Physics 101 Discussion Week 12 Explanation (2011)

D12-1 Horizontal oscillation

Q0. This is obviously about a harmonic oscillator. Can you write down Newton's second law in the x (horizontal) direction? Let x be the displacement from the equilibrium state (no force no velocity condition), and a be the acceleration in the x direction.

Now, we must respect the direction of the force, so Hooke's law for the x-component of the force due to the spring reads $F = -kx$. Thus,[1]

$$Ma = -kx.$$

You'd better recognize this as

$$a = -\omega^2 x,$$

where ω is the angular frequency of the oscillation. If you find a is proportional to x with a negative proportionality constant, the motion is always a simple harmonic oscillation.

Q1. What is the most important characteristic feature of simple harmonic oscillation?

It is a motion obtained by projecting a constant angular speed circular motion onto a line (say, the x-axis) as illustrated below (or on p114). The radius A of the circle corresponds to the amplitude, and the constant angular speed ω of the circular motion corresponds to the angular frequency of the oscillation.

Notice that this projection relation works even for acceleration: the centripetal acceleration of the circular motion projected on the same line above is the acceleration a of the oscillating mass.

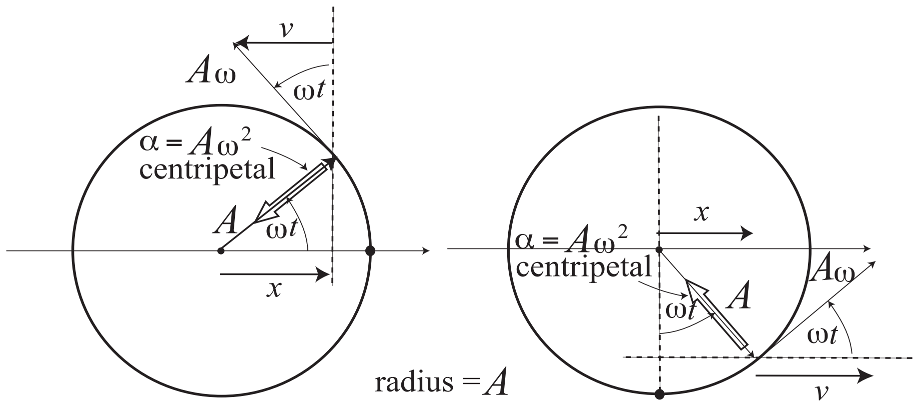
[1]If you know calculus, this reads

$$\frac{d^2x}{dt^2} = -\omega^2 x,$$

and its general solution is $x(t) = A\sin\omega t + B\cos\omega t$ or $x(t) = A\cos(\omega t + \theta)$.

Understand the following table and the illustration below:

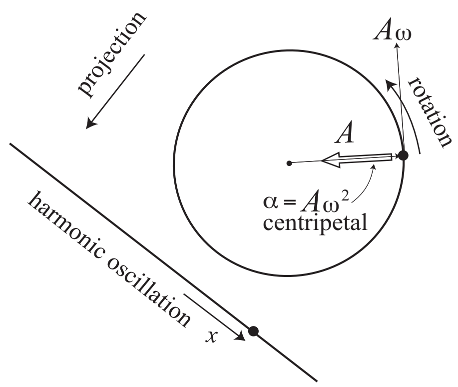
start from $x = A$ at $t = 0$ with speed 0	start from $x = 0$ at $t = 0$ with speed $v(0) = A\omega$
$x(t) = [A]\cos(\omega t)$ $v(t) = -[A\omega]\sin(\omega t)$ $a(t) = -[A\omega^2]\cos(\omega t)$	$x(t) = [A]\sin(\omega t)$ $v(t) = [A\omega]\cos(\omega t)$ $a(t) = -[A\omega^2]\sin(\omega t)$



Left: The case starting at $x = A$ with speed 0;
Right: The case starting at $x = 0$ with the max speed $v = A\omega$.
In both cases, the dot denotes the starting point.

Clearly recognize that

the maximum displacement A = the amplitude = the radius of the circle,
the maximum speed ωA = the tangential speed of the circular motion,
the maximum acceleration $\omega^2 A$ = the centripetal acceleration of the circular motion.



The dot is the starting point.

1: *A*

Q2. Can you imagine what happens when the force is removed (without giving any speed to the block)?

The spring 'wishes to' return to its original length with $x = 0$, so the block should start to move to the right. Then, due to inertia, it keeps moving to the right, stretching the spring. When the spring is stretched by the same amount of displacement from the $x = 0$ position, the mass cannot move further to the right. Remember, this must be true due to conservation of energy. Thus, the magnitude of the initial displacement must be the amplitude of the oscillation.

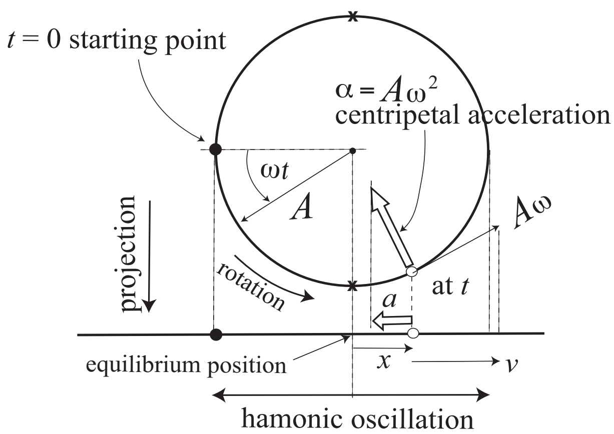
Q3. Find the amplitude.

If the force F is removed, then the spring cannot become shorter, so the magnitude of the displacement of the mass from its equilibrium point due to F must be the amplitude as explained in the answer to **Q1**.

$$|F| = k|x|, \text{ so } x = F/k = 2.5/20 = 0.125 \text{ m}.$$

Therefore, the amplitude must be $A = 0.125$ m.

Q4. Can you actually relate the oscillating motion of the block with the corresponding circular motion in this particular case just as discussed generally above?



2: f

Q5. What is the relation between the frequency f and the angular frequency ω?

$\omega = 2\pi f$.

Q6. Since we have written down Newton's second law, you can read ω^2 off. What is the formula for ω? Then, finish **2**.

$\omega = \sqrt{k/M}$.

3: a

Since we know the relation between a and x (Newton's second law applied to a simple oscillator), the answer should be immediate: $a = A\omega^2$. We know $A = F/k$, and $\omega^2 = k/M$. Then, what is $A\omega^2$? Do you see that $a = A\omega^2$ is a very natural relation?

$a = A\omega^2 = (F/k)(k/M) = F/M.$

From this you should have realized that a can be obtained simply by Newton's second law.

When the force F is applied, the block is stationary, because it is pulled in the opposite direction by the force due to the spring. Therefore, immediately after the force F is removed, only the force due to the spring (its magnitude is F) acts on the block. Therefore, Newton's second law tells us that the equation of motion is $Ma = F$. Hence, $a = F/M$.

5: max speed time

Q7. What is the period T of this oscillator in terms of f or ω?

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/M}} = 2\pi\sqrt{\frac{M}{k}}.$$

Q8. Mark the location(s) on the circle where the oscillator speed is the largest.

Actually, it is already marked with x.

Q9. What is the answer in terms of T?

$T/4$. This implies $2\pi/(4\omega) = \pi/2\omega = (\pi/2)\sqrt{M/k}$.
If this is not obvious, plot out the position and velocity functions we supplied you with on page 2 of this Explanation.

D12-2 Vertical Oscillation

Q0. This is another simple oscillator problem. What is the key point of the vertical oscillation?

The important point is: the displacement due to gravity simply defines the new origin of the oscillation (the new equilibrium point of the oscillator), so you can ignore it.[2] Now, you must realize that D12-2 is actually the same problem as D12-1.

1: A

Q1. What is the displacement x due to F from the natural hanging stationary state?

Just $F = kx$, so $F/k = 23/59 = 0.39$ m.

Therefore, you know the answer just as in **D12-1**: $A = 0.39$ m.

2: max speed, max energy

Q2. Write down the max speed in terms of A, m and k.

We have derived $\omega = \sqrt{k/m}$.
The max speed is $A\omega = A\sqrt{k/m} = 0.39 \times \sqrt{59/2.1} = 2.067$ m/s.

Now, let us use the conservation of energy to obtain the max speed.
Q3. Write down the conservation of energy in terms of $v(t)$ and $x(t)$.

$$E = \frac{1}{2}mv^2(t) + \frac{1}{2}kx^2(t).$$

Q4. When is the maximum kinetic energy realized?

Obviously, at the time when the oscillator is the fastest, when the potential energy is zero.

Q5. Calculate the total energy, or calculate the max speed.

[2]If you are skeptical, read the following:
Let the vertical coordinate be y measured from the natural end of the spring. The equation of motion reads $ma = -ky + F - mg$. Initially, the system is stationary, so $y_i = (F - mg)/k$ is the displacement from its natural state. When the force is removed, the equation of motion is $ma = -ky - mg$, so the oscillation is around $y_e = -mg/k$. Relative to this, the initial displacement is $y_i - y_e = F/k$ (i.e., the initial position is F/k higher than its equilibrium point). This must be the amplitude.

The maximum kinetic energy is, according to $mv^2/2 = mA^2\omega^2/2 = kA^2/2$, 4.487 J. This must be equal to the max potential energy (relative to the equilibrium point) $F^2/2k = 4.48$ J (more accurate).

D12-3 Pendulum and peg

1: lowest

Q1. What is the period T of this pendulum (without the peg)?

$\omega = \sqrt{g/L} = \sqrt{9.8} = 3.13$ rad/s. Therefore, the period $T = 2\pi/\omega = 2.0$ sec.

Q2. In terms of T, what is the required time?

$T/4$. So the answer = 0.5 sec.

2: max height

Q3. Is there any conserved quantity?

Mechanical energy is conserved, so, in particular, the potential energy at the highest points must be the same.

Q4. What does this mean for the height of the ball after touching the peg?

The same as the starting height illustrated in A.

Q5. Calculate the height from the lowest point.

$1 \times (1 - \cos 5°) = 0.038$ m. That is, 3.8 cm. (Here, the outside '1' stands for the length 1 m.)

3: return time

Q6. What is the period T of the pendulum of length 2/3 m?

$T = 2\pi\sqrt{L/g} = 1.64$ sec.

Q7. How long does it take for the pendulum to return to the bottom after touching the peg?

This must be half the period of the pendulum with length 2/3 m, so $1.64/2 = 0.82$ sec.

Q8. So, the total time needed to return to A is ...?

$(2.0/2 =)$ 1. sec + 0.82 sec = 1.82 sec.

D12-4 Pendulum in elevator

1: a

Q0. Suppose the elevator is 'perfect' and you cannot feel any noise or vibration nor see anything outside. Then, you can imagine that you are on a different planet with a different acceleration of gravity g' (equivalence principle $\ast$). Let the mass of the pendulum be m (= 3 kg), and the length be L (= 2 m). Write down its period T in terms of these symbols.

$\ast$ Why is this true? If the elevator is accelerating upward you would feel heavier. If the elevator were allowed to fall freely you would feel no gravity. In the sealed elevator you could never know anything about the acceleration causing it. You would feel gravity acting differently on you. Don't scoff at this; Einstein's general relativity has come from this sort of intuition.

Formula Sheet tells us that $T = 2\pi\sqrt{L/g'}$.

Q1. Obtain g' symbolically. Then, using $L = 2$ m and $T = 3$ s, compute g' numerically.

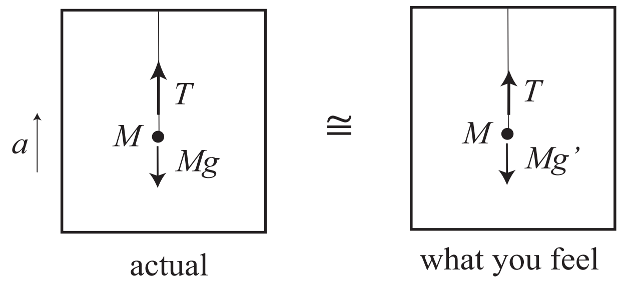
$g' = 4\pi^2 L/T^2 = 8.8$ m/s^2.

Q2. Actually, we are in an accelerating elevator with acceleration a (+ is chosen to be upward) on the earth. To relate a to g' let us study the tension in the string of the pendulum at the resting position with the aid of Newton's second law. See the figure.

Newton's second law applied to the situation left in the above illustration is (the positive direction is upward)

$$Ma = T - Mg \quad \Rightarrow \quad T = M(a + g).$$

However, we feel this T is due to an effective gravity (the situation illustrated in the right above), so $T = Mg'$. That is $g' = a + g$.



Thus, we can find a.

$$a = g' - g = 8.8 - 9.8 = -1.0 \text{ m/s}^2.$$

Q3. Can you tell the moving direction of the elevator?

Moving direction is determined by its velocity, not by its rate of change. Perhaps, the elevator is going down and speeding up, or going up and is reducing its speed. $a < 0$ implies downward speed is increasing, or upward speed is decreasing.

2: L

Q4. What can you change in order to change the period?

We can alter m or L, but we know m is unrelated.

Q5. Then, we must change L to L' to keep the period constant. Find the relation between g, g', L and L'. Then, compute L'.

Since T is proportional to $\sqrt{L/g}$, maintaining the period implies $L/g' = L'/g$ or $L' = Lg/g' = 2.2$ m.

D12-5 Pendulum energy

1: max K

Q0. What is the key fact/principle of this question?

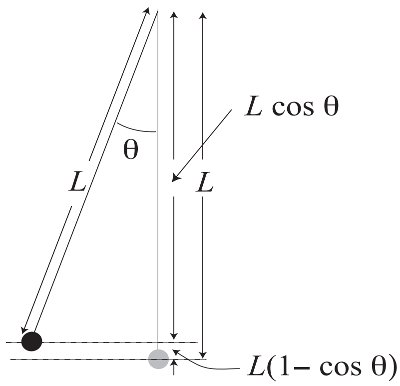
The total mechanical energy = kinetic energy + potential energy is conserved.

Q1. Using the tangential speed v and the height h at the same time relative to the lowest position, write down the total kinetic energy.

$$E = \frac{1}{2}mv^2 + mgh.$$

Q2. What is the maximum value of h?

It is the initial height relative to the lowest point: $h_{max} = L - L\cos 8°$. See the illustration on the next page.



calculation of the relative height

Q3. In terms of h_{max} write E. Then, numerically compute E.

$$E = mgh_{max} = mgL(1 - \cos 8°) = 2 \times 9.8 \times 1.5(1 - \cos 8°) = 0.286 \text{ J}.$$

Let us finish the problem **1**. Obviously, $K_{max} = 0.28$ J.

2: tension

Q4. Can you write down the radial equation of motion for the mass when it is at the bottom in terms of m, v, g, L and the tension T in the string?

The radial acceleration is v^2/L whose direction is upward (to the center of the rotation!). Therefore, Newton's second law in the vertical direction reads (upward is positive,because it happens to be the direction to the center)

$$m\frac{v^2}{L} = T - mg$$

Q5. What is mv^2 in the present context? Or, what is the relation between mv^2 and the total mechanical energy E?

Since v at the lowest point is the maximum speed of the pendulum, it is just $2E$.

Q6. Find T.

From the answer to **Q4**, we get $T = mv^2/L + mg$, but $mv^2 = 2E$, so $T = 2E/L + mg = 2 \times 0.28/1.5 + 2 \times 9.8 = 19.98$ N.

3: doubling mass

Q7. Write down the formula for T above with the aid of the explicit formula for E obtained in **Q3**.

$T = mg + 2mg(1 - \cos\theta)$.

Q8. So, what is your answer?

Exactly doubled.