Physics 101: Lecture 15
Rolling Objects

- Today's lecture will cover Textbook Chapter 8.5-8.7

Overview

- Review
  - $K_{rotation} = \frac{1}{2} I \omega^2$
  - Torque = Force that causes rotation

    $\tau = F r \sin\theta$
  - Equilibrium

    $\Sigma F = 0$

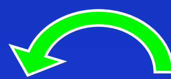
    $\Sigma \tau = 0$
- Today
  - $\Sigma \tau = I \alpha$
  - Energy conservation revisited

Linear and Angular

	Linear	Angular
Displacement	x	θ
Velocity	v	ω
Acceleration	a	α
Inertia	m	I
KE	½ m v2	$\frac{1}{2} I \omega^2$
N2L	F = ma	$\tau = I\alpha$
Momentum	p = mv	

Today

Rotational Form Newton's 2nd Law

- $\Sigma \tau = I \alpha$
  - Torque is amount of twist provide by a force
    - » Signs: positive = CCW 
  - Moment of Inertial like mass. Large I means hard to start or stop from spinning.
- Problems Solved Like N2L
  - Draw FBD
  - Write N2L

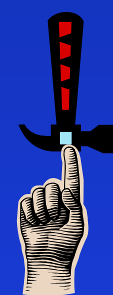
The Hammer!

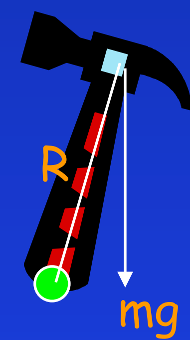
You want to balance a hammer on the tip of your finger, which way is easier

A) Head up

B) Head down

C) Same





Example: Falling weight & pulley

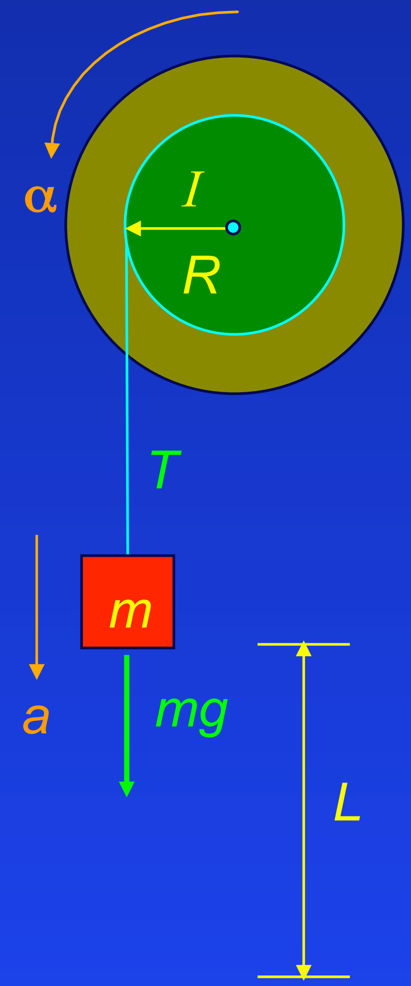
A mass m is hung by a string that is wrapped around a pulley of radius R attached to a heavy flywheel. The moment of inertia of the pulley + flywheel is I. The string does not slip on the pulley. Starting at rest, how long does it take for the mass to fall a distance L.

(no numbers $\Rightarrow$ algebra)

What method should we use to solve this problem

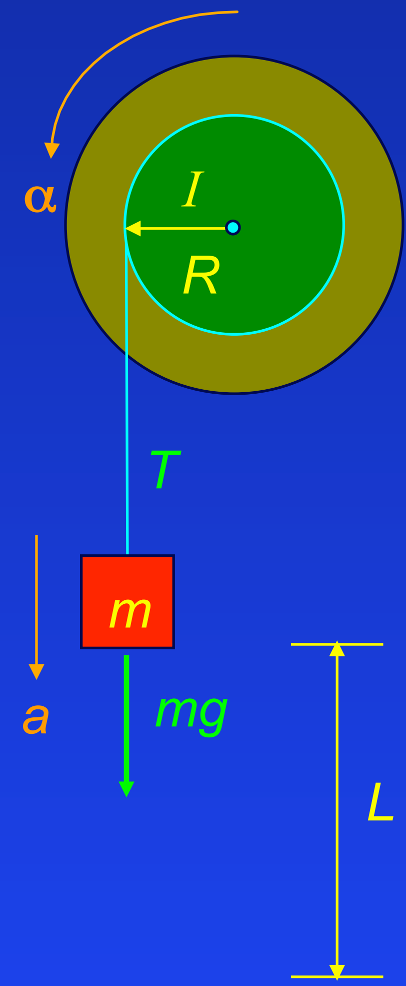
A) Conservation of Energy (including rotational)

B) $\Sigma\tau = I\alpha$ and then use kinematics



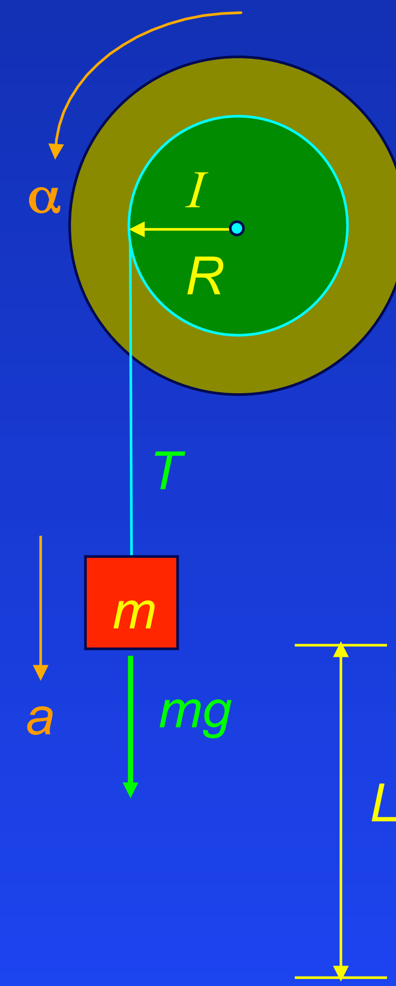
Falling weight & pulley...

- For the hanging mass use $\Sigma F = ma$
- For the flywheel use $\boldsymbol{\Sigma\tau} = I\alpha$
- Realize that $a = \alpha R$
- Now solve for a using the above equations.

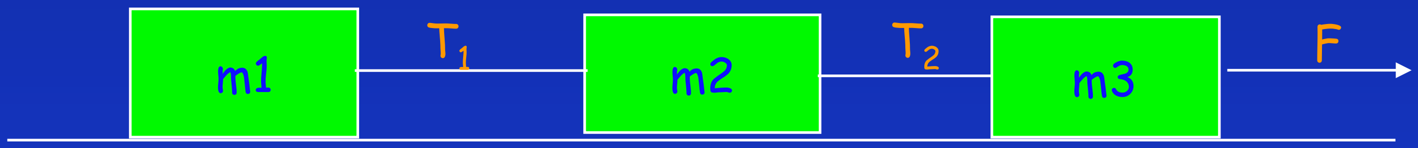


Falling weight & pulley...

- Using 1-D kinematics we can solve for the time required for the weight to fall a distance L:



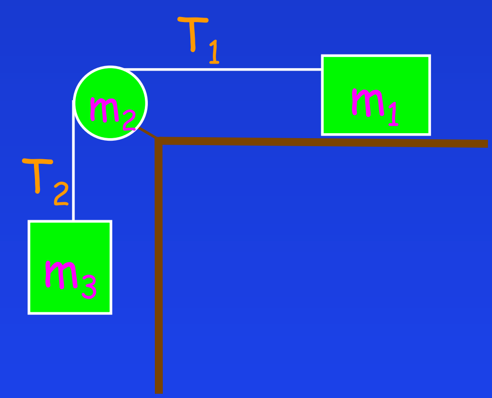
Tension.....



Compare the tensions T_1 and T_2 as the blocks are accelerated to the right by the force F.

A) $T_1 < T_2$ B) $T_1 = T_2$ C) $T_1 > T_2$

$T_1 < T_2$ since $T_2 - T_1 = m_2 a$. It takes force to accelerate block 2.

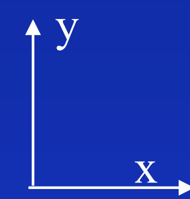


Compare the tensions T_1 and T_2 as block 3 falls

A) $T_1 < T_2$ B) $T_1 = T_2$ C) $T_1 > T_2$

$T_2 > T_1$ since $RT_2 - RT_1 = I_2 \alpha$. It takes force (torque) to accelerate the pulley.

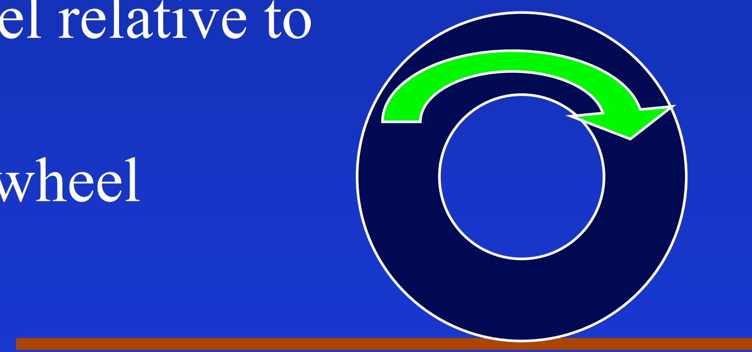
Rolling on a surface ACT



A wheel is spinning clockwise such that the speed of the outer rim is 2 m/s.

What is the velocity of the top of the wheel relative to the ground?

What is the velocity of the bottom of the wheel relative to the ground?



You now roll the wheel to the right at 2 m/s.

What is the velocity of the top of the wheel relative to the ground?

A) -4 m/s   B) -2 m/s   C) 0 m/s   D) +2m/s   E) +4 m/s

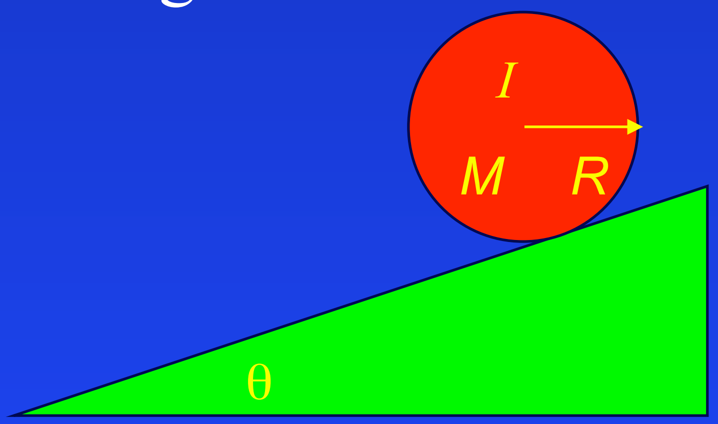
What is the velocity of the bottom of the wheel relative to the ground?

A) -4 m/s   B) -2 m/s   C) 0 m/s   D) +2m/s   E) +4 m/s

Example: Rolling down a plane

- An object with mass M, radius R, and moment of inertia I rolls without slipping down a plane inclined at an angle θ with respect to horizontal. What is its acceleration?

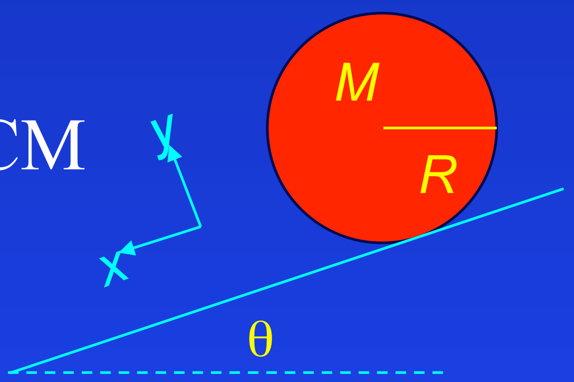
- Consider CM motion and rotation about the CM separately when solving this problem



Example: Rolling down a plane…

- Static friction *f* causes turning. It is an unknown, so we must solve for it.
- First consider the free body diagram of the object and use $\Sigma F = Ma_{cm}$:
- Now consider rotation about the CM and use $\Sigma\tau = I\alpha$ realizing that

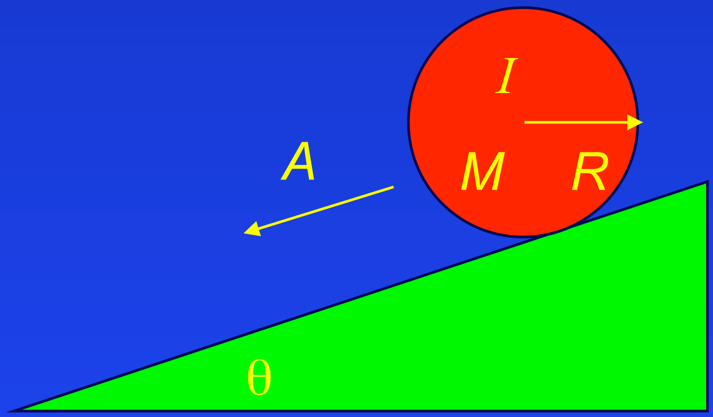
  $\tau = Rf$ and $a = \alpha R$



Example: Rolling down a plane...

- We have two equations:

- We can combine these to eliminate *f*:



Energy Conservation!

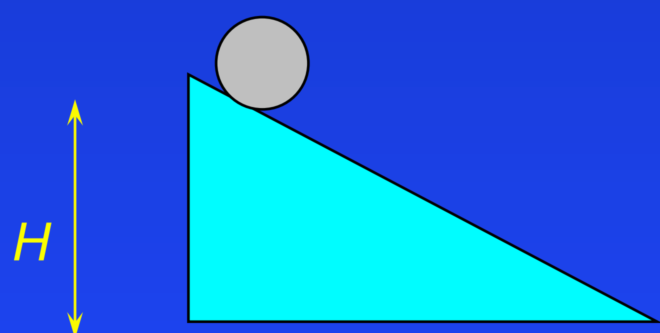
- Friction causes object to roll, but if it rolls w/o slipping friction does NO work!
  - ➡ $W = F\, d \cos\theta$ d is zero for point in contact

- No dissipated work, energy is conserved

- Need to include both translation and rotation kinetic energy.
  - ➡ $K = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

Translational + Rotational KE

- Consider a solid cylinder with radius R and mass M, rolling w/o slipping down a ramp. Determine the ratio of the translational to rotational KE.

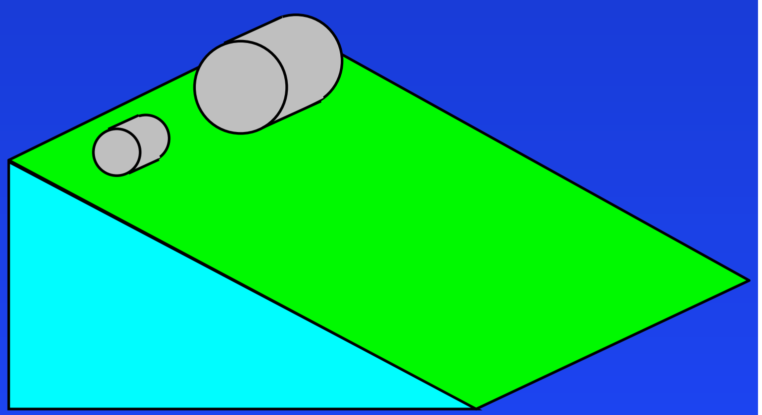


Rolling ACT

- Two uniform cylinders are machined out of solid aluminum. One has twice the radius of the other.

  If both are placed at the top of the same ramp and released, which is moving faster at the bottom?

(a) bigger one (b) smaller one (c) same



Summary

- $\tau = I\,\alpha$

- Energy is Conserved
  - Need to include translational and rotational