Physics 101: Lecture 10

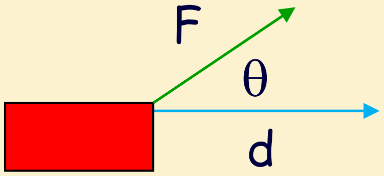
Potential Energy & Energy Conservation

Announcements

- Exam 1 this week, Wed-Fri (hope you signed up)
- Formula sheet has been posted on the class home page.
- HW deadline for exams weeks has been pushed back to Sat. 8am.
- Review session in Altgeld Hall, Room 314 **TODAY (Monday) at 6pm.** Bring problems & questions. If there are no questions then we will solve problems from spring 2007 exam (available in course web site).

Review

- Work: Transfer of Energy by Force
  - $W_F = F\, d \cos\theta$



- Kinetic Energy (Energy of Motion)
  - $K = ½\, mv^2$

- Work-Kinetic Energy Theorem:
  - $W_{Net} = \Delta K$ (work done ON object by all forces = change in kinetic energy)
    $= K_f - K_i$

Today:

- Potential (Stored) Energy: U

Work Done by Gravity 1

- Example 1: Drop ball

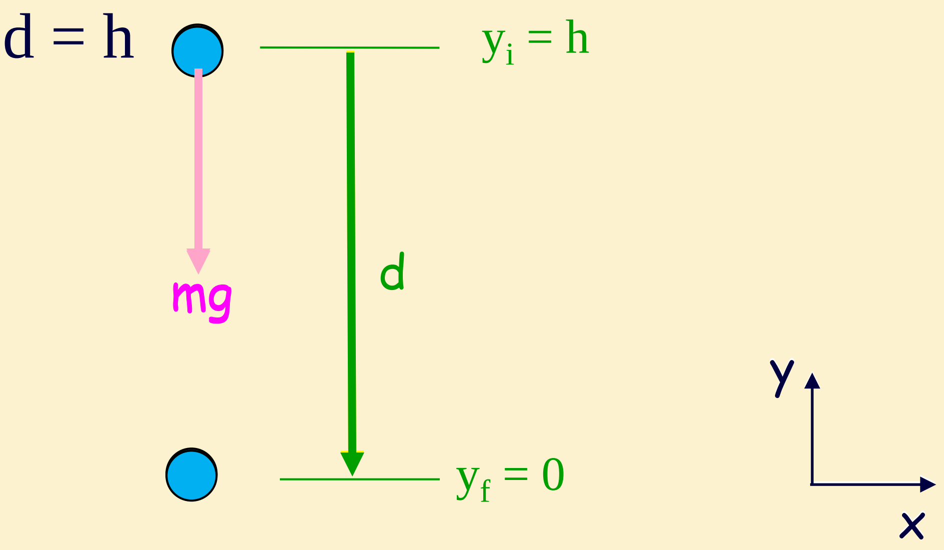
$W_g = Fd\cos\theta$

$W_g = (m\ g)(d)\cos(0^0)$

$W_g = m\ g\ h$

$\Delta y = y_f - y_i = -h$

$$\boxed{W_g = -m\ g\ \Delta y}$$



Note: This work is positive overall

Work Done by Gravity 2

- Example 2: Toss ball up

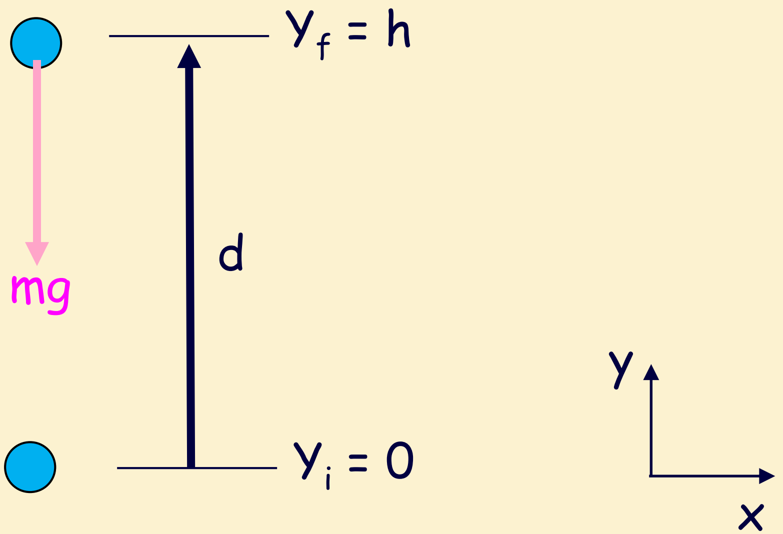
$W_g = (mg)(d)\cos\theta$

$d = h$

$W_g = m\,g\,h\cos(180^0) = -m\,g\,h$

$\Delta y = y_f - y_i = +h$

$\boxed{W_g = -m\,g\,\Delta y}$

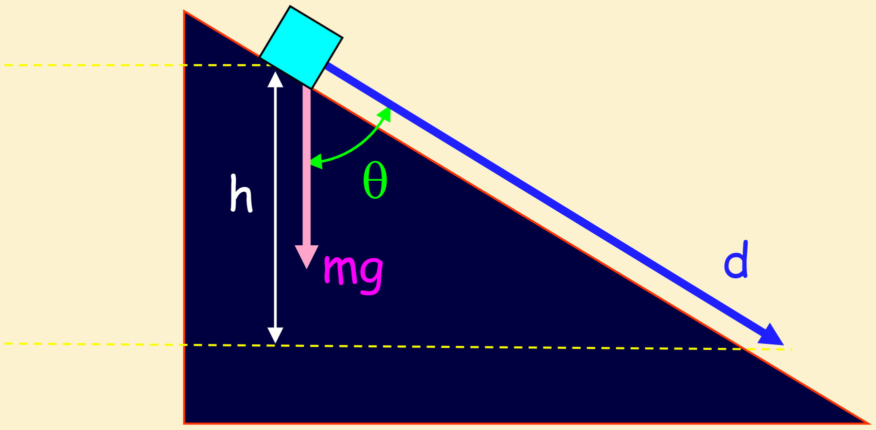


Note: This work is negative overall

Work Done by Gravity 3

A) W>0   B) W=0   C) W<0

- Example 3: Slide block down incline



$$W_g = (mg)(d)\cos\theta$$

$$d\cos\theta = h$$

Work and Potential Energy

- Work done by gravity is ***independent of path***
  - $W_g = -m\,g\,(y_f - y_i) = -mg\Delta y$

- True for any CONSERVATIVE force, like gravitation and springs (all others are non-cons.)

- Define potential energy $U_g = m\,g\,y$

$$W_{cons} = -\Delta U = -(U_f - U_i) = -(m\,g\,y_f - m\,g\,y_i)$$

- Work-Energy theorem

$$W_{net} = W_{cons} + W_{nc} = \Delta K$$

$$W_{nc} = \Delta K - W_{cons} = \Delta K + \Delta U$$

Work done by non-conservative forces

(e.g., frictional force)

How gravitational potential energy and work done by gravity "works"

Example: Raise brick of mass M from floor to height H.

Work done by gravity is: $-M\,g\,H$

(F is down and "d=H" is up so angle is 180^o, so

$W_g = (Mg)\,d\cos 180^o = -M\,g\,H$)

Change in potential energy is $Mg(h_f - h_i) = M\,g\,H$

So… $W = -\Delta U$.

NOTE: Your hand does positive work $= M\,g\,H$

What if we lower the brick from H to the floor?

Energy Conservation

$$W_{nc} = \Delta K + \Delta U = \Delta(K + U) = \Delta E$$

Define MECHANICAL energy E, as the sum of kinetic and potential energies:

$$E = K + U$$

If there is no work done by non-conservative forces, then

$$W_{nc} = \Delta E = 0$$

Thus TOTAL mechanical energy does not change. E is CONSERVED when non-conservative forces do no work.

$$E_i = E_f$$

Skiing Example (no friction)

A skier goes down a 78 meter high hill with a variety of slopes. What is the maximum speed she can obtain if she starts from rest at the top?

No friction ⇒ Mechanical Energy is conserved!

Total Mechanical Energy Before:

$E_i = K_i + U_i = \frac{1}{2} m v_i^2 + mgy_i = mgy_i$

Total Mechanical Energy After:

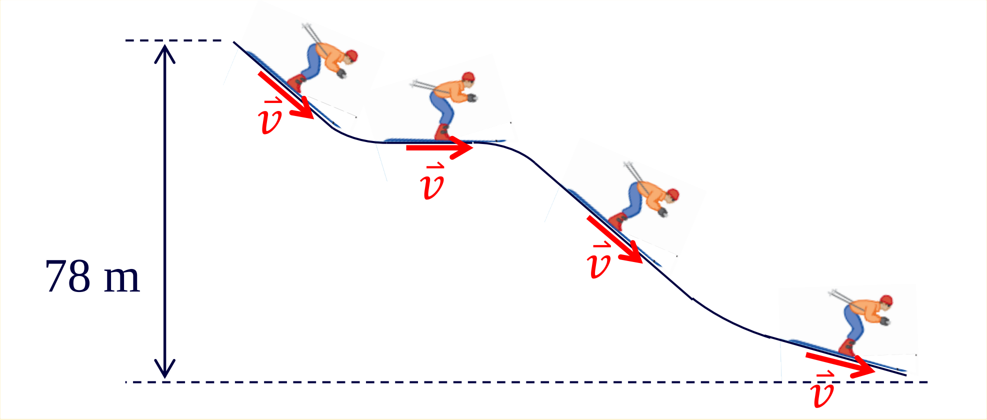
$E_f = K_f + U_f = \frac{1}{2} m v_f^2 + mgy_f = \frac{1}{2} m v_f^2$

Conserve E!

$E_i = E_f$

$mgy_i = \frac{1}{2} m v_f^2$

$v_f = \text{sqrt}(2gy) = 39 \text{ m/s}$



Pendulum Demo

Conservation of Energy (E_0=E_f)

Total Mechanical Energy Before:

$E_i = K_i + U_i = mgy_0$

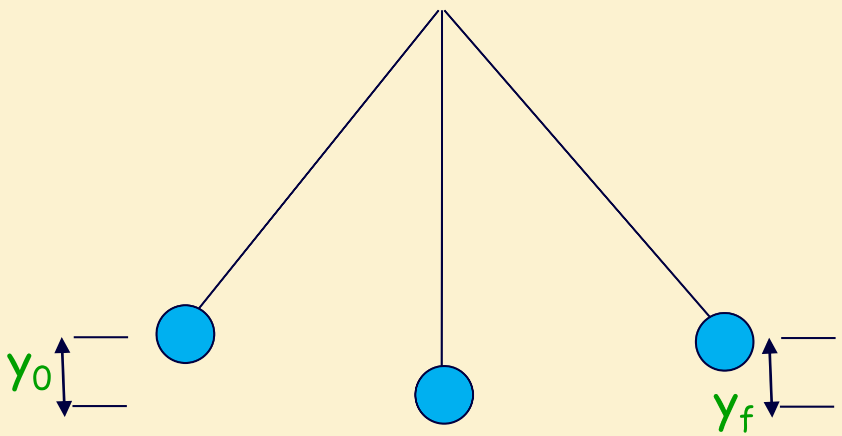
Total Mechanical Energy After:

$E_f = K_f + U_f = mgy_f$

Conserve E!

$E_i = E_f$ $\qquad$ $mgy_0 = mgy_f$

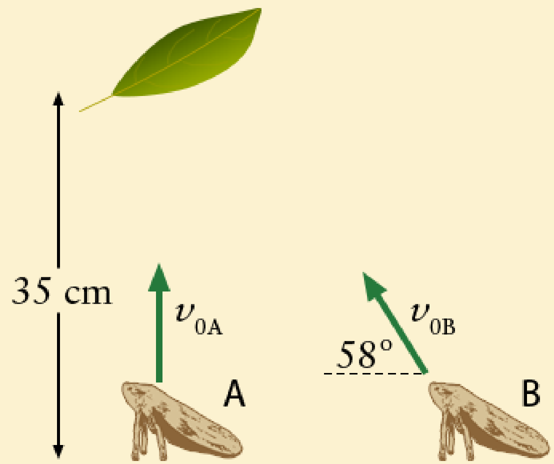
$y_f = y_0$



Checkpoint 1

The average froghopper insect, the "world's greatest leaper", is about 6 mm (0.2 inches) long and has a mass of 12.3 milligram. Its highest jumps reach 70 cm (27 inches).
Two froghoppers sitting on the ground aim at the same leaf, located 35 cm above the ground. Froghopper A jumps straight up while froghopper B jumps at a takeoff angle of 58^{o} above the horizontal. Which froghopper experiences the greatest change in kinetic energy from the start of the jump to when it reaches the leaf?

A) froghopper A
B) froghopper B
C) same for both



Potential energy stored in springs

Spring force is **conservative**.

Thus work done by springs can be written as the negative change in potential energy:

$$W_{spring} = -\Delta U_{spring}$$

The potential energy in a spring at a distance *d* from its resting position is:

$$U_{spring} = \frac{1}{2} kd^2$$

Checkpoint 2

A box of mass m sliding with an initial speed of v_i on a rough horizontal surface runs into a fixed spring of elastic constant k, compressing it a distance x_1 from its relaxed position while momentarily coming to rest. Which equation best represents the energy conversion during the process of the box compressing the spring?

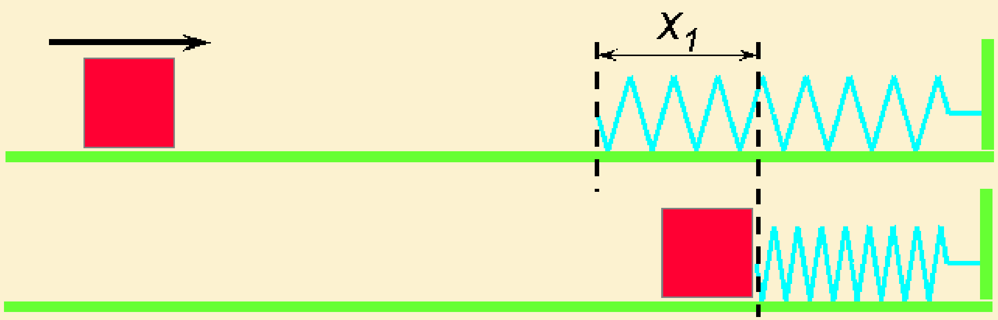
A) $\frac{1}{2}mv_i^2 + \frac{1}{2}kx_1^2 + W_{due\ to\ friction\ with\ surface} = 0$

B) $\frac{1}{2}mv_i^2 + W_{due\ to\ friction\ with\ surface} = \frac{1}{2}mv_f^2 - \frac{1}{2}kx_1^2$

C) $\frac{1}{2}mv_i^2 = \frac{1}{2}kx_1^2$

D) $W_{due\ to\ friction\ with\ surface} = \frac{1}{2}kx_1^2$

E) $\frac{1}{2}mv_i^2 + W_{due\ to\ friction\ with\ surface} = \frac{1}{2}kx_1^2$



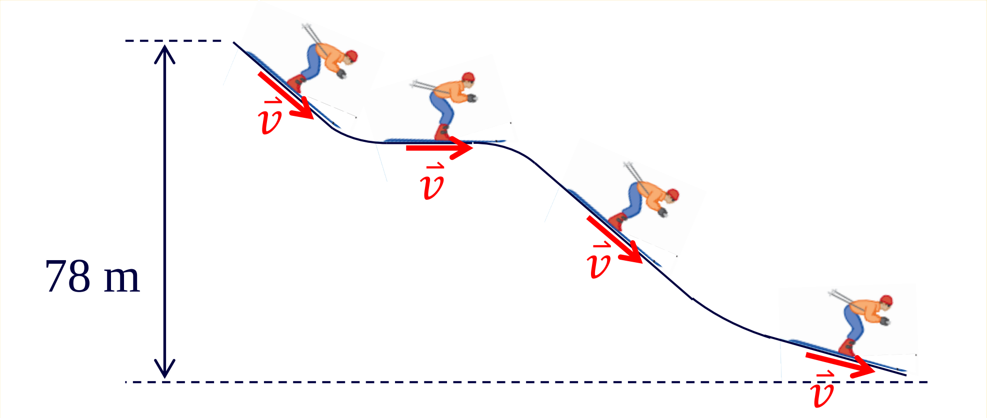
Skiing w/ Friction

A 50 kg skier goes down a 78 meter high hill with a variety of slopes. She finally stops at the bottom of the hill. If friction is the force responsible for her stopping, how much work does it do?

Total Energy changes when friction is present!
(friction is NONCONSERVATIVE)

Total Energy Before:

$E_i = K_i + U_i = mgy_i$



Total Energy After:

$E_f = K_f + U_f = 0$

Change in Energy is work done by friction!

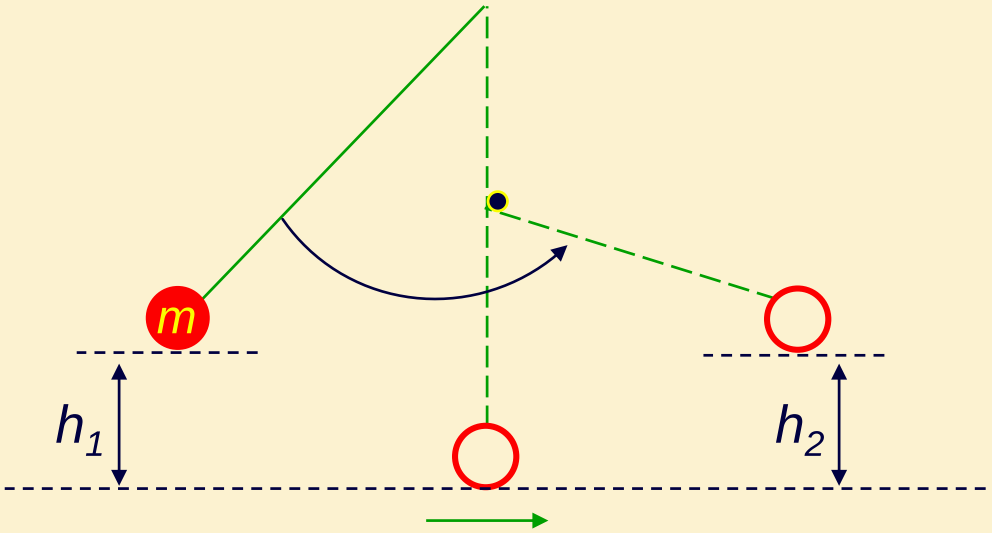
$W_{nc} = \Delta E = 0 - mgy_i$

$= -38200$ Joules

Clicker Q

How high will the pendulum swing on the other side now?

A) $h_1 > h_2$ B) $h_1 = h_2$ C) $h_1 < h_2$



Power (Rate of Work)

- $P = W / \Delta t$
  - ➔ Units: Joules/Second = Watt

- How much power does a 70 kg student expend while running up the stairs in 141 Loomis (5 meters in 7 sec)?

$P = W / t$

$= m\,g\,h\ / t$

$= (70\text{ kg})\ (9.8\text{ m/s}^2)\ (5\text{ m}) / 7\text{ s}$

$= 490\text{ J/s}$ or 490 Watts

Example

A block of mass M slides down a frictionless ramp from height H, then enters a rough horizontal region, then compresses a spring having spring constant k a distance D. How much work was done by kinetic friction in terms of M, k, D, and H?

Big Idea: Apply the Work-Kinetic Energy Theorem

Justification: W-KE Theorem relates work to K and U; these are related to M, k, D & H

Plan: 1) Apply W-KE Theorem: $W_{nc} = \Delta E$

2) Friction is only non-conservative force doing work, $W_{nc}=W_{fric}$

3) Write E=K+U in final and initial states. Subtract them. Initial state has only gravitational U. Final state has only spring U.

$$W_{fric} = E_f - E_i$$

$$= (K_f + U_f) - (K_i + U_i) = U_{spring,f} - U_{grav,i}$$

$$= (1/2)kD^2 - MgH$$

Summary

- Conservative Forces
  - » Work is independent of path
  - » Define Potential Energy U
    - $U_{gravity} = m\,g\,y$
    - $U_{spring} = \frac{1}{2}\,k\,x^2$
- Work-Energy Theorem

$$W_{nc} = \Delta E = \Delta(K + U)$$
$$= \Delta K + \Delta U$$
$$W_{nc} + W_{cons} = W_{tot} = \Delta K$$

- Power: $P = W / \Delta t$