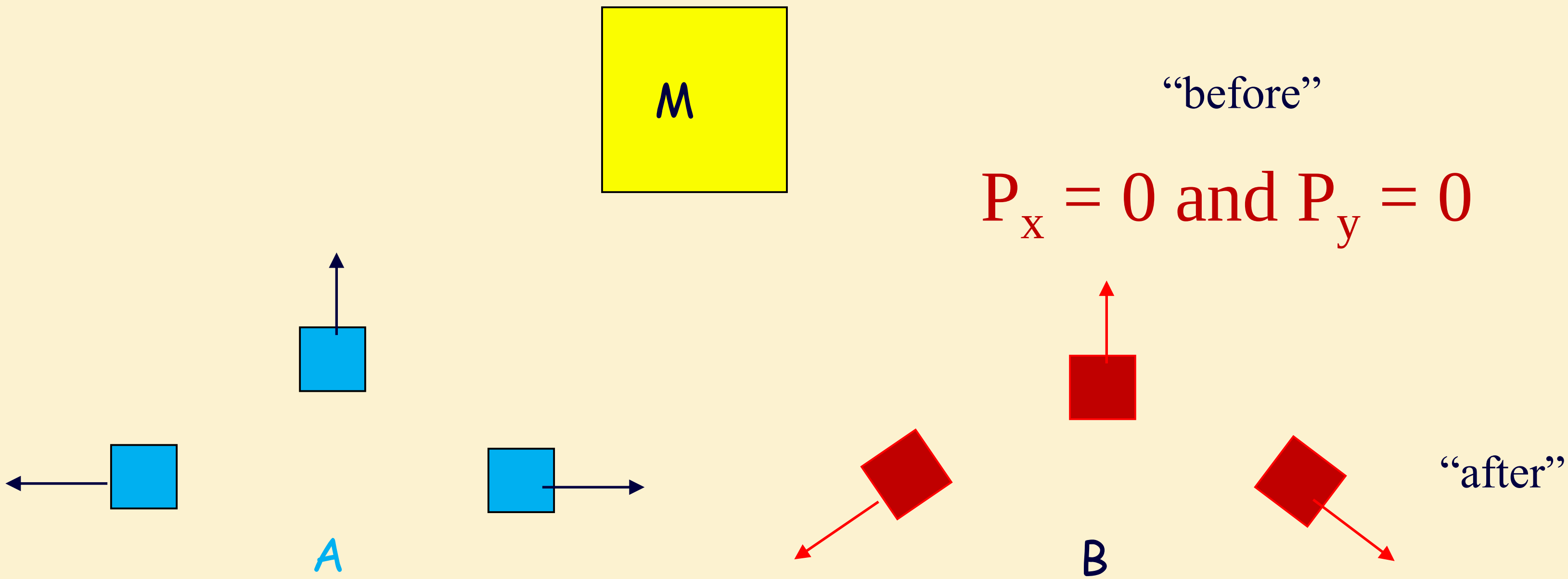
Physics 101: Lecture 13

Rotational Motion, Kinetic Energy and Rotational Inertia

Explosions Clicker Q



$P_{Net,\,x} = 0$, but $P_{Net,\,y} > 0$

$P_{Net,\,x} = 0$, and $P_{Net,\,y} = 0$

Which of these is possible? (Ignore friction and gravity)

A

B

C = both

D = Neither

Center of Mass

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{\sum m_i}$$

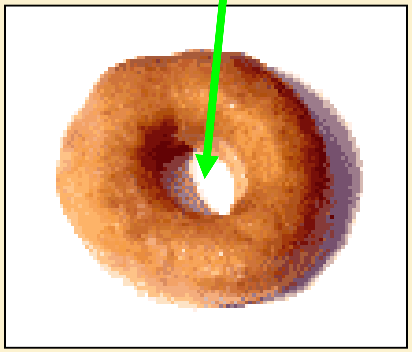
Center of Mass = Balance point

In practice do the above in x and y directions separately

$$x_{cm} = \frac{m_1x_1 + m_2x_{2+}\ldots}{\sum m_i} \qquad y_{cm} = \frac{m_1y_1 + m_2y_{2+}\ldots}{\sum m_i}$$

Center of Mass!

Shown is a yummy doughnut. Where would you expect the center of mass of this breakfast of champions to be located?



Center of Mass

$P_{tot} = M_{tot} V_{cm}$

$(P_{tot})/M_{tot} = V_{cm}$

$F_{ext} \Delta t = \Delta P_{tot} = M_{tot} \Delta V_{cm}$

So if $F_{ext} = 0$ then V_{cm} is constant

Also: $F_{ext} = M_{tot} a_{cm}$

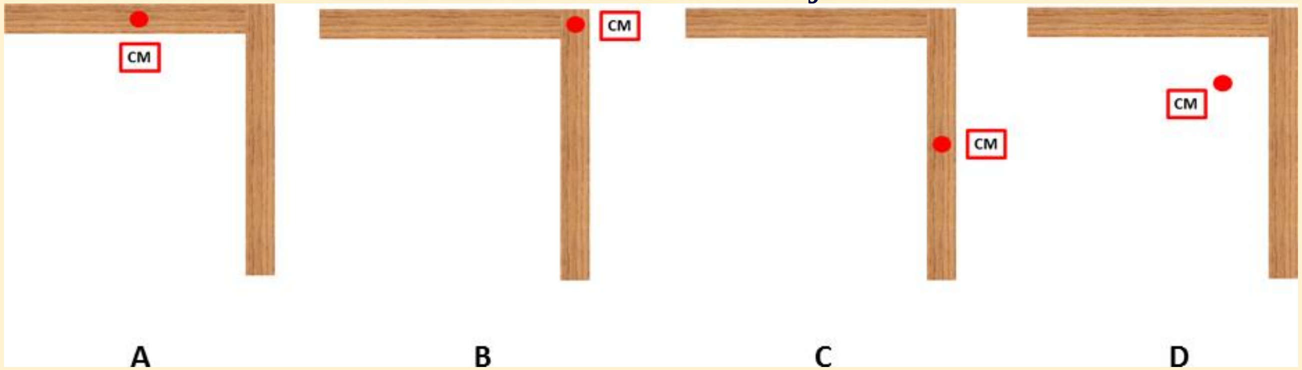
Center of Mass of a system behaves in a SIMPLE way

- moves like a point particle!
- velocity of CM is unaffected by collision if $F_{ext} = 0$

(pork chop demo)

Checkpoint 3 / Lect 12

An object is designed from two sheets of oak wood having the same mass and length, as shown in the figure. Where is the center of mass of the object located?



Recall from a previous commentary (Lec 5):

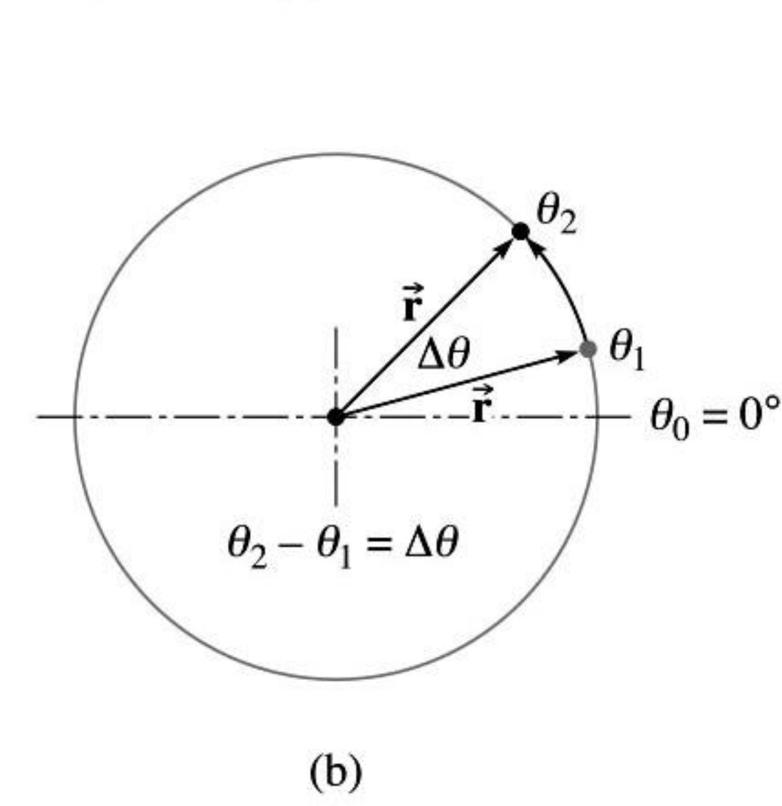
One of you said: This stuff… has the potential to be cool.

I responded: I agree. You can even make money with it—more later

Making money with physics
(aka: Making money at bars)

Lesson #1: The center of mass of humans

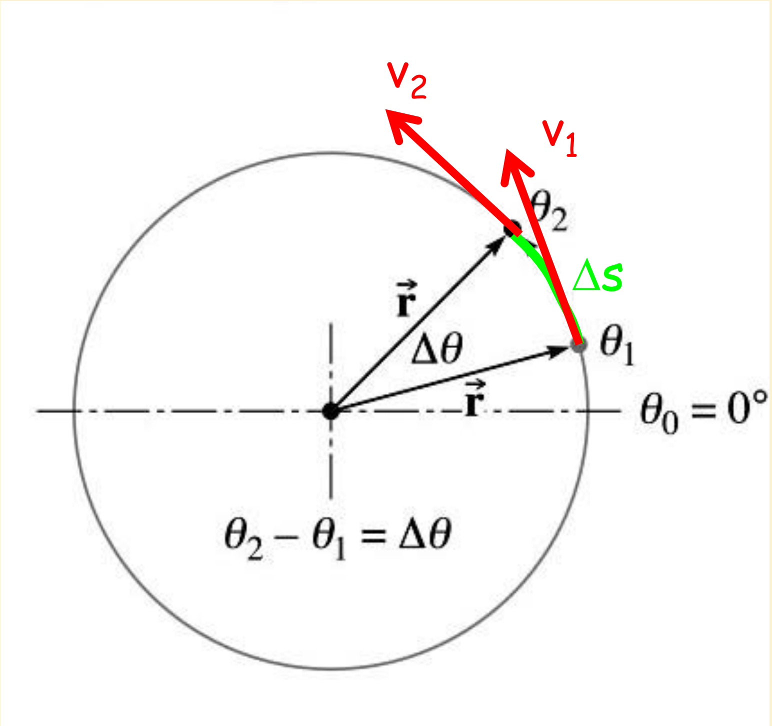
Circular Motion



- Angular displacement $\Delta\theta = \theta_2 - \theta_1$
  - ➔ How far it has rotated
  - ➔ Units: radians (2π = 1 revolution)
- Angular velocity $\omega = \Delta\theta/\Delta t$
  - ➔ How fast it is rotating
  - ➔ Units: radians/second
- Frequency measures revolutions per second: $f=\omega/2\pi$
- Period =1/frequency $T = 1/f = 2\pi / \omega$
  - ➔ Time to complete 1 revolution

Circular to Linear

- Displacement $\Delta s = r\,\Delta\theta$ (θ in radians)
- Speed $|v| = \Delta s/\Delta t = r\,\Delta\theta/\Delta t = r\omega$
- Direction of v: tangent to circle

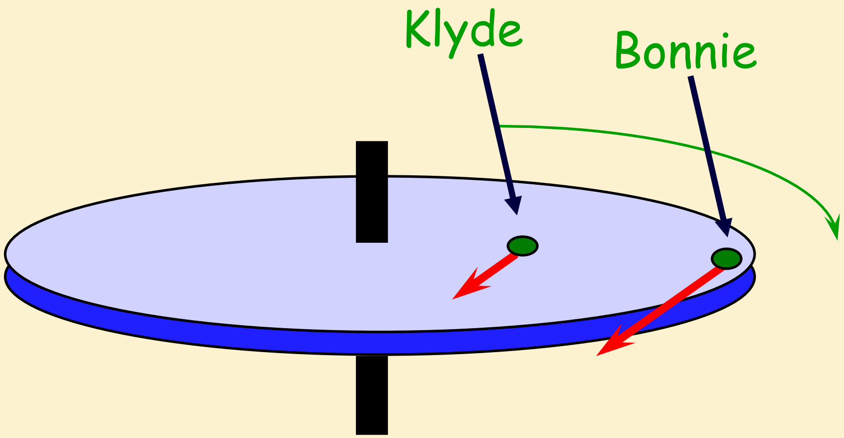


(b)

Merry-Go-Round Clicker Q1

- Bonnie sits on the outer rim of a merry-go-round with radius 3 meters, and Klyde sits midway between the center and the rim. The merry-go-round makes one complete revolution every two seconds (demo).

  ➔Klyde's speed is:



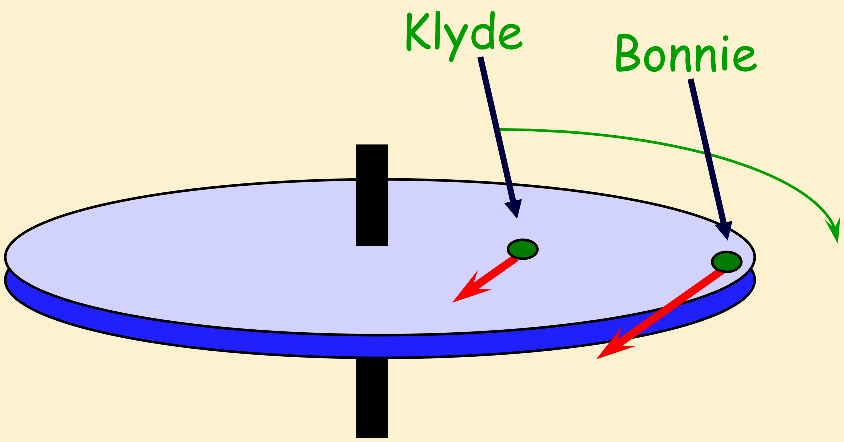
(a) **the same as Bonnie's**

(b) **twice Bonnie's**

(c) **half Bonnie's**

Merry-Go-Round Clicker Q2

- Bonnie sits on the outer rim of a merry-go-round, and Klyde sits midway between the center and the rim. The merry-go-round makes one complete revolution every two seconds.
  - ➔Klyde's angular velocity is:



(a) **the same as Bonnie's**

(b) **twice Bonnie's**

(c) **half Bonnie's**

Angular Acceleration

- Angular acceleration is the change in angular velocity ω divided by the change in time.

$$\bar{\alpha} \equiv \frac{\omega_f - \omega_0}{\Delta t}$$

- Example: If the speed of a roller coaster car is 15 m/s at the top of a 20 m loop, and 25 m/s at the bottom. What is the car's average angular acceleration if it takes 1.6 seconds to go from the top to the bottom?

$$\omega = \frac{V}{R} \qquad \omega_f = \frac{25}{10} = 2.5 \qquad \omega_0 = \frac{15}{10} = 1.5$$

$$\bar{\alpha} \equiv \frac{2.5 - 1.5}{1.6} = 0.64 \text{ rad/s}^2$$

Linear and Angular Motion

	Linear	Angular	
Displacement	x	θ	
Velocity	v	ω	
Acceleration	a	α	
Inertia	m	I	Today
KE	$\frac{1}{2} m v^2$	$\frac{1}{2} I\omega^2$	Today
Newton's 2^{nd}	$F=ma$	coming	
Momentum	$p = mv$	coming	

Angular kinematic equations (with comparison to 1-D kinematics)

Angular	Linear
α=constant	a=constant
$\omega = \omega_o + \alpha t$	$v = v_o + at$
$\theta = \theta_0 + \omega_o t + ½\alpha t^2$	$x = x_o + v_o t + ½at^2$
$\omega^2 = \omega_o^2 + 2\alpha\,\Delta\theta$	$v^2 = v_o^2 + 2a\,\Delta x$

$x \to \theta$

$v \to \omega$

$a_t \to \alpha$

$x = R\theta$ $\quad v = \omega R$ $\quad a_t = \alpha R$ $\quad a_c = v^2/R = R\,\omega^2$

CD Player Example

- The CD in a disk player spins at about 20 radians/second. If it accelerates uniformly from rest with angular acceleration of 15 rad/s^2, how many revolutions does the disk make before it reaches its final angular speed of 20 radians/second?
- Plan: Use angular kinematics first to find θ in radians, and then convert to revolutions using 1 rev = 2π rad

$$\omega_0 = 0$$
$$\omega_f = 20 \text{ rad/s}$$
$$\alpha = 15 \text{ rad/s}^2$$
$$\Delta\theta = ?$$

$$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\frac{\omega_f^2 - \omega_0^2}{2\alpha} = \Delta\theta$$

$$\frac{20^2 - 0^2}{2 \times 15} = \Delta\theta$$

Δθ = 13.3 radians

1 Revolution = 2 π radians

Δθ = 13.3 radians

= 2.12 revolutions

Axes and sign
(i.e. what is positive and negative)

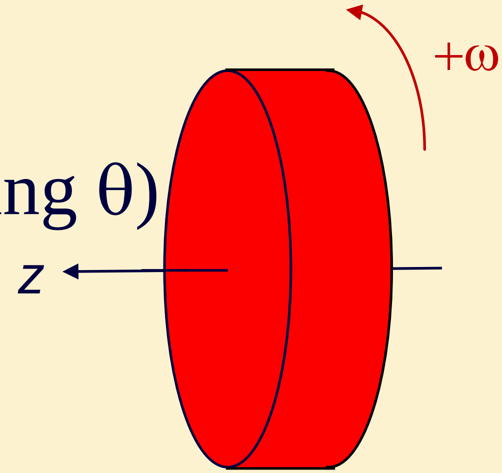
Whenever we talk about rotation, it is implied that there is a rotation "axis". We need a way of distinguishing + from – rotations.

This is typically called the "z" axis (we usually omit the z subscript for simplicity).

Counter-clockwise rotations: (increasing θ) will be positive

Clockwise rotations: (decreasing θ) will be negative.

[demo: bike wheel].



Energy Clicker Q and demo

When the bucket reaches the bottom, its potential energy has decreased by an amount mgh. Where has this energy gone?

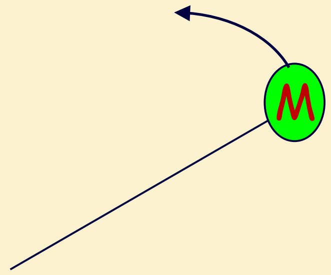
A) Kinetic Energy of bucket

B) Kinetic Energy of flywheel

C) Both 1 and 2.

Rotational Kinetic Energy

- Consider a mass M on the end of a string being spun around in a circle with radius r and angular frequency ω [demo]
  - ➔ Mass has speed $v = \omega r$
  - ➔ Mass has kinetic energy
    - » $K = \frac{1}{2} M v^2 = \frac{1}{2} M (\omega r)^2$
    - » $= \frac{1}{2} M \omega^2 r^2$
    - » $= \frac{1}{2} (M r^2) \omega^2$
    - » $\boxed{= \frac{1}{2} I \omega^2}$



I is "moment of inertia" and is the equivalent of mass for rotational motion (don't confuse I w/ impulse)

- Rotational Kinetic Energy is energy due to circular motion of object.

Rotational Inertia I
(or *moment of inertia*)

- Tells how "hard" it is to get an object rotating. Just like mass tells you how "hard" it is to get an object moving.

  ➔ $K_{tran} = ½ \, m \, v^2$ Linear Motion
  ➔ $K_{rot} = ½ \, I \, \omega^2$ Rotational Motion

- $I = \Sigma \, m_i r_i^2$ (units: kg m^2; I plays same role in rotational motion that mass plays in linear motion)

- Note! rotational inertia (or *moment of inertia*) changes if the axis of rotation changes.

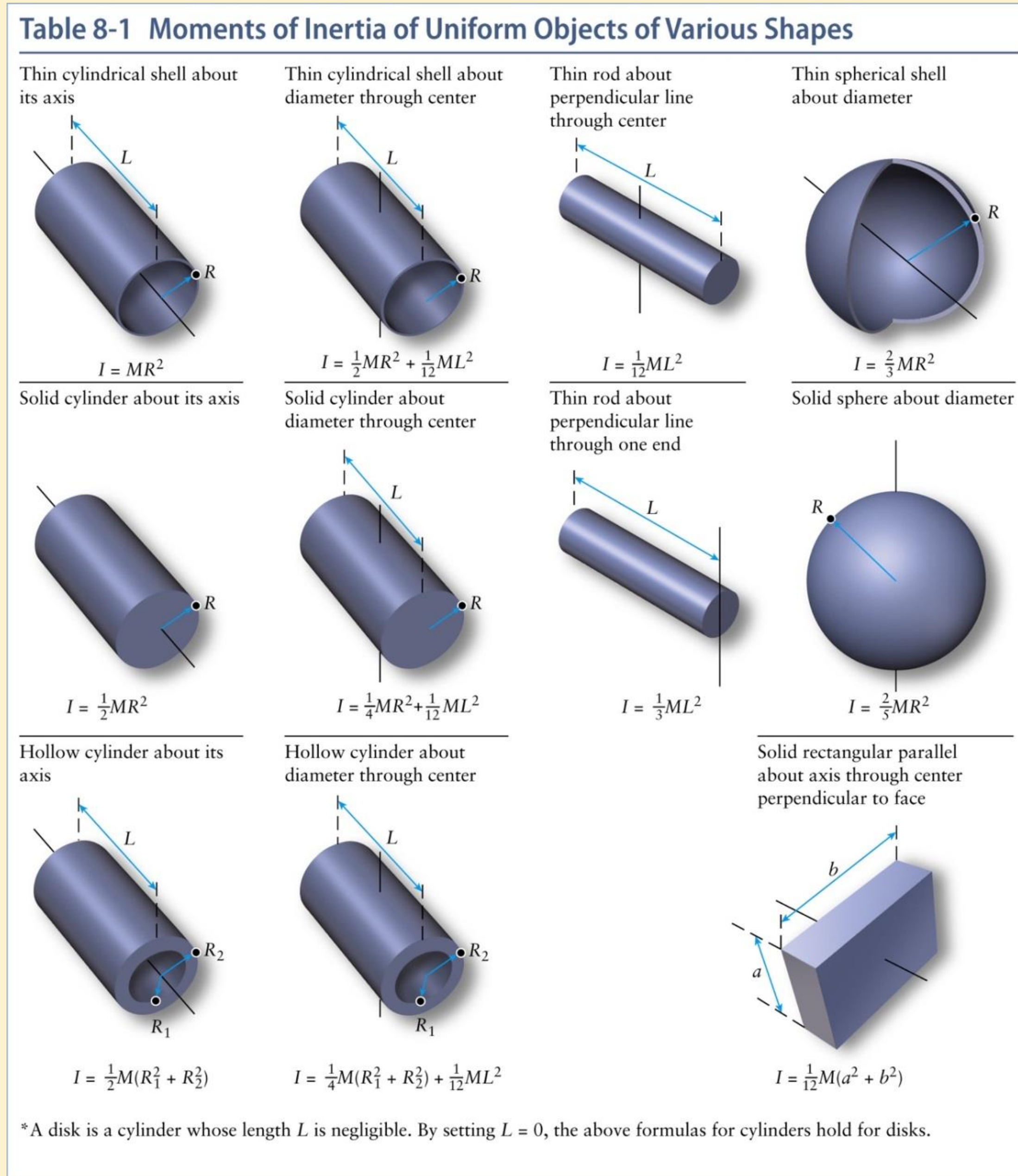
Moment of Inertia Table

Different shapes have different I.

For point masses, use $I = \Sigma\, m\, r^2$.

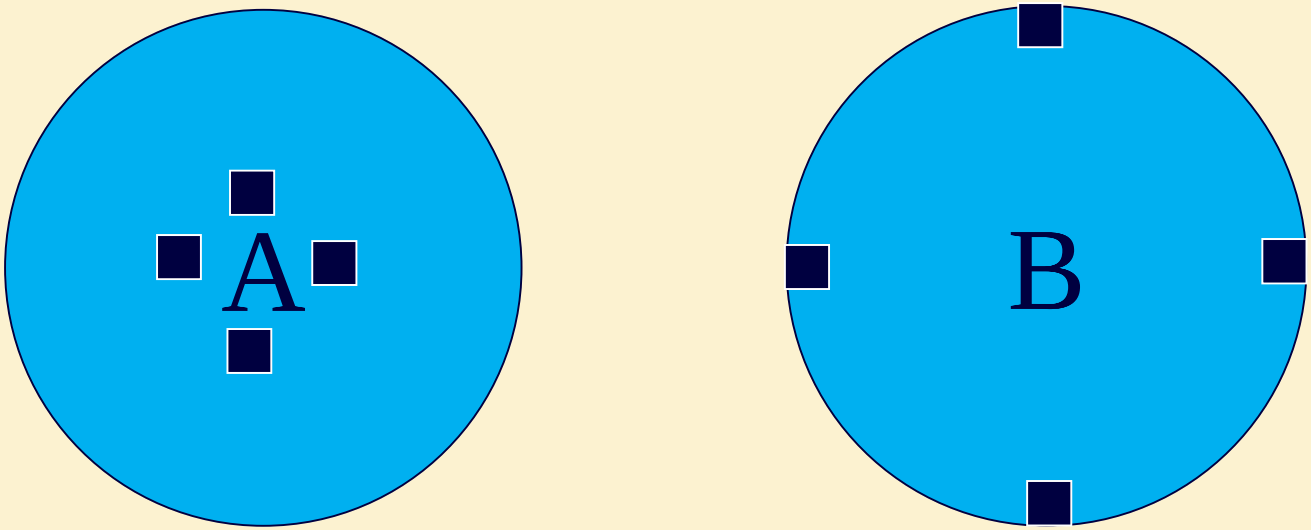
For extended objects, use this table.

These are computed with calculus.



Merry Go Round Clicker Q

Four kids (mass m) are riding on a (light) merry-go-round rotating with angular velocity ω=3 rad/s. In case A the kids are near the center (r=1.5 m), in case B they are near the edge (r=3 m). Compare the kinetic energy of the kids on the two rides.



A) $K_A > K_B$ B) $K_A = K_B$ C) $K_A < K_B$

Inertia Rods Clicker Q

Two batons have equal mass and length.
Which will be "easier" to spin?

A) Mass on ends

B) Same

C) Mass in center

Main Ideas

- Center of Mass
- Rotational Kinematics is just like linear kinematics with parallel equations of motion
- Rotating objects have kinetic energy
  - ➔ $KE = \frac{1}{2} I \omega^2$
- Moment of Inertia $I = \Sigma mr^2$
  - ➔ Depends on Mass
  - ➔ Depends on axis of rotation
- Energy is conserved but need to include rotational energy too: $K_{rot} = \frac{1}{2} I \omega^2$