Physics 101 Lecture 4
Kinematics: Projectile and Circular Motion

What concepts did you find most difficult, or what would you like to be sure we discuss in lecture?

- More conceptual problems … sometimes the problems without physical numbers can be the most challenging.
- First part
- None
- the trigonometry.
- what would happen if the horizontal component was altered?
- can we go over more practice problems for circular motion.

Review: 1-dimensional Kinematics Example

- A car is traveling 30 m/s and applies its breaks (constant deceleration) to stop after a distance of 150 m.
- How fast is the car going after it has traveled ½ the distance (75 meters) ?

A) $v < 15$ m/s B) $v = 15$ m/s C) $v > 15$ m/s

Note: It's NOT half, so relation is not linear!

Let's think about a plan for solving this problem

Plan:
1. First use kinematics to find acceleration from first problem statement
2. Use kinematics again to find speed at $x = 75$ m

Review: 1-dimensional Kinematics Example

- A car is traveling 30 m/s and applies its breaks to stop after a distance of 150 m.
- How fast is the car going after it has traveled ½ the distance (75 meters) ?
- $x = x_0 + v_0 t + \frac{1}{2} a t^2$
- $v = v_0 + at$
- $v^2 = v_0^2 + 2a(x - x_0)$

Plan:
1. Find acceleration: $0 = (30 \text{ m/s})^2 + 2a(150 \text{ m})$, so $a = -3 \text{ m/s}^2$
2. Use kinematics again to find speed at $x = 75$ m:

$$v^2 = \left(30 \frac{m}{s}\right)^2 + 2(-3 \text{ m/s}^2)(75\ m), \text{ so v} = 21.2 \text{ m/s}$$

Important Concepts for Motion in 2 Dimensions

- X and Y directions are **Independent**!
- Position, velocity and acceleration are vectors (they have directions and magnitudes)
- Vectors have special rules

Kinematics in Two Dimensions: Equations and Facts

Must be able to identify variables in these equations!

$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$	$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$
$v_x = v_{0_x} + a_x t$	$v_y = v_{0_y} + a_y t$
$v_x^2 = v_{0x}^2 + 2a_x \Delta x$	$v_y^2 = v_{0y}^2 + 2a_y \Delta y$

Remember: x and y directions are *independent.*

Independent means:

Calculate the x-direction by itself and the y-direction by itself, then use math to combine if needed

Demo: Ball shot vertically from moving train

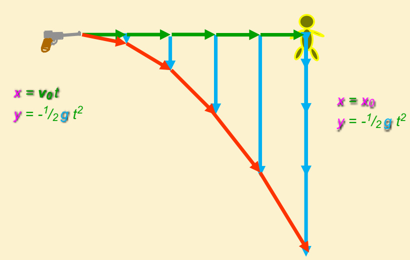
This demo illustrates the independence of x and y motion.

Projectile Motion: A Special Case

$a_x = 0$	$a_y = -g$
$x = x_0 + v_{0x} t$	$y = y_0 + v_{0y} t - ½ g t^2$
$v_x = v_{0x}$	$v_y = v_{0y} - g t$
	$v_y^2 = v_{0y}^2 - 2g\Delta y$

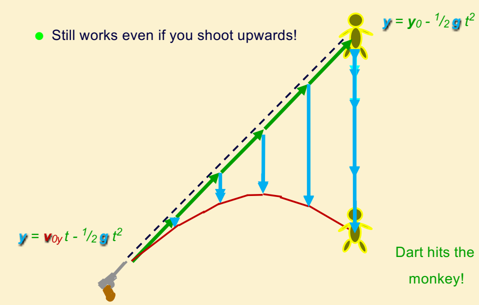
- *Procedure*:
  - Choose standard coordinate system (that's how + and – are determined)
  - Solve kinematics equations in each direction separately.
  - As time evolves, motion in each direction proceeds independently

Demo: Shooting the Monkey...



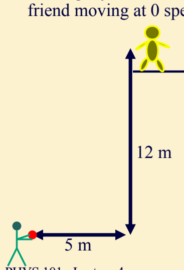
Demo: Shooting the Monkey...

- Still works even if you shoot upwards!



Ex: Throw ball to your friend at a window

You throw a ball to your friend at a window of a building 12 meters above and 5 meters to the right of you. Determine the speed and angle you should throw it such that the ball "just reaches" your friend moving at 0 speed in y-direction.



y-direction (v_{0y})

$v_{fy}{}^2 = v_{0y}{}^2 + 2(-g)\Delta y$, with $v_{fy}=0$

$v_{0y} = \sqrt{2\times 9.8\times 12} = 15.3\ m/s$

The reason why $v_{fy}=0$ is that the vertical velocity when ball gets to your friend is 0 m/s.

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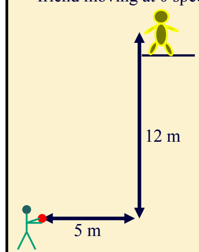

x-direction (v_{0x}):

First find the time it takes ball to get to your friend (this is a *y-direction question*):

$v_{fy} = v_{0y} - gt$, with $v_{fy}=0$

$t = v_{0y}/g = (15.3\ \text{m/s})/g = 1.56\ \text{s}.$

Then use the time to find the required horizontal speed (*x-direction question*):

$x = xo + v_{ox}t + ½a_x t^2$

$v_{ox} = 5\ m\ /\ 1.56\ \text{s}$

$= 3.2\ \text{m/s}$

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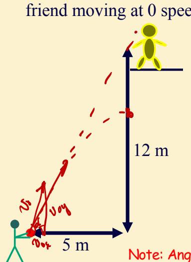


Speed and Angle:

First use the x- and y-direction velocities you calculated to find the angle for the *initial velocity vector*:

$$\alpha = \tan^{-1}\frac{v_{0y}}{v_{0x}} = \tan^{-1}\frac{15.3\ m/s}{3.2\ m/s} = 78.2°$$

Then use these values to find the magnitude (speed) of the initial velocity vector (Pythagorean Thm):

$$|v| = \sqrt{v_{0_x}^2 + v_{0_y}^2} = \sqrt{3.2^2 + 15.3^2} = 15.6\ m/s$$

Note: Angle is not the same as throwing directly to your friend. The "direct" angle is 63.4°.

Projectile Motion: Summary

- Velocity, position, and acceleration are vectors
  - They have *both* magnitude and direction
  - Vector magnitude: $|A| = \sqrt{A_x^2 + A_y^2}$
  - Vector direction (described by an angle): $\theta = \tan^{-1}\frac{A_y}{A_x}$
- x- and y-directions are *independent*
- Kinematic Equations for 2-D: *Must be able to identify variables in these equations!*

$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$
$v_x = v_{0_x} + a_x t$	$v_y = v_{0_y} + a_y t$
$v_x{}^2 = v_{0x}{}^2 + 2a_x\Delta x$	$v_y{}^2 = v_{0y}{}^2 + 2a_y\Delta y$

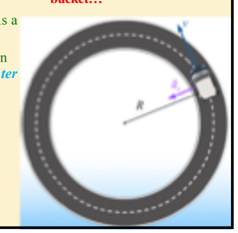
- *Projectile Motion*: a special case where $a_x = 0$ and $a_y = -g$

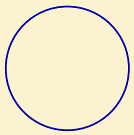
Motion in a Circle with Constant Speed: Uniform circular motion

(Here "uniform" means "constant speed")

- If an object moves ***with constant speed v*** in a perfect circle of radius *r* then:
  - Its velocity vector is constantly changing direction (though its speed is constant). As a result, it must be ***accelerating***.
  - The magnitude of the object's acceleration is $a = v^2/r$ and is directed *towards the* ***center of the circle***. *(Centripetal Acceleration)*
- Unless the acceleration is v^2/r, the motion will not be circular with constant speed.
- Note: A car *could* also have a "tangential acceleration" in addition to it centripetal acceleration.

Demo: Consider the wine glass on a plate, water in bucket…





Summary of Concepts

- Projectile Motion
  - Kinematic Equations for 2-D: *Must be able to identify variables in these equations!*

$x = x_0 + v_{0x}\, t + \frac{1}{2} a_x t^2$	$y = y_0 + v_{0y}\, t + \frac{1}{2} a_y t^2$
$v_x = v_{0_x} + a_x t$	$v_y = v_{0_y} + a_y t$
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  - *Projectile Motion*: a special case where $a_x = 0$ and $a_y = g$
- Uniform Circular Motion
  - Speed is constant
  - Direction is changing
  - Acceleration toward center $a_c = v^2 / r$, and $a_t = 0$