Physics 101: Lecture 15
Torque, F=ma for rotation, and Equilibrium

Strike (Day 10)

- Prelectures, checkpoints, lectures continue with no change.
- Take-home quizzes this week. See Elaine Schulte's email.
- HW deadlines now re-set.
  HW6 DUE TOMORROW!
  HW7 & 8 DUE NEXT THURSDAY!
- Labs & pre-labs continue.
- Help Room is 204 Loomis. Open 9am-6pm, M-F.

Exam 2: Mar. 28-30
covers Lectures 9-15
Sign up for exam ASAP!

Linear and Angular Motion

	Linear	Angular
Displacement	x	θ
Velocity	v	ω
Acceleration	a	α
Inertia	m	I
KE	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$
Force	**F**	**τ (torque)**
Newton's 2nd	**F=ma**	**$\tau = I\alpha$**
Momentum	p = mv	coming

$x = R\theta$

$v = \omega R$

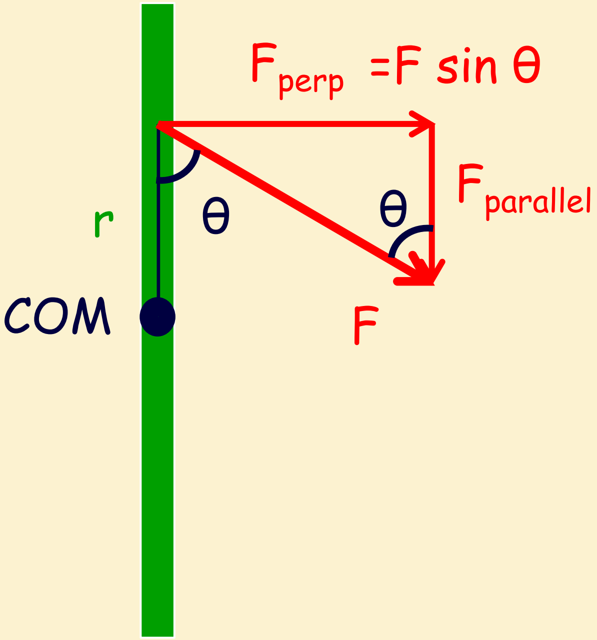
$a_t = \alpha R$

today

Torque Definition

- A TORQUE is a *force x distance* that causes rotation. It tells how effective a force is at twisting or rotating an object.

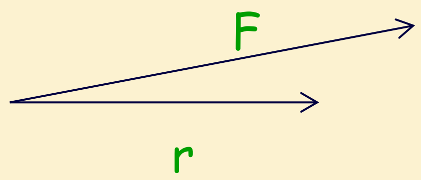
- $\tau = r\,F_{perpendicular} = r\,F\sin\theta$
  - ➡Units N m
  - ➡Sign: CCW rotation is positive
    CW rotation is negative



Two ways to compute torque:

1. Put r and F vectors tail-to-tail and compute

$$\tau = rF\sin\theta.$$



2. Decompose F into components parallel and perpendicular to r, and take:

$$\tau = rF_{\perp}$$

If rotation is clockwise, torque is negative, and if rotation is counterclockwise torque is positive.

Note: If F and r are parallel or antiparallel, the torque is 0.
(e.g., can't open a door if pushing or pulling toward the hinges)

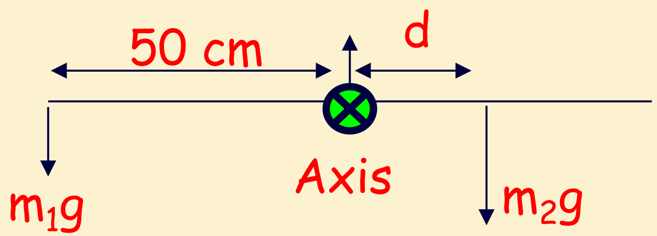
Equilibrium

- Conditions for Equilibrium
  - $F_{Net} = ma = 0$ Translational *a* of CM must be 0
  - $\tau_{Net} = I\alpha = 0$ Rotational α about any axis must be 0
    - » Choose axis of rotation wisely to make problems easier!
    - » But as long as you're consistent everything will be OK!

- A meter stick is suspended at the center. If a 1 kg weight is placed at x=0. Where do you need to place a 2 kg weight to balance it?

A) x = 25 B) x=50 C) x=75 D) x=100

E) 1 kg can't balance a 2 kg weight.



Compute torque about axis and set =0:

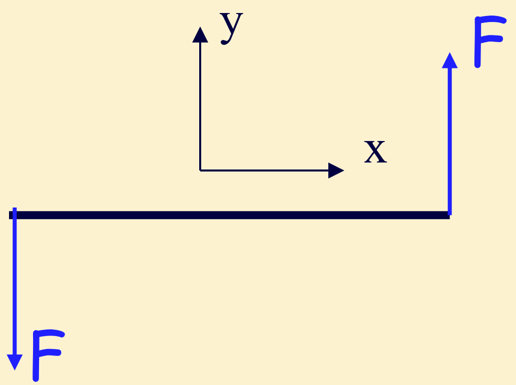
Balance Demo

Equilibrium: $a = 0, \alpha = 0$

- A rod is lying on a table and has two equal but opposite forces acting on it. The net force on the rod is:

Y direction: $F_{net\ y} = ma_y$

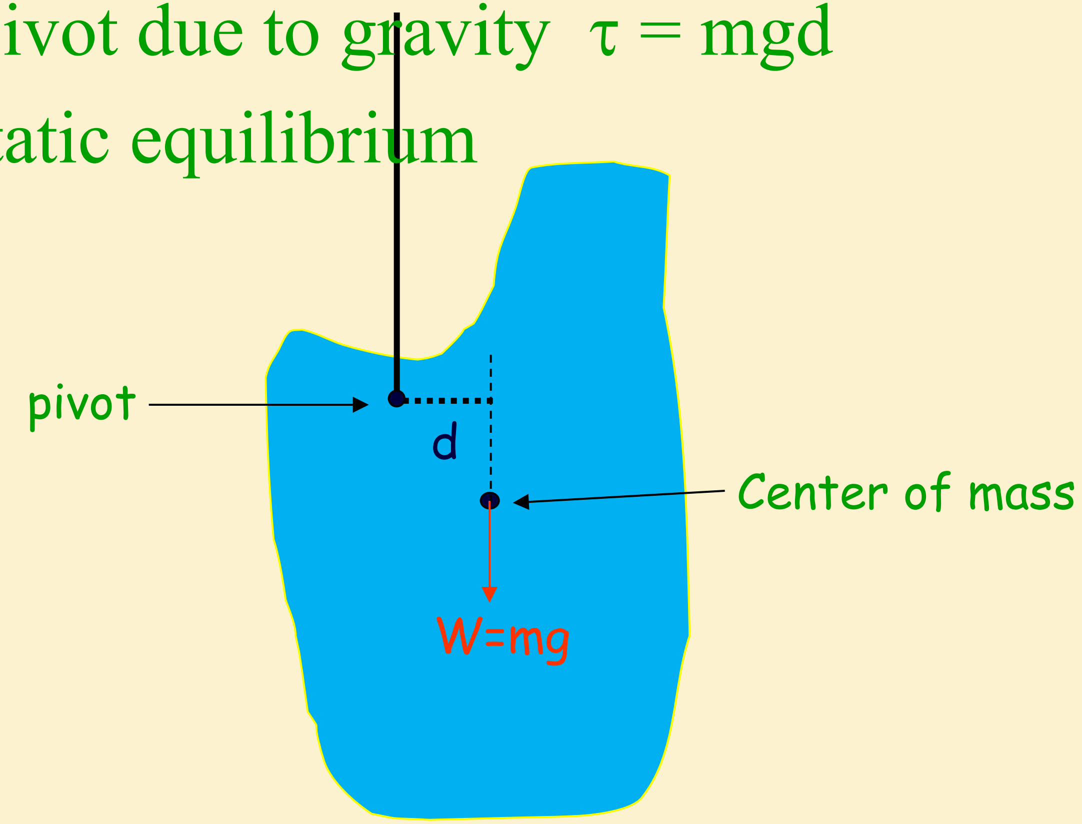
$+F - F = 0 = ma_y$



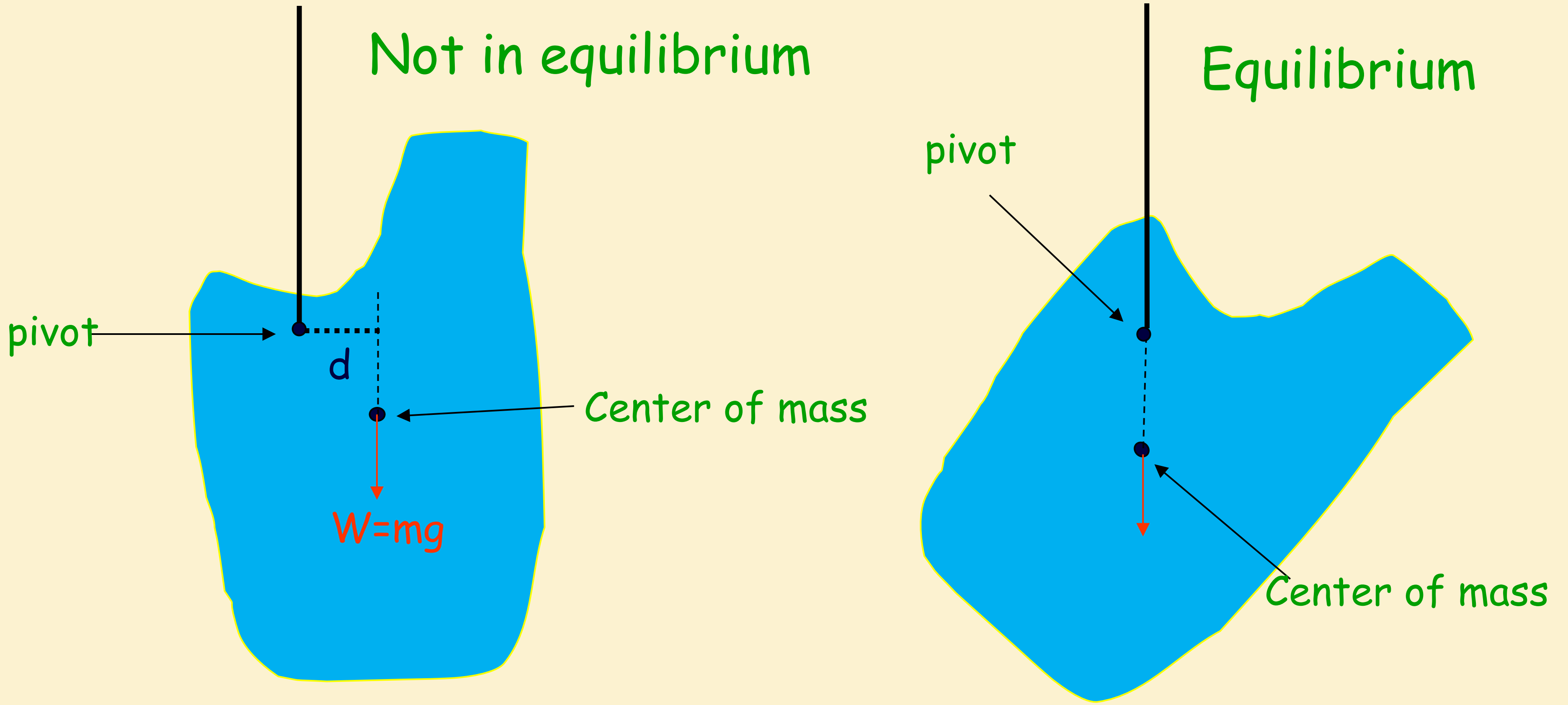
- The rod has no *a* in linear direction, so it **won't translate.** However, the rod will have a non-zero torque, hence a non-zero α and **will rotate.**

Static Equilibrium and Center of Mass

- Gravitational Force   Weight = mg
  - Acts as force at center of mass
  - Torque about pivot due to gravity $\tau = mgd$
  - Object not in static equilibrium



Static Equilibrium



Torque about pivot $\neq 0$

Torque about pivot $= 0$

A method to find center of mass of an irregular object

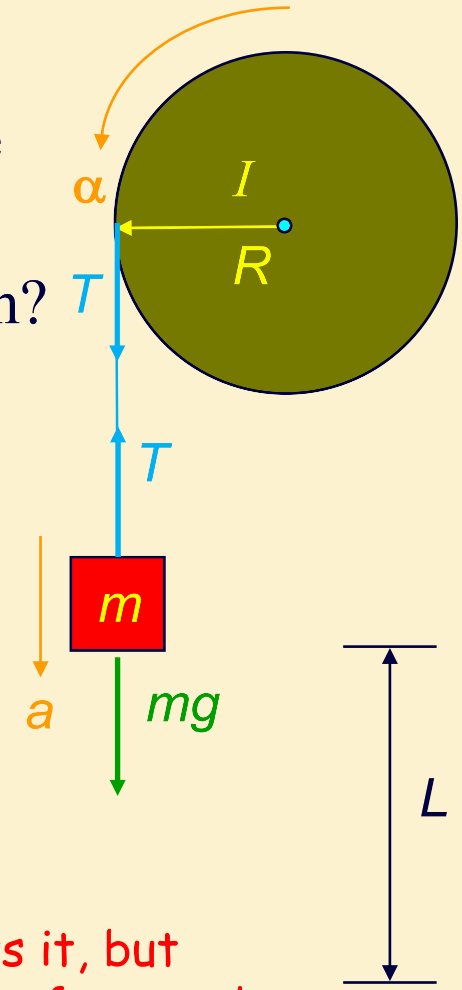
Rotational Newton's 2^{nd} Law

- $\tau_{Net} = I\,\alpha$
  - Torque is amount of twist provided by a force
    - » Signs: positive = CCW 
    - » negative = CW 
  - Moment of Inertia = rotational mass. Large I means hard to start or stop rotation.
- Problems Solved Like Newton's 2nd
  - Draw FBD
  - Write Newton's 2^{nd} Law in linear and/or rotational form, then use algebra.

Falling weight & pulley example

- A mass m is hung by a string that is wrapped around a disk of radius R and mass M. The moment of inertia of the disk is $I=1/2\ MR^2$. The string does not slip on the disk.

What is the acceleration, a, of the hanging mass, m?



What method should we use to solve this problem?

A) Conservation of Energy (including rotational)

B) $\tau_{Net} = I\alpha$ and $F=ma$

Either can be applied here in the sense that physics allows it, but Cons. of E gives you speed, and Newton's Second in angular form and linear form lets you solve for a, so we will use B.

Falling weight & pulley... (need to find a)

Big Idea: N#2 in linear form for m and angular form for disk.

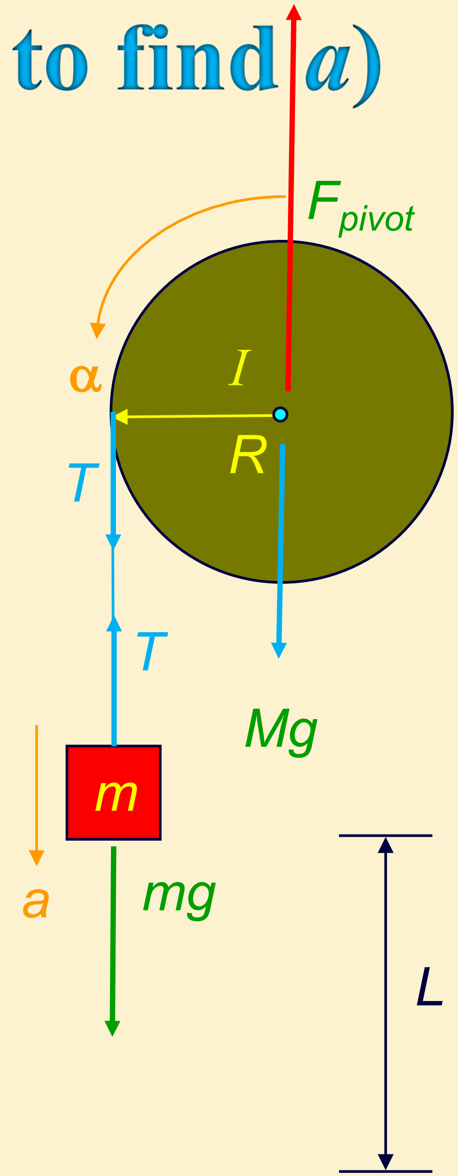
Justification: N#2 good for finding a and t.

Plan: 1. Draw a Free-Body Diagram

2. For the hanging mass apply $F_{Net} = ma$ and for disk apply $\tau = I\alpha$

3. Relate a and α using $a = \alpha R$ *(see slide 2)*

4. Use algebra to solve 3 equations in 3 unknowns, T, a, α.



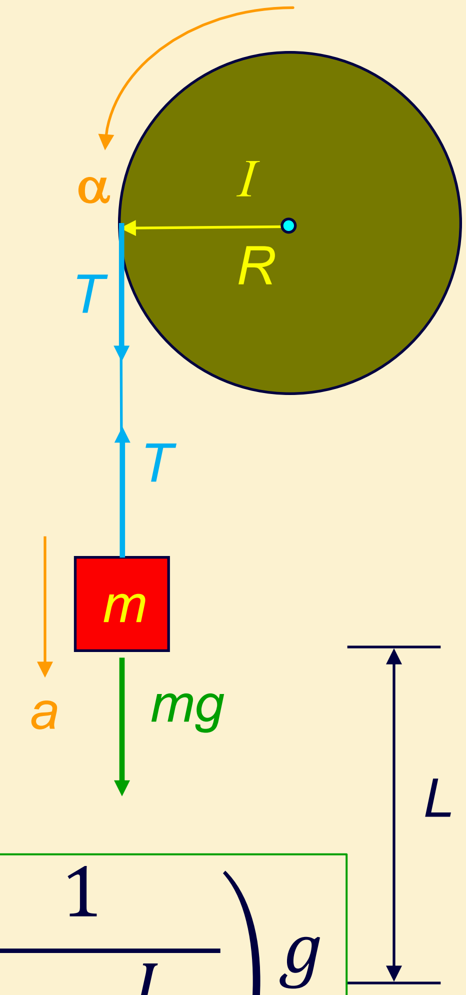
Falling weight & pulley... (need to find *a*)

2: $T - mg = -ma$

$TR\sin(90) = I\alpha$ ($I=1/2\ MR^2$)

3: $a = \alpha R$

4: *Use algebra to solve for a*

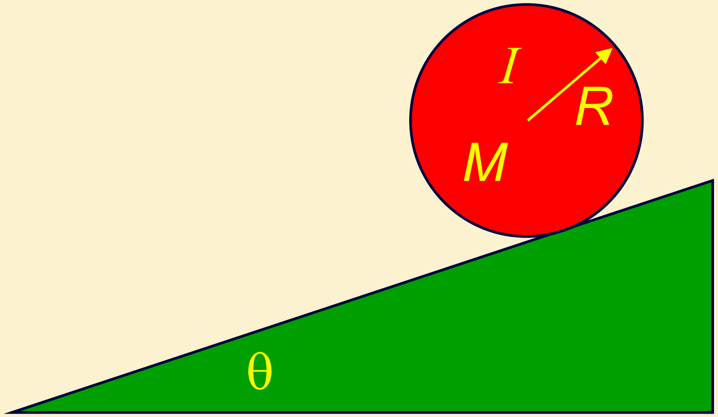


$$a = \left(\frac{1}{1+\frac{I}{mR^2}}\right)g$$

Rolling

- An object with mass *M*, radius *R*, and moment of inertia *I* rolls without slipping down a plane inclined at an angle θ with respect to horizontal. What is its acceleration?

- Consider CM motion and rotation about the CM separately when solving this problem



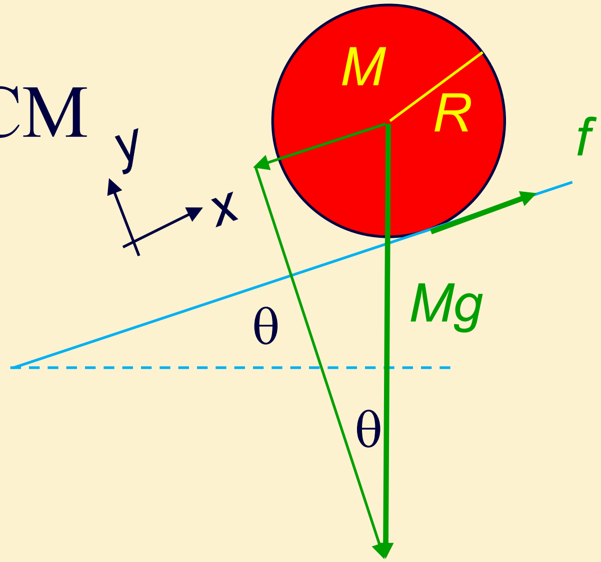
Rolling...

- Static friction f causes rolling. It is an unknown, so we must solve for it.
- First consider the free body diagram of the object and use $F_{NET} = Ma_{cm}$:

  In the x direction: $- Mg \sin\theta + f = - Ma_{cm}$

- Now consider rotation about the CM and use $\tau_{Net} = I\alpha$ realizing that

  $\tau = Rf \sin 90 = Rf$ and $a = \alpha R$



Rolling...

- We have 3 equations in 3 unknowns, a, α and *f*:

From F=ma applied to CM: $-Mg\sin\theta + f = -Ma$

From τ=Iα applied about CM: $fR\sin 90 = fR = I\alpha$

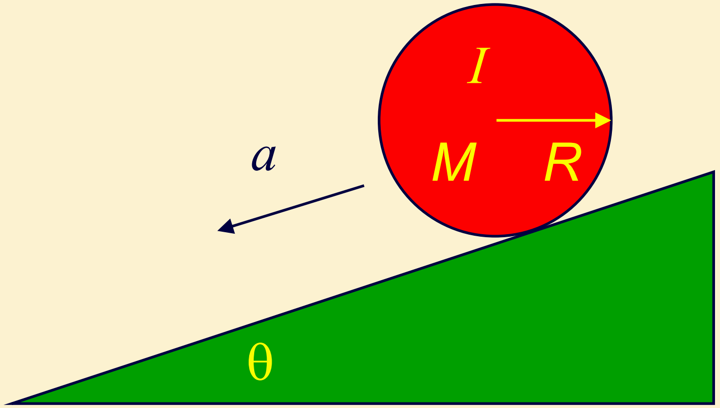
From relationship between a and α: $R\alpha = a$

- Use algebra to combine these to eliminate *f*, and solve for *a*:

$$a = g\left(\frac{MR^2 \sin\theta}{MR^2 + I}\right)$$

For a sphere:

$$a = g\left(\frac{MR^2 \sin\theta}{MR^2 + \frac{2}{5}MR^2}\right) = \frac{5}{7} g \sin\theta$$



Work Done by Torque

- Recall $W = F\, d \cos\theta$
- For a wheel
  - Work: $W = F_{tangential}\, s$

    $= F_{tangential}\, r\, \theta$ (s = r θ, θ in radians)

    $= \tau\, \theta$

Summary

- Torque = Force that causes rotation
  - $\tau = F\, r \sin\theta$
  - F=m*a* for rotation: $\tau = I\alpha$.
  - Work done by torque $W = \tau\,\theta$
- Use F=ma, $\tau = I\alpha$ to solve for *a*, α, tension, time. Use conservation of energy to solve for speed.
- Equilibrium
  - $\Sigma F = 0$
  - $\Sigma \tau = 0$
    - » Can choose any axis or pivot around which to compute torques. Trick of the trade: If there is a force on the pivot, the torque it produces is 0!