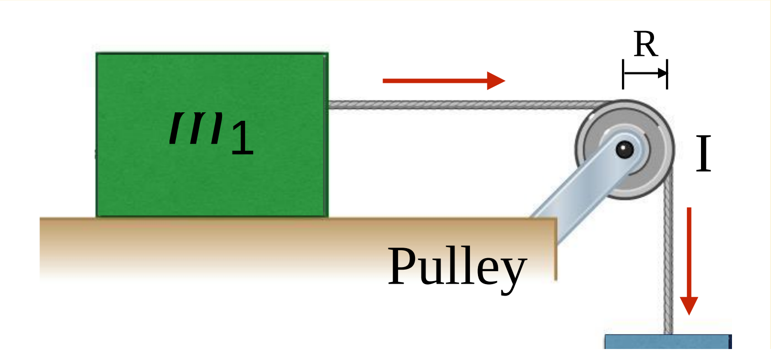
Physics 101: Lecture 15

Torque, F=ma for rotation, and Equilibrium

Massless Pulley, no friction Example

Consider the two masses connected by a pulley as shown. Use conservation of energy to calculate the speed of the blocks after m_2 has dropped a distance h. Assume the pulley is massless.



$$W_{nc} = \Delta E = (K_f + U_f) - (K_i + U_i)$$

Big Idea: Conservation of mechanical energy

$$U_{initial} + K_{initial} = U_{final} + K_{final}$$

Justification: Non-conservative forces do no work, so E conserved

$$m_2 gh = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

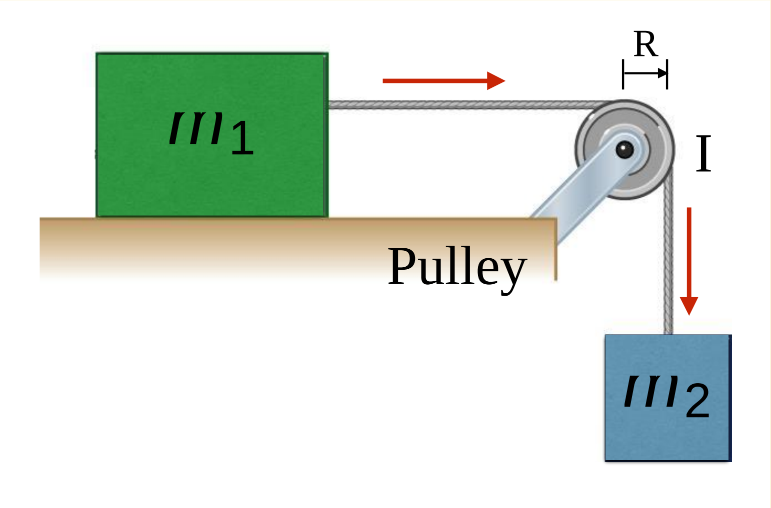
$$2m_2 gh = m_1 v^2 + m_2 v^2$$

Plan: 1) Set $E_i = E_f$
2) Solve for v

$$v = \sqrt{\frac{2m_2 gh}{m_1 + m_2}}$$

Massive Pulley, no friction Clicker Q

Consider the two masses connected by a pulley as shown. If the pulley is massive, after m_2 drops a distance h, the blocks will be moving



A) faster than

B) the same speed as

C) slower than

if it was a massless pulley

$$U_{initial} + K_{initial} = U_{final} + K_{final}$$

$$m_2 gh = +\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{1}{4}Mv^2$$

$$m_2 gh = \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{1}{2}I\omega^2$$

$$m_2 gh = +\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2$$

Linear and Angular Motion

	Linear	Angular
Displacement	x	θ
Velocity	v	ω
Acceleration	a	α
Inertia	m	I
KE	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$
Force	**F**	**τ (torque)**
Newton's 2nd	**F=ma**	**$\tau = I\alpha$**
Momentum	p = mv	L = Iω

$x = R\theta$

$v = \omega R$

$a_t = \alpha R$

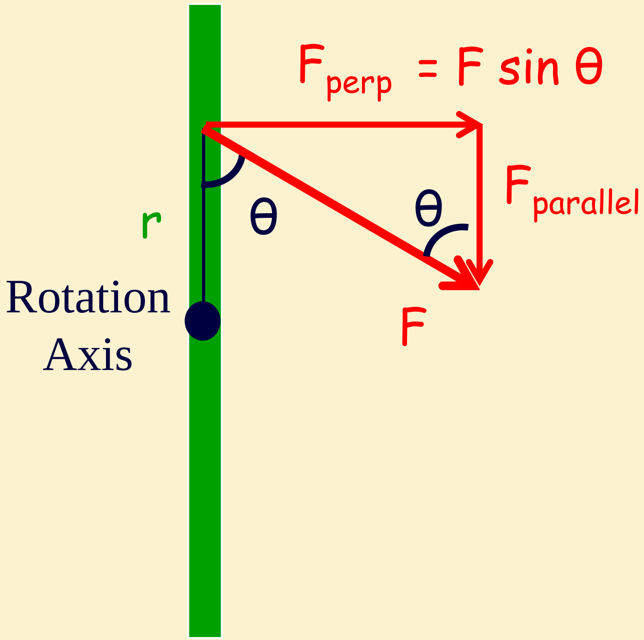
today

Torque Definition

- A TORQUE is a *force x distance* that causes rotation. It tells how effective a force is at twisting or rotating an object.

- $\tau = r\, F_{perpendicular} = r\, F \sin\theta$
  - ➔ Units N m
  - ➔ Sign: CCW rotation is positive

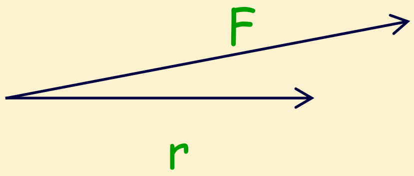
    CW rotation is negative



Equivalent ways to find torque:

1. Put r and F vectors *tail-to-tail* and compute

$$\tau = r\,F\,\sin\theta.$$



2. Decompose F into components parallel and perpendicular to r, and take:

$$\tau = r\,F_{\perp}$$

If rotation is clockwise, torque is negative, and if rotation is counterclockwise torque is positive.

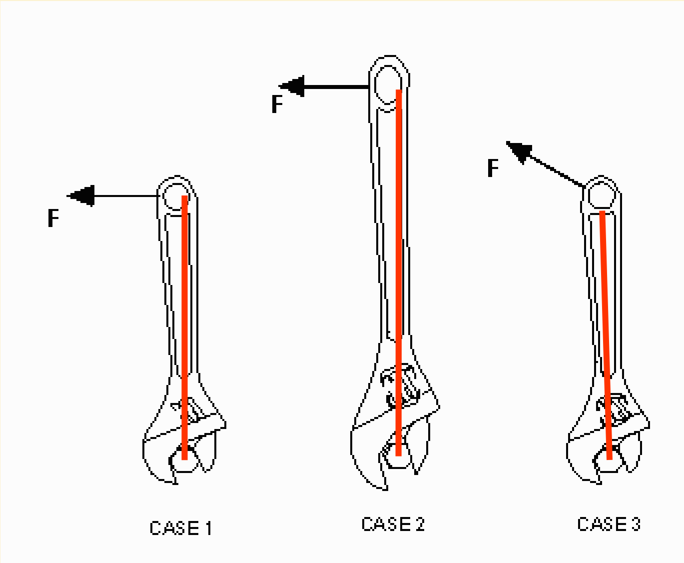
Note: If F and r are parallel or antiparallel, the torque is 0.
(can't open a door if pushing or pulling toward the hinges)

Wrench Clicker Q

The picture below shows three different ways of using a wrench to loosen a stuck nut. Assume the applied force F is the same in each case.

In which of the cases is the torque on the nut the biggest?

A. Case 1
B. Case 2
C. Case 3



Demo

Newton's Second Law for Rotation

- $\tau = I\alpha$ (compare to $F = ma$)
- Note analogy:

$F \longrightarrow \tau$,

$m \longrightarrow I$,

$a \longrightarrow \alpha$.

Equilibrium

- Conditions for Equilibrium
  - ➔ $F_{Net} = ma = 0$ Translational a of CM must be 0
  - ➔ $\tau_{Net} = I\alpha = 0$ Rotational α about any axis must be 0
    - » Choose axis of rotation wisely to make problems easier!
    - » But as long as you're consistent everything will be OK!

- A meter stick is suspended at the center. If a 1 kg weight is placed at x=0. Where do you need to place a 2 kg weight to balance it?

A) x = 25 B) x=50 C) x=75 D) x=100

E) 1 kg can't balance a 2 kg weight.

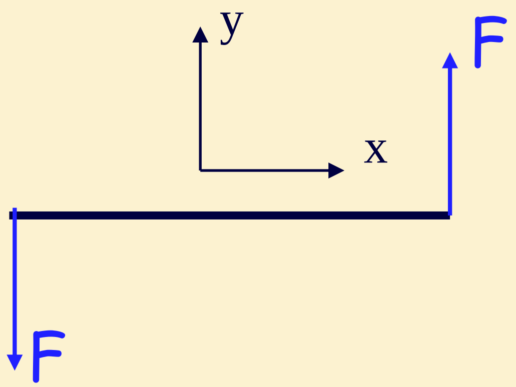
Balance Demo

Equilibrium: $a = 0, \alpha = 0$

- A rod is lying on a table and has two equal but opposite forces acting on it. The net force on the rod is:

Y direction: $F_{net\ y} = ma_y$

$+F - F = 0 = ma_y$

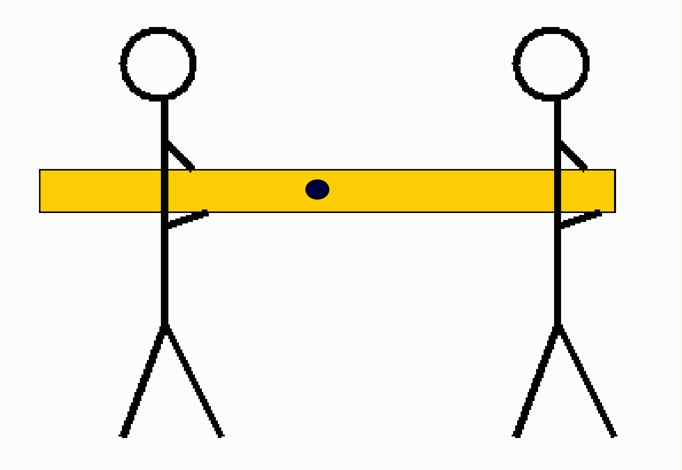


- The rod has no *a* in linear direction, so it **won't translate.** However, the rod will have a non-zero torque, hence a non-zero α and **will rotate.**

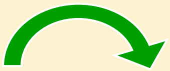
Clicker Q

The picture below shows two people lifting a heavy log. Which of the two people is supporting the greatest weight?

1. The person on the left is supporting the greatest weight
2. The person on the right is supporting the greatest weight
3. They are supporting the same weight



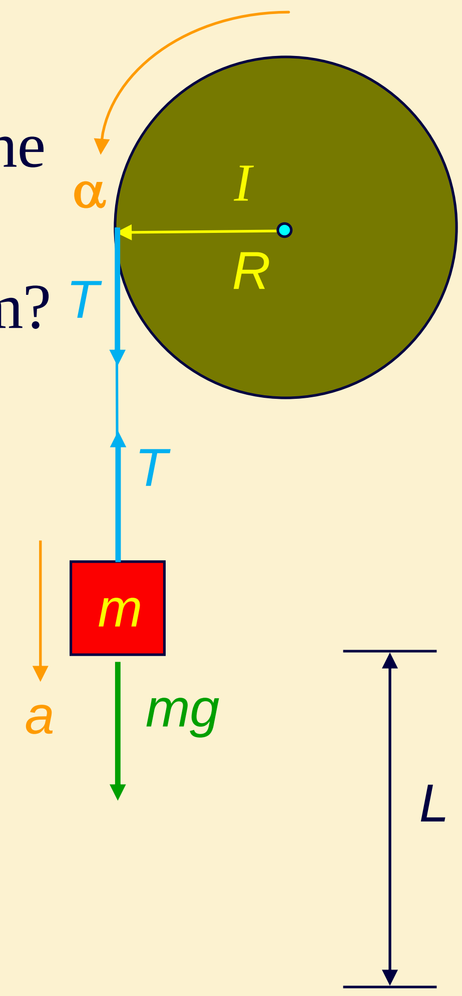
Rotational Newton's 2nd Law

- $\tau_{Net} = I\,\alpha$
  - ➔ Torque is amount of twist provided by a force
    - » Signs: positive = CCW 
    - » negative = CW 
  - ➔ Moment of Inertia = rotational mass. Large I means hard to start or stop rotation.
- Problems Solved Like Newton's 2nd
  - ➔ Draw FBD
  - ➔ Write Newton's 2nd Law in linear and/or rotational form, then use algebra.

Falling weight & pulley example

- A mass *m* is hung by a string that is wrapped around a disk of radius *R* and mass M. The moment of inertia of the disk is $I=(1/2)\,MR^2$. The string does not slip on the disk.

What is the acceleration, *a*, of the hanging mass, m?



What method should we use to solve this problem?

A) Conservation of Energy (including rotational)

B) $\tau_{Net} = I\alpha$ and $F=ma$

Falling weight & pulley... (need to find a)

Big Idea: N#2 in linear form for m and angular form for disk.

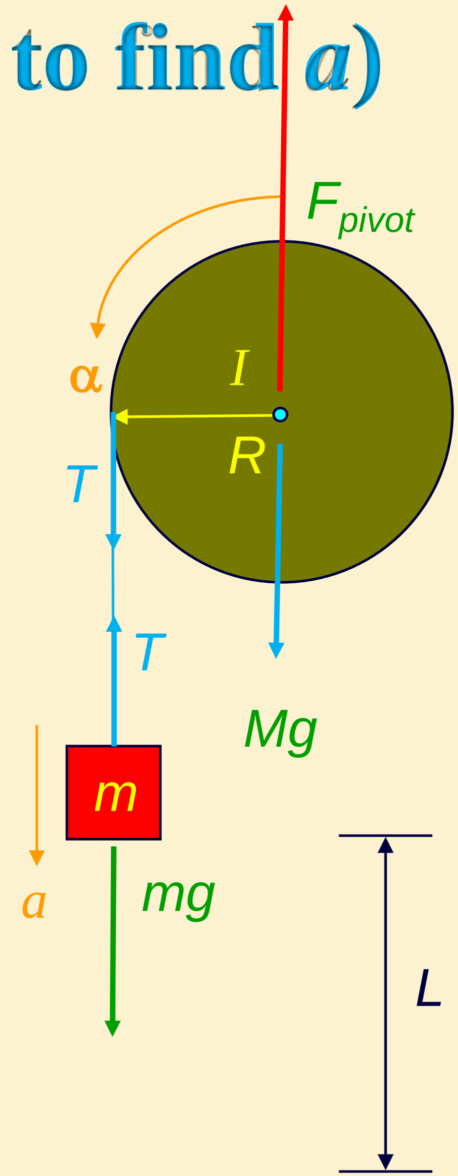
Justification: N#2 good for finding a and t.

Plan: 1. Draw a Free-Body Diagram

2. For the hanging mass apply $F_{Net} = ma$ and for disk apply $\tau = I\alpha$

3. Relate a and α using $a = \alpha R$ *(see slide 4)*

4. Use algebra to solve 3 equations in 3 unknowns, T, a, α.



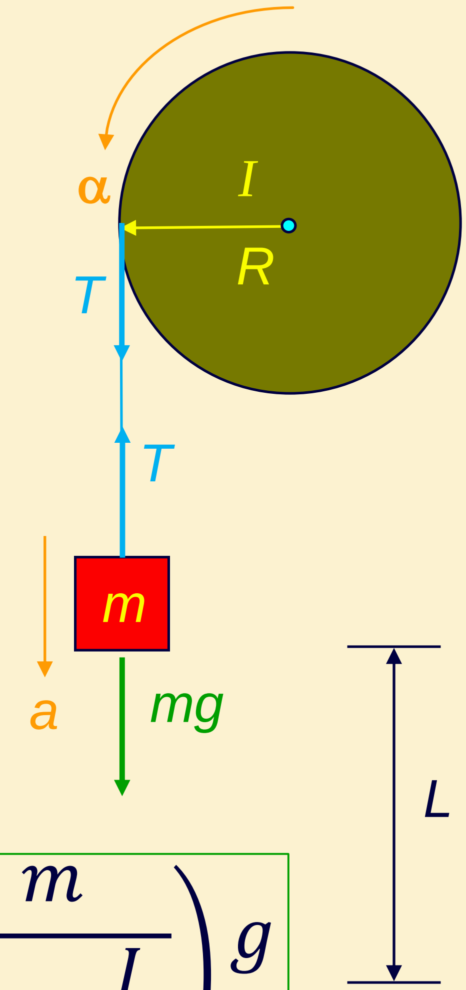
Falling weight & pulley... (need to find *a*)

$$\boxed{2: \quad T - mg = -ma}$$

$$\boxed{TR\sin(90) = I\alpha} \quad (I=1/2\ MR^2)$$

$$3: \quad \boxed{a = \alpha R}$$

4: math it to find *a*.

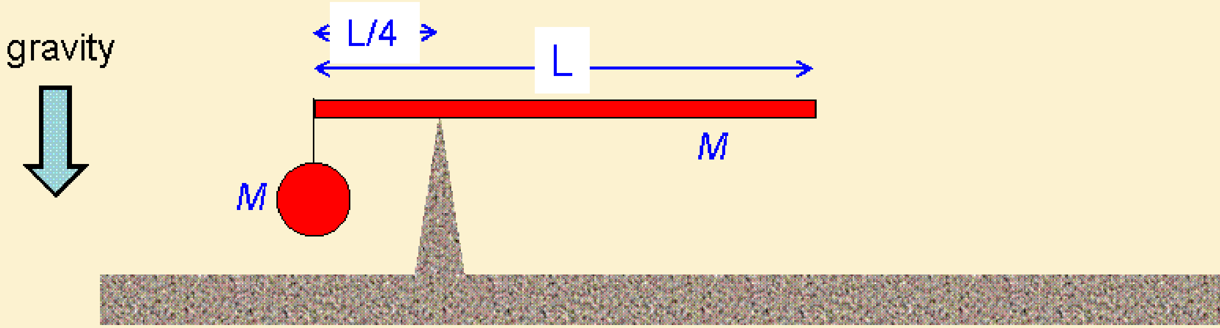


$$\boxed{a = \left(\frac{m}{m + \frac{I}{R^2}}\right) g}$$

Checkpoint 1

An object is made by hanging a ball of mass M from one end of a plank having the same mass and length L. The object is then placed on a support at a point a distance $L/4$ from the end of the plank supporting the ball, as shown. Is the object balanced?

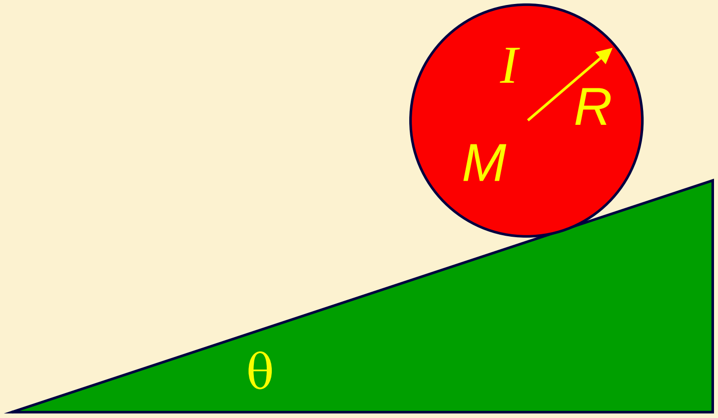
A) No, it will fall to the left.
B) No, it will fall to the right.
C) Yes



Rolling

- An object with mass *M*, radius *R*, and moment of inertia *I* rolls without slipping down a plane inclined at an angle θ with respect to horizontal. What is its acceleration?

- Consider translational CM motion, and rotation about the CM separately when solving this problem



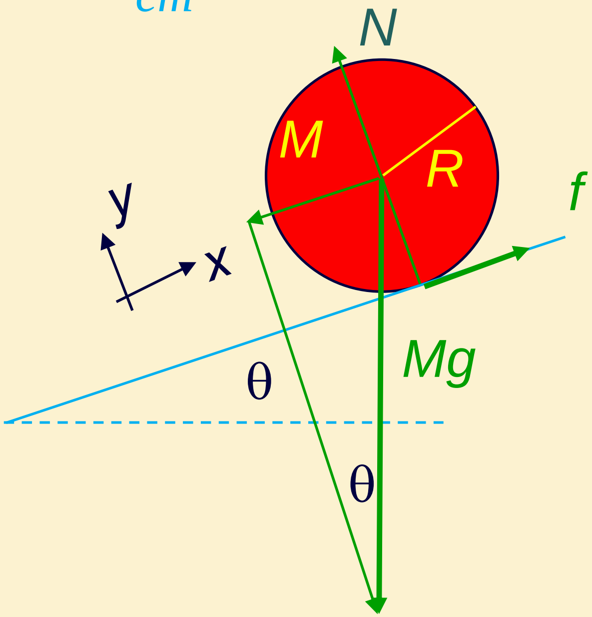
Rolling...

- Static friction f causes rolling. It is an unknown, so we must solve for it.
- First consider the free body diagram of the object and use $F_{NET} = Ma_{cm}$:

  In the x direction: $-Mg\sin\theta + f = -Ma_{cm}$

- Now consider rotation about the CM and use $\boldsymbol{\tau}_{\mathbf{Net}} = I\alpha$ realizing that

  $\tau = Rf\sin 90 = Rf$ and $a = \alpha R$



Rolling...

- We have 3 equations in 3 unknowns, a, α and *f*:

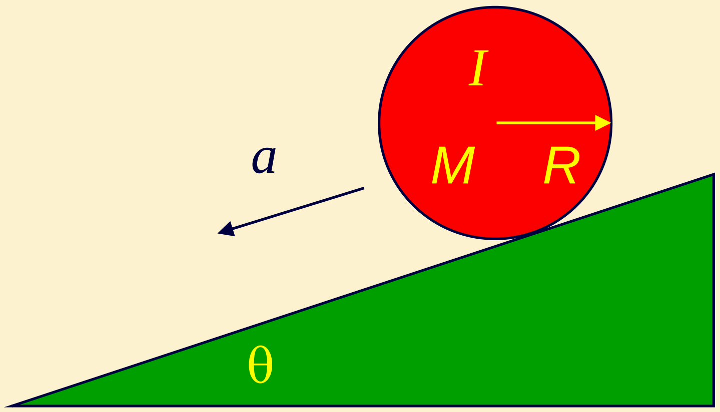
From F=m*a* applied to CM: $-Mg\sin\theta + f = -Ma$

From τ=Iα applied about CM: $fR\sin 90 = fR = I\alpha$

From relationship between *a* and α: $R\alpha = a$

- Use algebra to combine these to eliminate *f*, and solve for *a*:

$$\boxed{a = g\left(\frac{MR^2\sin\theta}{MR^2+I}\right)}$$



For a sphere:

$$a = g\left(\frac{MR^2\sin\theta}{MR^2+\frac{2}{5}MR^2}\right) = \frac{5}{7}g\sin\theta$$

Checkpoint 2

In which of the following cases is the torque about the shoulder due to the weight of the arm the greatest?

Case 1: A person holds her arm at an angle of 30^o above the horizontal (hand is higher than shoulder).

Case 2: A person holds her arm straight out parallel to the ground.

Case 3: A person holds her arm at an angle of 30^o below the horizontal (hand is lower than shoulder).

A) Case 1
B) Case 2
C) Case 3
D) Case 1 and 3
E) Same for all

Checkpoint 2b

If the person lets her arm swing freely from an initial straight-out parallel-to-the-ground position, when is the angular acceleration, α, of the arm about the shoulder the greatest?

A) Immediately after her arm begins to swing.
B) When the arm is vertical.
C) The angular acceleration is constant.

Work Done by Torque

- Recall $W = F\, d \cos\theta$
- For a wheel
  - ➔ Work: $W = F_{tangential}\, s$

    $= F_{tangential}\, r\ \theta$ (s = rθ, θ in radians)

    $= \tau\, \theta$

Summary

- Torque = Force that causes rotation
  - ➔ $\tau = F\, r \sin\theta$
  - ➔ F=m*a* for rotation: $\tau = I\alpha$.
  - ➔ Work done by torque $W = \tau\,\theta$
- Use F=ma, $\tau = I\alpha$ to solve for *a*, α, tension, time. Use conservation of energy to solve for speed.
- Equilibrium
  - ➔ $\Sigma F = 0$
  - ➔ $\Sigma \tau = 0$
    - » Can choose any axis or pivot around which to compute torques. Trick of the trade: If there is a force on the pivot, the torque it produces is 0!