acting on the body.

The initial angular speed $\omega_0 = 400$ rpm $= x$ rad/s or

$$x \times \frac{\text{rad}}{\text{s}} = 400 \times \frac{\text{revolution}}{\text{min}}.$$

Therefore,

$$\begin{aligned} x &= 400 \times \frac{\text{revolution}}{\text{min}} \times \frac{\text{s}}{\text{rad}} = 400 \times \frac{\text{revolution}}{\text{rad}} \frac{\text{s}}{\text{min}} \\ &= 400 \times \frac{2\pi \text{ rad}}{\text{rad}} \frac{\text{s}}{60 \text{ s}} = 400 \times \frac{2\pi}{60} = \frac{40}{3}\pi = 41.9 \end{aligned}$$

That is, $\omega_0 = 41.9$ rad/s.

2: I

$I = MR^2 = 2 \times 0.6^2 = 18/25 = 0.72$ kg·m^2.

3: $\boldsymbol{K}$

$K = (1/2)I\omega^2 = (1/2)0.72 \times 41.9^2 = 632$ J.

4: deceleration

Q1. There are at least two ways to solve this problem. What do you think they are?

(1) Use Newton's second law or τ-α relation. To this end we need α.
(2) Use the work/energy theorem. The work done by the torque must cancel the total kinetic energy just computed above.

Let us try both.

Q2. (2) is easier, actually. What is the total work W done by the torque τ due to the force F on the hoop? Write down W in terms of τ and the total angular displacement $\Delta\theta$ (= the total angular 'distance' the hoop rotates by the time it comes to a halt). How is it related to the formula of the work done by the force F and the displacement L?

$$W = \tau\Delta\theta$$

Since $\tau = FR$ and since $R\Delta\theta = L$ is the actual distance the rim of the hoop runs by the time it comes to a halt, the above formula is nothing but $W = (FR)\Delta\theta = F(R\Delta\theta) = FL$.

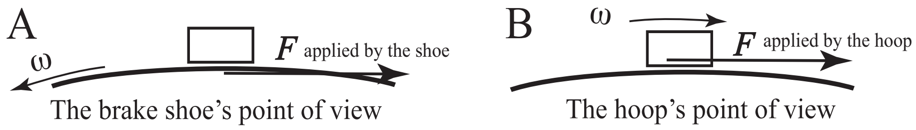
Q3. We need the angular displacement $\Delta\theta$. What is it? Then, obtain W.

$\Delta\theta = 10$ revolutions $= 20\pi$. Therefore, the *hoop does* the work $+20\pi\tau$. That is, the hoop loses this much of energy.

Notes on the sign of work: [This is a bit complicated, so read very slowly, checking each step.]

When you compute work, you must clearly designate who is doing work on whom.

In this problem something is touching on the rim of the rotating hoop to decelerate it just as in the case of the brake of a car. The object touching on the rotating part is called the brake shoe. So let us call this touching something denoted by a block in the following figure a brake shoe.



A: From the brake shoe's point of view, the frictional force F the shoe is applying on the rim of the hoop and the moving direction of the hoop are opposite, so the work done *by the shoe* on the hoop (the work gained by the hoop) is **negative**. That is, the shoe is gaining energy (a negative expenditure is an income!). This energy is of course supplied by the hoop which does positive work (giving away energy) as we see just below.

B: From the hoop's point of view, the brake shoe is running (think the relative velocity) in the same direction of the frictional force F acting on the shoe. That is, the direction of the force and the motion are the same, so W *done by the hoop* to the brake shoe is **positive**. That is, the hoop is losing energy to the brake shoe.

This may sound complicated, but it should be intuitively obvious who is losing and who is gaining (mechanical) energy. Then, look at the signs of the work: the work *done by* the hoop *(to) the brake shoe is* $W > 0$*; the work* done by *the brake shoe (*to) the hoop is $W < 0$.

These signs are easy to understand if you calculate the energy the hoop has at the start E_i and the energy it has at the end, E_f. The work/energy theorem tells us that W(done onto the hoop) $= E_f - E_i$. Here, for the hoop $E_f = 0$, and E_i is positive, so $W = -E_i < 0$. This says that the work done to the hoop is negative, which is the same as saying the hoop does (positive) work to the 'shoe,' i.e., gives energy away.

Let us finish the calculation of τ.

We apply the work/energy theorem: The energy change $\Delta E = W$: work done to the system. In our case $\Delta E = -632$ J. The work done to the system (the hoop) by the brake shoe is $-20\pi\tau$:

$$-20\pi\tau = -632$$

That is, $\tau = 10.1$ N·m.

Now, let us use (1). This approach is not recommended, but it is a good exercise.

Q4. Let us write the angular acceleration as α (rad/s^2). What is the relation between the torque τ and α?

$I\alpha = MR^2\alpha = \tau$

Q5. We need α. Using a kinematic relation, obtain it.

We use

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta.$$

Here, $\omega_0 = 41.9$ rad/s, $\omega = 0$, and $\Delta\theta = 20\pi$, so $\alpha = \omega_0^2/2\Delta\theta = 41.9^2/40\pi = 13.97$ rad/s^2.

Now, we can finish the problem.

$\tau = I\alpha = MR^2\alpha = 2 \times 0.6^2 \times 13.97 = 10.06$ N·s.

Needless to say, the answer agrees with the one obtained by (1) (up to the roundoff error).