Physics 101: Lecture 20
Oscillations: Simple Harmonic Motion

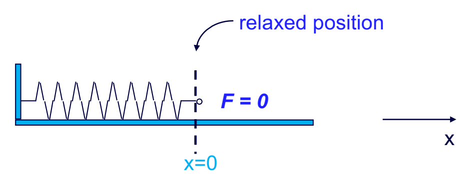
Overview

- Springs
  - ➡Force is proportional to displacement
  - ➡F = -k x (- means if you pull in +x direction spring pulls back in –x direction)
  - ➡U = ½ k x^2 (potential energy stored in spring; spring forces are conservative)
- Today
  - ➡Simple Harmonic Motion
  - ➡Springs Revisited
- Note: In the prelecture for Wed, we will not cover "the physical pendulum", "damped oscillators", and "driven oscillators".

Springs

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

$F = -k\,x$ Where x is the displacement from the relaxed position and k is the constant of proportionality.

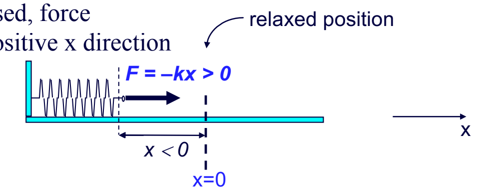


Springs

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

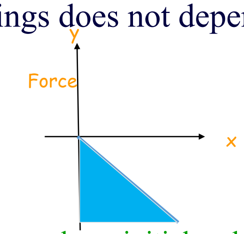
$F = -kx$ Where x is the displacement from the relaxed position and k is the constant of proportionality.

If compressed, force points in positive x direction



Potential Energy in Spring

- Hooke's Law force is Conservative: Work done by springs does not depend on path
  - ➡$F = -k\,x$
  - ➡$W = -1/2\ k\,x^2$

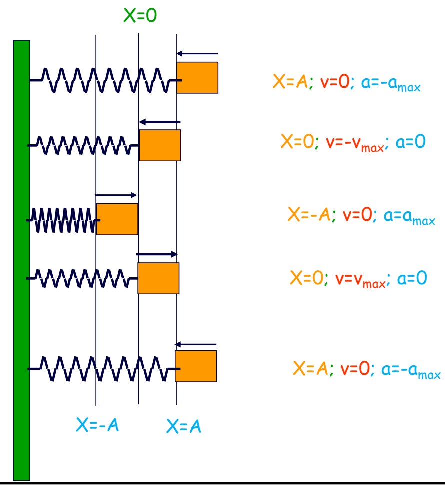


  - ➡Work done only depends on initial and final position
  - ➡Define Potential Energy $U_{spring} = ½\ k\,x^2$

Simple Harmonic Motion

- Vibrations
  - ➡Vocal cords when singing/speaking
  - ➡String/rubber band
- Simple Harmonic Motion
  - ➡Restoring force proportional to displacement
  - ➡Springs $F = -kx$

Springs and Simple Harmonic Motion



Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$ OR $x(t) = [A]\sin(\omega t)$

$v(t) = -[A\omega]\sin(\omega t)$ OR $v(t) = [A\omega]\cos(\omega t)$

$a(t) = -[A\omega^2]\cos(\omega t)$ OR $a(t) = -[A\omega^2]\sin(\omega t)$

$x_{max} = A$

$v_{max} = A\omega$

$a_{max} = A\omega^2$

Period = T (seconds per cycle)

Frequency = f = 1/T (cycles per second)

Angular frequency = $\omega = 2\pi f = 2\pi/T$

For spring: $\omega^2 = k/m$, $T = 2\pi\sqrt{m/k}$

Energy

- A mass is attached to a spring and set to motion. The maximum displacement is $x=A$
  - Apply Work-Kinetic Energy Thm: $W_{nc} = \Delta E = \Delta(K + U)$
  - $0 = \Delta(K + U)$ or Energy, U+K, is constant!

    Energy $= \frac{1}{2} m v^2 + \frac{1}{2} k x^2$
  - At maximum displacement $x=A$, $v = 0$

    Energy $= \frac{1}{2} k A^2 + 0$
  - At zero displacement $x = 0$

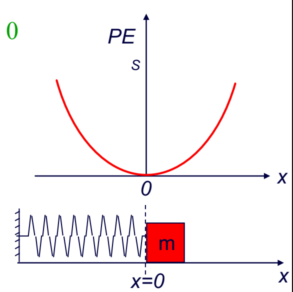
    Energy $= 0 + \frac{1}{2} m v_{max}^2$

  Since Total Energy is same

  $\frac{1}{2} k A^2 = \frac{1}{2} m v_{max}^2$

  $v_{max} = A\ \mathrm{sqrt}(k/m) = A\omega$

  Same as in last slide



Summary

- Springs
  - $F = -kx$
  - $U = \frac{1}{2} k x^2$
  - $\omega = \text{sqrt}(k/m)$, $\omega = 2\pi f = 2\pi/T$, $f = 1/T$

- Simple Harmonic Motion
  - Occurs when have linear restoring force $F= -kx$
  - $x(t) = [A] \cos(\omega t)$ or $[A] \sin(\omega t)$
  - $v(t) = -[A\omega] \sin(\omega t)$ or $[A\omega] \cos(\omega t)$
  - $a(t) = -[A\omega^2] \cos(\omega t)$ or $-[A\omega^2] \sin(\omega t)$

50