Physics 101: Lecture 14
Parallel Axis Theorem, Rotational Energy, Conservation of Energy Examples, and a Little Torque

Review

- Rotational Kinetic Energy $K_{rot} = \frac{1}{2} I \omega^2$
- Rotational Inertia $I = \Sigma m_i r_i^2$ for point masses. For continuous objects use table (you need calculus to compute I for continuous objects)

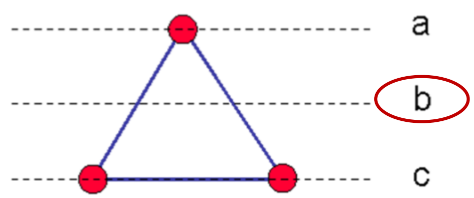
Strike (Day 8)

- Prelectures, checkpoints, lectures continue with no change.
- Please come to your discussion section. We are trying something different with quizzes this week. More coming by email.
- HW deadlines now re-set.
  HW6 DUE THIS WEEK!
  HW7 & 8 DUE NEXT WEEK!
- Labs & pre-labs continue.
- Help Room is 204 Loomis. Open 9am-6pm, M-F.

Exam 2: Mar. 28-30
covers Lectures 9-15
Sign up for exam ASAP!

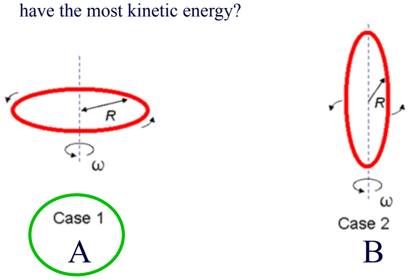
Checkpoint 2 / Lecture 13

A triangular-shaped toy is made from identical small but relatively massive red beads and identical rigid and lightweight blue rods as shown in the figure. The moments of inertia about the a, b, and c axes are I_a, I_b, and I_c, respectively. The b axis is half-way between the a and c axes. Around which axis will it be easiest to rotate the toy?



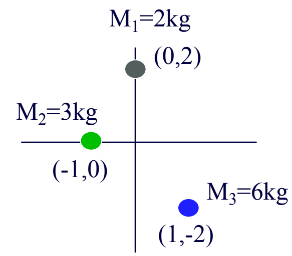
Checkpoint 3 / Lecture 13

In both cases shown below a hula hoop with mass M and radius R is spun with the same angular velocity about a vertical axis through its center. In Case 1 the plane of the hoop is parallel to the floor and in Case 2 it is perpendicular. In which case does the spinning hoop have the most kinetic energy?



C. Same for both

Moment of Inertia Example: 3 masses



Find moment of inertia, I, about an axis perpendicular to page going through the origin.

What would change if we computed I about (3,0)?

Parallel Axis Theorem

- If you know the moment of inertia of a body about an axis through its center of mass, then you can find its moment of inertia about any axis parallel to this axis using the *parallel axis theorem*.

$$I = I_{CM} + Mh^2$$

(h is distance from cm to axis)

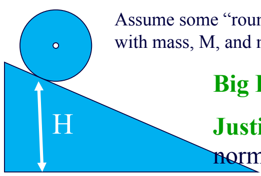
Example: Moment of inertia of stick about one end

- $I = I_{CM} + Mh^2$

From the (last) prelecture or Table 8-1 in book, you know that the moment of inertia of a uniform stick about its CM is: $(1/12)ML^2$. Let's use this to find I about one end:

Note: We can find I about any parallel axis

How would we find the speed of these rolling objects at the bottom?



Assume some "round" rolling object of radius R, at height H, with mass, M, and moment of inertia, I.

Big Idea: Conservation of mechanical energy, E

Justification: Non-conservative forces (friction and normal) do no work so E conserved

Plan: 1. Write E_i (all potential)
2. Write E_f and don't forget K of rotation
3. Set $E_i = E_f$ and solve for v by *relating v to ω with v= ωR*

V_{ball} = SQRT(10/7 gh); $V_{cylinder}$ = SQRT(4/3 gh); V_{hoop} = SQRT(gh)

a) Object w/ smaller I goes faster at bottom, b) both objects have same K at bottom, c) Bigger I means more energy goes into rotation than translation relatively speaking

Energy Conservation!

- Friction causes object to roll, but if it rolls without slipping, friction does NO work!
  - $W = F\, d \cos\theta$ d is zero for point of contact
- No sliding means friction does no work so energy is conserved
- Need to include both translational and rotational kinetic energy.
  - $K = ½\, m\, v^2 + ½\, I\, \omega^2$
- Setting (initial total energy) = (final total energy) is a good method for finding speed (not *a* or t)

Distribution of Translational & Rotational KE for a solid disk

- Consider a **solid disk** with radius R and mass M, rolling w/o slipping down a ramp. Determine the ratio of the translational to rotational KE.

Translational: $K_T = \frac{1}{2} M v^2$

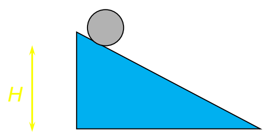
Rotational: $K_R = \frac{1}{2} I \omega^2$

use $I = \frac{1}{2} M R^2$ and $\omega = \frac{v}{R}$

Rotational: $K_R = \frac{1}{2} (\frac{1}{2} M R^2) (V/R)^2$

$= \frac{1}{4} M v^2$

$= \frac{1}{2} K_T$

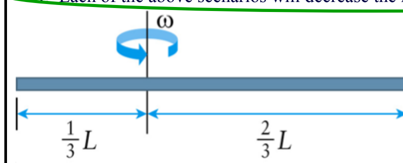


Twice as much Kinetic energy is in translation than in rotation **for a disk**

Checkpoint 1 / Lecture 13

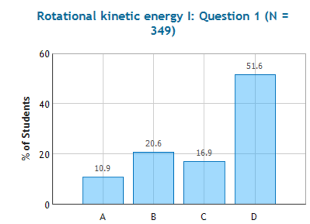
A thin rod of length *L* and mass *M* rotates around an axis that passes through a point one-third of the way from the left end, as shown in the Which of the following will decrease the rotational kinetic energy of the rod by the greatest amount?

A. Decreasing the mass of the rod by one fourth, while maintaining its length.
B. Decreasing the length of the rod by one half, while maintaining its mass.
C. Decreasing the angular speed of the bar by one half.
D. Each of the above scenarios will decrease the rotational kinetic energy by same amount



$K = (1/2) I \omega^2$

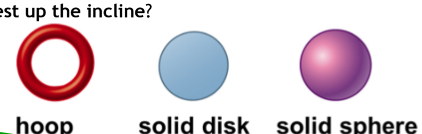
$I = (\text{factor}) ML^2$
$= (1/9) ML^2$

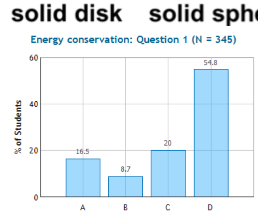


Checkpoint 4 / Lecture 13

A hoop, a solid disk, and a solid sphere, all with the same mass and the same radius, **are set rolling without slipping up an incline, all with the same initial kinetic energy. Which goes furthest up the incline?**

A. The hoop
B. The disk
C. The sphere
D. They all roll to the same height



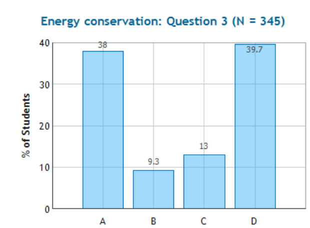


Conceptual thought Q: Initially since all three have the same K, will they have the same v?

Follow-Up to Checkpoint 4

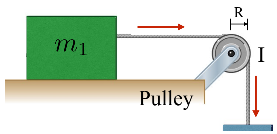
A hoop, a solid disk, and a solid sphere, all with the same mass and the same radius, **are set rolling without slipping up an incline, all with the same initial speed. Which goes furthest up the incline?**

A. The hoop
B. The disk
C. The sphere
D. They all roll to the same height



Massless Pulley, no friction Example

Consider the two masses connected by a pulley as shown. Use conservation of energy to calculate the speed of the blocks after m_2 has dropped a distance h. Assume the pulley is massless.



Big Idea: Conservation of mechanical energy

Justification: Non-conservative forces do no work, so E conserved

Plan: 1) Set $E_i = E_f$
2) Solve for v

$$W_{nc} = \Delta E = (K_f + U_f) - (K_i + U_i)$$

$$U_{initial} + K_{initial} = U_{final} + K_{final}$$

$$m_2 gh = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

$$2m_2 gh = m_1 v^2 + m_2 v^2$$

$$v = \sqrt{\frac{2m_2 gh}{m_1 + m_2}}$$

Summary

- Energy is conserved for rolling objects
- The amount of kinetic energy of a rolling object depends on its speed, angular velocity, mass and moment of inertia
- Parallel Axis Theorem lets you compute moment of inertia about any axis parallel to CM axis if you know I_{CM}.