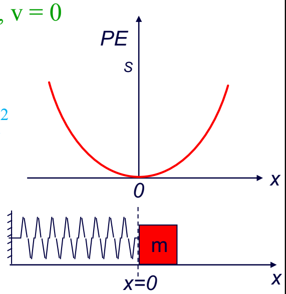
# Physics 101: Lecture 21

## Oscillations 2: More mass-at-end-of-spring oscillations & Pendula

## Review Energy in Simple Harmonic Motion

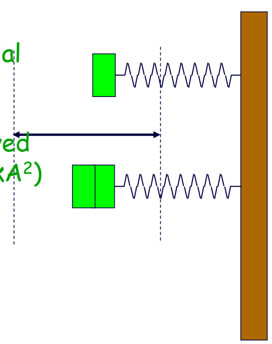
- A mass is attached to a spring and set to motion. The maximum displacement is $x=A$
  - Energy = U + K = constant!
    $= \frac{1}{2} k x^2 + \frac{1}{2} m v^2$
  - At maximum displacement $x=A$, $v = 0$
    Energy $= \frac{1}{2} k A^2 + 0$
  - At zero displacement $x = 0$
    Energy $= 0 + \frac{1}{2} m v_{max}^2$
    $\frac{1}{2} k A^2 = \frac{1}{2} m v_m^2$
    $v_{max} = A\ \mathrm{sqrt}(k/m)$
  - Analogy with gravity/ball



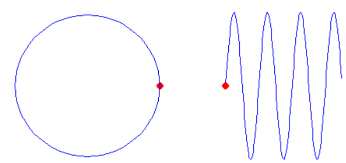
# Summary

In Case 1 a mass on a spring oscillates back and forth. In Case 2, the mass is doubled but the spring and the amplitude of the oscillation is the same as in Case 1.

- Both have the same maximum Potential Energy ($\frac{1}{2}kA^2$ is the same for both)
- Both have the same maximum Kinetic Energy (mechanical energy is conserved so K+U= $\frac{1}{2}kA^2$ and when U=0, $K_{max}$= $\frac{1}{2}kA^2$)
- Case 1 has larger max speed at x=0.
- Case 1 has larger $\omega$ since $\omega = \sqrt{k/m}$
- Case 2 has a longer Period since $T = 2\pi\sqrt{m/k}$



# Review: Simple Harmonic Motion



Period = T (seconds per cycle)

Frequency = f = 1/T (cycles per second)

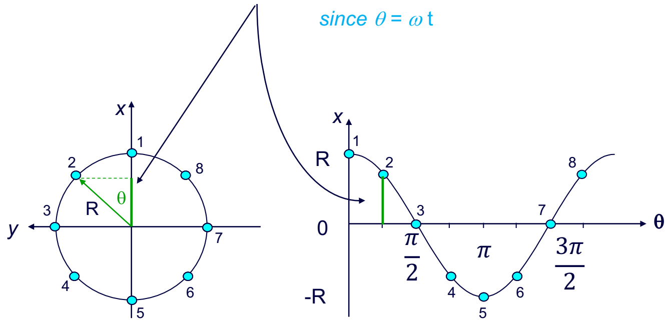
Angular frequency = $\omega = 2\pi f = 2\pi/T$ (radians per second)

17

What does *moving in a circle* have to do with moving back & forth *in a straight line* ??

$x = R \cos\theta = R \cos(\omega t)$

*since* $\theta = \omega t$




# Spring Oscillations

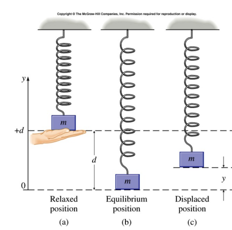
- Simple Harmonic Oscillator
  - $\omega = 2\pi f = 2\pi / T$
  - $x(t) = [A] \cos(\omega t)$
  - $v(t) = -[A\omega] \sin(\omega t)$
  - $a(t) = -[A\omega^2] \cos(\omega t) = -\omega^2 x(t)$
- Draw FBD write F=ma
  - $-k x = m a$
  - $-k A = m a_{max}$
  - $-A m\omega^2 = m a_{max}$
  - $a_{max} = -A\omega^2$

Demos:
A,m,k dependence

$$\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

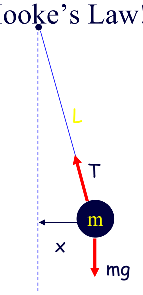
# Vertical Mass and Spring

- If we include gravity, there are two forces acting on mass. With mass, new equilibrium position has spring stretched d
  - $F_{Net,\,y} = 0$
  - $kd - mg = 0$
  - $d = mg/k$
- Now what happens when we compress spring a distance y?
  - $F_{net} = k(d-y) - mg$
  - $= kd - ky - mg$
  - Still Hook's Law! SHO
  - New equilibrium position



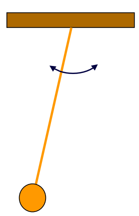
# Pendulum Motion

- For *small angles*
  - $T \cong mg$ (T is *approximately* equal to mg)
  - $F_{net,x} = T_x = -T \sin\theta$
    $= -mg\,(x/L) = -(mg/L)\,x$ Hooke's Law!
  - $F_{net,\,x} = m\,a_x$
    $-mg\,(x/L) = m\,a_x$
    $a_x = -(g/L)\,x$
  - Recall for SHO $a = -\omega^2 x$
    $\omega = \mathrm{sqrt}(g/L)$
    $T = 2\,\pi\,\mathrm{sqrt}(L/g)$
    Period does not depend on A, or m!



# Application

Imagine you have been kidnapped by space invaders and are being held prisoner in a room with no windows. All you have is a cheap digital wristwatch and a pair of shoes (including shoelaces of known length). Explain how you might figure out whether this room is on the earth or on the moon



$$\omega = \sqrt{\frac{g}{L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

$$g = (2\pi)^2 \frac{L}{T^2}$$

make a pendulum with the shoelace and shoes and use the wristwatch to determine the length of each period.

# Summary

- Simple Harmonic Motion
  - Occurs when have linear restoring force F= -kx
  - $x(t) = [A] \cos(\omega t)$
  - $v(t) = -[A\omega] \sin(\omega t)$
  - $a(t) = -[A\omega^2] \cos(\omega t)$
- Springs
  - $F = -kx$
  - $U = \frac{1}{2} k x^2$
  - $\omega = \text{sqrt}(k/m)$
- Pendulum (Small oscillations)
  - $\omega = \text{sqrt}(L/g)$