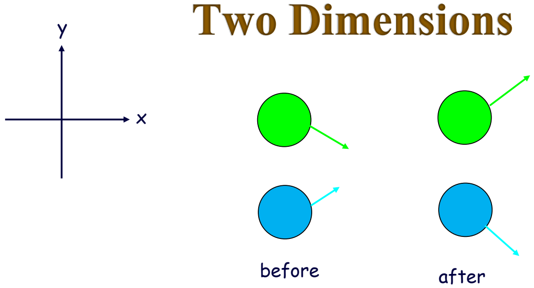
Physics 101: Lecture 13
Rotational Motion, Kinetic Energy and Rotational Inertia

Strike

Reminders:

- Prelectures, checkpoints, lectures continue with no change.
- Please come to your discussion section. No quiz, no participation points.
- HW deadlines extended beyond strike (exactly how far beyond TBA). Continue to do HW so you don't get behind!
- Labs & pre-labs continue.
- Help Room has been set up in 204 Loomis. Open 9am-6pm.

Collisions or Explosions in Two Dimensions



- $P_{total,x}$ and $P_{total,y}$ independently conserved

  $P_{total,x,before} = P_{total,x,after}$

  $P_{total,y,before} = P_{total,y,after}$

Center of Mass

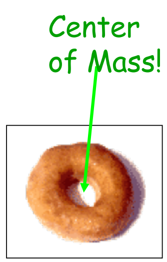
$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{\sum m_i}$$

Center of Mass = Balance point

In practice do the above in x and y directions separately

$$x_{cm} = \frac{m_1 x_1 + m_2 x_{2} + \ldots}{\sum m_i} \qquad y_{cm} = \frac{m_1 y_1 + m_2 y_{2} + \ldots}{\sum m_i}$$

Shown is a yummy doughnut. Where would you expect the center of mass of this breakfast of champions to be located?



Center of Mass

$P_{tot} = M_{tot}V_{cm}$

$(P_{tot})/M_{tot} = V_{cm}$

$F_{ext}\Delta t = \Delta P_{tot} = M_{tot}\Delta V_{cm}$

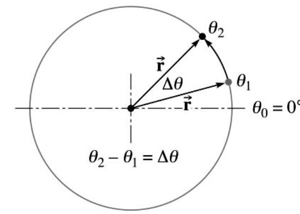
So if $F_{ext} = 0$ then V_{cm} is constant

Also: $F_{ext} = M_{tot}a_{cm}$

Center of Mass of a system behaves in a SIMPLE way
- moves like a point particle!
- velocity of CM is unaffected by collision if $F_{ext} = 0$

(pork chop demo)

Circular Motion

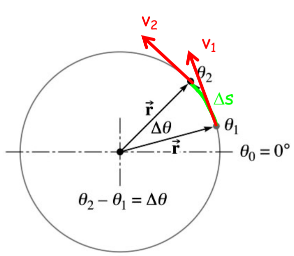


(b)

- Angular displacement $\Delta\theta = \theta_2 - \theta_1$
  - How far it has rotated
  - Units radians (2π = 1 revolution)
- Angular velocity $\omega = \Delta\theta/\Delta t$
  - How fast it is rotating
  - Units radians/second
- Frequency measures revolutions per second: $f=\omega/2\pi$
- Period =1/frequency $T = 1/f = 2\pi / \omega$
  - Time to complete 1 revolution

Circular to Linear

- Displacement $\Delta s = r\,\Delta\theta$ (θ in radians)
- Speed $|v| = \Delta s/\Delta t = r\,\Delta\theta/\Delta t = r\omega$
- Direction of v is tangent to circle



(b)

Angular Acceleration

- Angular acceleration is the change in angular velocity ω divided by the change in time.

$$\bar{\alpha} \equiv \frac{\omega_f - \omega_0}{\Delta t}$$

- Example: If the speed of a roller coaster car is 15 m/s at the top of a 20 m loop, and 25 m/s at the bottom. What is the car's average angular acceleration if it takes 1.6 seconds to go from the top to the bottom?

$$\omega = \frac{V}{R} \qquad \omega_f = \frac{25}{10} = 2.5 \qquad \omega_0 = \frac{15}{10} = 1.5$$

$$\bar{\alpha} \equiv \frac{2.5 - 1.5}{1.6} = 0.64 \text{ rad/s}^2$$

Linear and Angular Motion

	Linear	Angular
Displacement	x	θ
Velocity	v	ω
Acceleration	a	α
Inertia	m	I Today
KE	$\frac{1}{2} m v^2$	$\frac{1}{2} I\omega^2$ Today
Newton's 2nd	F=ma	coming
Momentum	p = mv	coming

Angular kinematic equations (with comparison to 1-D kinematics)

Angular	Linear
α=constant	a=constant
$\omega = \omega_o + \alpha t$	$v = v_o + at$
$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$	$x = x_o + v_o t + \frac{1}{2}at^2$
$\omega^2 = \omega_o^2 + 2\alpha\,\Delta\theta$	$v^2 = v_o^2 + 2a\,\Delta x$

$x \rightarrow \theta$

$v \rightarrow \omega$

$a_t \rightarrow \alpha$

$x = R\theta \qquad v = \omega R \qquad a_t = \alpha R \qquad a_c = v^2/R$

CD Player Example

- The CD in a disk player spins at about 20 radians/second. If it accelerates uniformly from rest with angular acceleration of 15 rad/s^2, how many revolutions does the disk make before it reaches its final angular speed of 20 radians/second?
- Plan: Use angular kinematics first to find θ in radians, and then convert to revolutions using 1 rev = 2π rad

$$\omega_0 = 0$$
$$\omega_f = 20 \text{ rad/s}$$
$$\alpha = 15 \text{ rad/s}^2$$
$$\Delta\theta = ?$$

$$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$$
$$\frac{\omega_f^2 - \omega_0^2}{2\alpha} = \Delta\theta$$
$$\frac{20^2 - 0^2}{2\times 15} = \Delta\theta$$

$\Delta\theta$ = 13.3 radians

1 Revolution = 2 π radians

$\Delta\theta$ = 13.3 radians

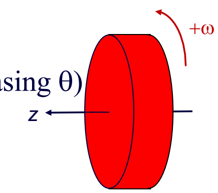
= 2.12 revolutions

Comment on axes and sign
(i.e. what is positive and negative)

Whenever we talk about rotation, it is implied that there is a rotation "axis". We need a way of distinguishing + from – rotations.

This is usually called the "z" axis (we usually omit the z subscript for simplicity).

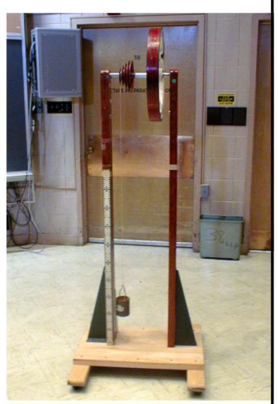
Counter-clockwise rotations: (increasing θ) will be positive
Clockwise rotations: (decreasing θ) will be negative.
[demo: bike wheel].



Energy Clicker Q and demo

When the bucket reaches the bottom, its potential energy has decreased by an amount mgh. Where has this energy gone?

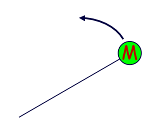
A) Kinetic Energy of bucket
B) Kinetic Energy of flywheel
C) Both 1 and 2.



Rotational Kinetic Energy

- Consider a mass M on the end of a string being spun around in a circle with radius r and angular frequency ω [demo]

  - Mass has speed $v = \omega r$
  - Mass has kinetic energy
    - » $K = \frac{1}{2} M v^2 = \frac{1}{2} M (\omega r)^2$
    - » $= \frac{1}{2} M \omega^2 r^2$
    - » $= \frac{1}{2} (M r^2) \omega^2$
    - » $= \frac{1}{2} I \omega^2$



I is "moment of inertia" and is the equivalent of mass for rotational motion (don't confuse I w/ impulse)

- Rotational Kinetic Energy is energy due to circular motion of object.

Rotational Inertia I (or moment of inertia)

- Tells how "hard" it is to get an object rotating. Just like mass tells you how much "hard" it is to get an object moving.
  - $K_{tran} = ½\ m\ v^2$ Linear Motion
  - $K_{rot} = ½\ I\ \omega^2$ Rotational Motion
- $I = \Sigma\ m_i r_i^2$ (units kg m^2; I plays role of mass in rotational motion)
- Note! Rotational Inertia (or "Moment of Inertia") depends on what axis you are rotating about (basically the r_i in the equation).

Main Ideas

- Rotational Kinematics is just like linear kinematics with parallel equations of motion
- Rotating objects have kinetic energy
  - $KE = \frac{1}{2} I \omega^2$
- Moment of Inertia $I = \Sigma mr^2$
  - Depends on Mass
  - Depends on axis of rotation
- Energy is conserved but need to include rotational energy too: $K_{rot} = \frac{1}{2} I \omega^2$