Physics 101 Discussion Week 3 Explanation (2011)

D3-1. Velocity and Acceleration

A.

1: average velocity.

Q1. What is the definition of the average velocity $\boldsymbol{v}$?

> Let $\Delta \boldsymbol{r}(t)$ be the total displacement vector in time t. Then, the average velocity $\boldsymbol{v}$ during this time span is $\boldsymbol{v} = \Delta \boldsymbol{r}(t)/t$.

Let us finish the problem:

> Our world in this problem is 1D, so the x coordinate is the displacement vector. After $t = 8$ s, $x \simeq 3.5$ m. Therefore, the average velocity is $3.5/8 = 0.44$ m/s.
>
> The average velocity vanishes when the displacement vanishes. Therefore, around $t = 8.5$ s this happens.

2: velocity.

Q2. What is the velocity at a certain time t geometrically?

> It is the slope in the 1D world.

At $t = 1$ s, the slope is approximately $10/1.5 = 6.7$ m/s.
At $t = 5$ s, the slope is approximately $-5/3 = -1.67$ m/s.

3: zero velocity moment.

Q3. What is the condition for the velocity to vanish geometrically?

> There is no slope to the graph of x vs. t.

This happens at around 4 s.

4: max speed.

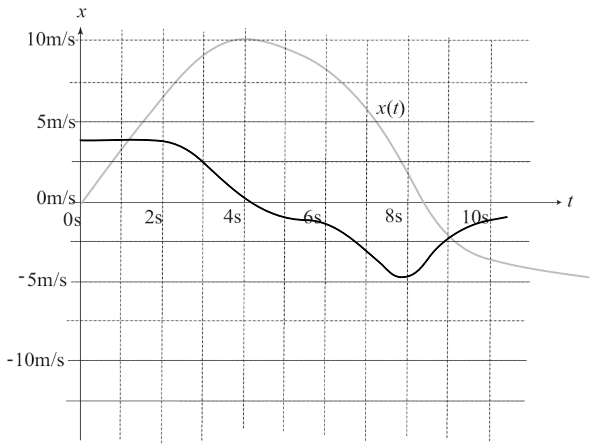
Q4. When is the speed maximum?

> When the magnitude of the slope is the largest; that is, at the steepest moment.

This seems to occur at around 8 s.

5: sketch v.

A crude sketch is as follows (not very quantitative).



B

1: net force

Q0. What is the relation between the net force and the slope of the given graph?

> The slope is the acceleration, so Newton's second law tells us that the slope times the mass is the net force acting on the body.

The net force vanishes when the slope vanishes or when the velocity changes its sign. This occurs at around 3 s and around 6.5 s.

2: max force.

The answer to **Q0** implies that this happens when the graph of the velocity is the steepest. This occurs at around 4.9 s (just before 5 s).

D3-2. Graphing motion

Q1. The speed of the car is given in km/hr, but you are asked to answer the problem using m and s. Find the speed of the car in m/s.

To this end, write down the equality, pretending that you know the answer that the speed is v m/s. Recall the method recommended in Week 1.

$$90\,\frac{\text{km}}{\text{hr}} = v\,\frac{\text{m}}{\text{s}}.$$

Q2. Unit symbols are just algebraic symbols. Solve the equation for v and obtain the numerical answer.

$$\begin{aligned} v &= 90 \times \frac{\text{km}}{\text{hr}} \times \frac{\text{s}}{\text{m}} = 90 \times \frac{\text{km}}{\text{m}} \times \frac{\text{s}}{\text{hr}} \\ &= 90 \times \frac{10^3\ \text{m}}{\text{m}} \times \frac{\text{s}}{3600\ \text{s}} = 90 \times 10^3/3600 = 90/3.6 = 25. \end{aligned}$$

Graph of acceleration a: Do not forget the reaction time of 0.2 s.

Graph of velocity v:

Q3. Suppose the initial velocity in the x-direction is v_0 and the uniform acceleration (in the same direction) is a (+ is accelerating, − is decelerating). After time τ (tau), what is the velocity $v(\tau)$ in the x-direction?

$v(\tau) = v_0 + a\tau.$

Q4. What is v_0 and a numerically (with units)?

$v_0 = 25$ m/s and $a = -2.5$ m/s^2. Do not forget the negative sign, because it is a deceleration (an acceleration in the opposite direction of the velocity).

Q5. How many seconds after she started to apply the brakes does the car stop?

Since τ is the time measured from the point of starting to apply the brakes,

$$v(\tau) = 25 - 2.5\tau.$$

$v = 0$ if $\tau = 10$ s. That is, the answer is 10 seconds later. When you graph v, you are asked to plot $v(t)$. Do not forget the dead time of 0.2 sec.

Graph of displacement x:

Q6. Write down the formula for the displacement $x(\tau)$ τ seconds after the brakes are applied. Use a and v_0.

$$x(\tau) = v_0\tau + \frac{1}{2}a\tau^2.$$

Q7. How far did the car travel (or what is its displacement) while the brakes were applied?

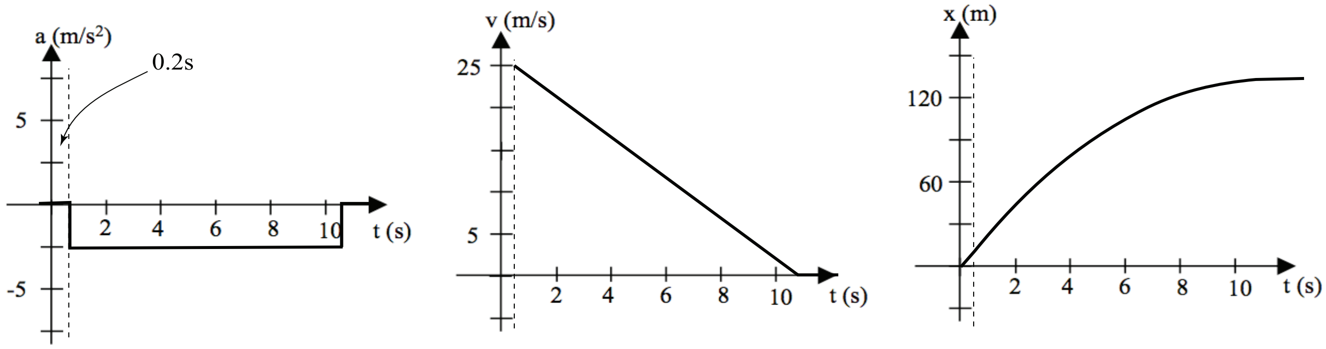
We know the car comes to a halt in 10 seconds, $v_0 = 25$ m/s and $a = -2.5$ m/s^2.

$$x(10) = 25 \times 10 - \frac{1}{2}2.5 \times 10^2 = 250 - 125 = 125.$$

125 m.

Q8. The answer in Q7 does not take into account the reaction time, during which the car is of course moving at 25 m/s. You must add this distance to the above answer to obtain the total displacement. Thus, the total displacement is?

$125 + 0.2 \times 25 = 130$ m. To draw the graph of $x(t)$, you must add the constant velocity motion during the initial 0.2 s to the graph of the displacement discussed in Q7.



D3-3. Graph and area
O
1: displacement as area.

Let us look at the first 10 seconds. The speed is constant, and 10 m/s, so $10 \times 10 = 100$ m is the displacement during the first 10 seconds. Notice that this is the area of the rectangle below the graph up to time $t = 10$ s.

During the time 10 s and 20 s, the average velocity is obviously 15 m/s, so the displacement during this time period must be $15 \times 10 = 150$ m, which is again the area below the graph between 10 and 20 seconds.

We can apply the above logics to the remaining time period, so the total displacement must be the area below the graph: $100 + 150 + 200 + 100 = 550$ m.

II A.

1: square.

The square is 5 m/s $\times$ 5 s, so it corresponds to 25 m displacement.

2: $x(10)$.

As discussed already, we have only to calculate the area below the graph to obtain the displacement: $25 \times 3 = 75$ m. The initial position was $x(0) = 5$ m, so $x(10) = 5 + 75 = 80$ m.

3: $x(30)$.

We have only to compute the area below the graph up to $t = 30$. An easy way is to count the number of squares below the graph: 13 squares, so $13 \times 25 = 325$ m. Therefore, the average velocity is $325/30 = 10.83$ m/s.

4: $t > 30$

Now the velocity is negative, so the total displacement should diminish. The signed area should be interpreted as the displacement.

II B.

The total displacement up to time $t = 30$ s is: 4.5 square ($= 4.5 \times 25 = 112.5$ m) + negative 2.5 squares, so the net displacement is $4.5 - 2.5 = 2$ squares $= 50$ m.

The total displacement up to time $t = 40$ s is: 4.5 square + negative 6.5 squares < 0, so there must have been a time when the particle returned to the origin.

D3-4. Thrown ball

1: Maximum height

Q0. What is the problem about, kinematics, dynamics, or something else? What are the useful formulas in the formula sheet?

> The kinematics of a 1 dimensional constant acceleration motion. Relevant formulas in the formula sheets are:
> $v = v_0 + at$, $x = x_0 + v_0 t + (1/2)at^2$, and $v^2 = v_0^2 + 2a\Delta x$.

There at at least two different ways to solve the problem. Let us first solve the problem step by step, studying the actual motion of the ball.

Q1. Suppose the initial height is h, and the initial upward speed (the y-component of the initial velocity) is v_0, what is its y coordinate at time t? Use g (as a number it is positive and +9.8 m/s) for the acceleration of gravity.

> This is essentially the formula discussed in **Q6** of **D3-2** (or the second equation listed in **Q0** just above):
>
> $$y(t) = h + v_0 t - \frac{1}{2}gt^2. \tag{1}$$
>
> Here, the acceleration of gravity is in the negative y direction, so do not forget the minus sign (because g is considered to be a positive number 9.8).

Q2. To use the formula above to obtain the highest point, we need the time when the ball reaches the highest point. Let us find the special time when the ball is at the highest point.

When the ball is at the highest point,? What is the y-component of its velocity at that time?

> Zero, because it is changing the direction of motion from upward to downward. This is the time when the ball reaches the highest point.

Q3. To find the time numerically in **Q2** we need the formula for the y-component of the velocity of the ball. Express the velocity $v(t)$ at time t in terms of v_0 and g. Then, use appropriate numbers and find the time when the ball reaches the highest point.

> $$v(t) = v_0 - gt = 10 - 9.8t. \tag{2}$$
>
> Therefore, at $t = v_0/g = 10/9.8 = 1.02$ sec the ball reaches the highest point.

Q4. Let us finish **1** by obtaining the y position at $t = 1.02$ s: $y(1.02)$.

With the aid of (1):

$$y(t) = h + v_0 t - \frac{1}{2} g t^2 = 100 + 10t - 4.9t^2. \tag{3}$$

Setting $t = 1.02$, $y(1.02) = 100 + 10.2 - 5.1 = 105.1$m.

There is a way to obtain h_{max} at once with the aid of the third equation (almost an incantation) listed in **Q0**:

$$v^2 = v_0^2 + 2a\Delta x.$$

Q5. How do you identify the quantities v, v_0, and Δx?

The initial velocity is $v_0 = 10$ m/s. The final velocity is $v = 0$ (because the ball is at the highest point). $a = -g = -9.8$ m/s^2, and $\Delta x = h_{max} - h$.

Q6. Why don't you obtain h_{max} with one step?

We get

$$h_{max} - h = v_0^2/2g,$$

so $h_{max} = 100 + 100/19.6 = 100 + 5.1 = 105.1$ m.

2: speed at the ground v_G

There are several ways to solve this question.

(I) First is to rely on a sort of incantation:

Q1. What quantities do we know? The initial velocity v_0, the acceleration $-g$, and the initial height h. Can you find a formula in the formula sheet that utilizes these quantities?

$$v^2 = v_0^2 + 2a\Delta x.$$

We must adapt this formula to our case:

Q2. Adapt the formula to our case and find v_G. The choice of Δx may be a bit tricky.

$a = -g$ and $\Delta x = -h$ (watch out for the signs);

$$v_G^2 = v_0^2 + 2gh.$$

Notice that it does not matter whether you throw the ball upward or downward as long as the initial speed is the same. Thus, $v_G = \sqrt{10^2 + 1960} = 45.4$ m/s.

(II) Using the same formula, but perhaps a bit less tricky:

Q3. The final velocity when it hits the ground will be the same as that of a ball dropped with zero initial velocity from the highest point ($H = 105.1$ m). That is, we have only to study the free-falling[1] ball from the highest point without any initial velocity. Adapt the above formula to the present case using v_G, g and H. Then, obtain the numerical answer.

$$v_G^2 = 2gH.$$

Thus, $v_G = \sqrt{2gH} = \sqrt{2 \times 9.8 \times 105.1} = 45.4$ m/s.

Remark If you learn the conservation of energy, the incantation becomes a very natural formula that does not even require memorization.

(III) Or, we can 'honestly' solve the problem: If we know when (at time t_G) the ball reaches the ground, we can use $v(t)$ to obtain $v_G = v(t_G)$.

Q4. We need the time t_G when the ball reaches the ground $y = 0$. Obtain this time. Then, obtain $v_G = v(t_G)$

From (3) $100 + 10t - 4.9t^2 = 0$. Solving this, we get $t = 5.65$ sec (the other root is negative). The velocity at this time is (see (2))

$$v_G = 10 - 9.8 \times 5.65 = -45.4.$$

That is, 45.4m/s downward.

3: The total time in the air

If you used **III** above, you know the answer already: 5.65 sec. However, you may wish to use a more elementary method:

Q5. We know the time needed for the ball to reach the highest point from **Q3** for **1**. Therefore, to answer the question we have only to find the time t_G required for the ball to fall from the highest point to the ground. One way is to use the knowledge of the speed v_G at the ground. Find t_G following this idea.

The motion is a free fall without any initial speed, so (this is the relation between magnitudes, so you may forget the signs.)

$$v_G = gt_G.$$

That is, $t_G = v_G/g = 45.4/9.8 = 4.62$ s. This + the time needed to go up to the highest point 1.02 sec must be the answer: 5.64 s.

[1]Free falling means falling only with the effect of gravity. The initial speed may not be zero. That is, a ball thrown vertically down ward at a certain initial speed still experiences a free fall.

Q6. Another approach is to compute honestly the time t_G needed for a free falling object without any initial speed to reach the ground. Let us write the initial height as H. Write down the formula for the height $y(t)$ at time t. Then, use $y(t_G) = 0$ to obtain t_G in terms of H and g.

$$y(t) = H - \frac{1}{2}gt^2.$$

Therefore, $t_G = \sqrt{2Hg}$.

Since we know $H = 101.5$ m, we get $t_G = \sqrt{2 \times 101.5 \times 9.8} = 4.62$ sec. Adding the time to climb up to H, we get the answer.

4: throwing down

1 is too trivial, since the ball starts at the highest point.
3 must be obvious: smaller.
2 is the only meaningful question, but we already discussed the answer in **Q2**.
No change!