# Physics 101: Lecture 20

## Oscillations: Simple Harmonic Motion

# Overview

- Springs
  - ➔ Force is proportional to displacement
  - ➔ $F = -k\ x$ (- means if you pull in +x direction spring pulls back in –x direction)
  - ➔ $U = \frac{1}{2}\ k\ x^2$ (potential energy stored in spring; spring forces are conservative)

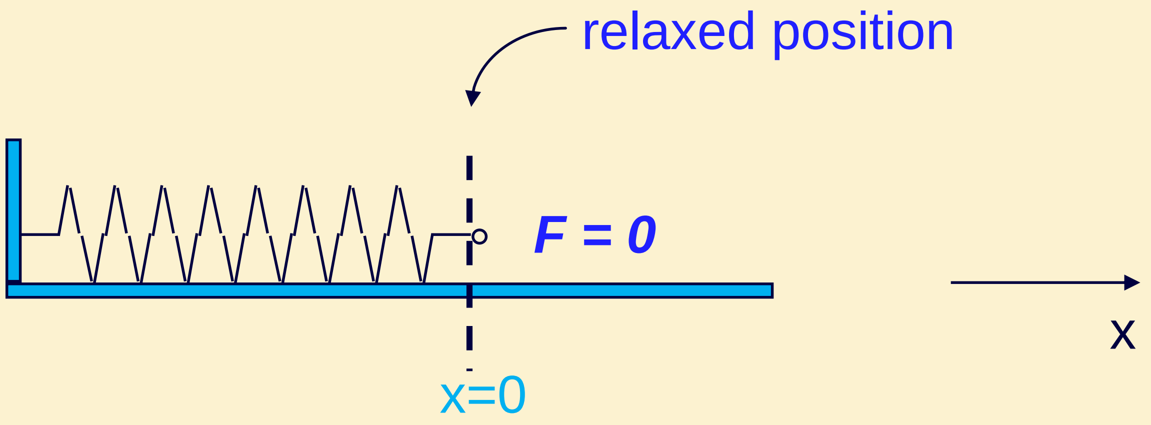
- Today
  - ➔ Simple Harmonic Motion with springs

  - ➔ On Wednesday: Simple Harmonic Motion with pendula

# Springs

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

$F = -k\,x$ Where $x$ is the displacement from the relaxed position and $k$ is the constant of proportionality.



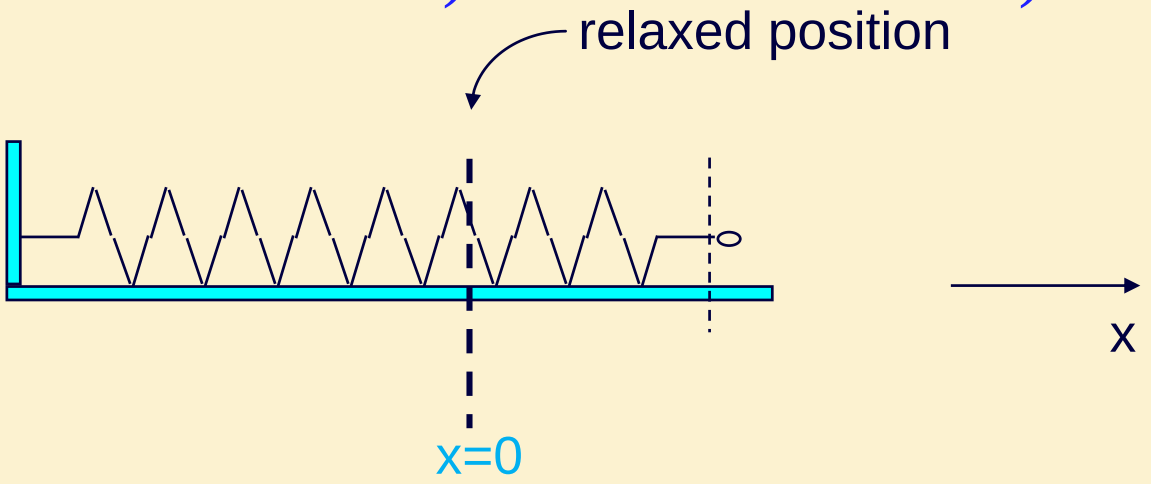
# Clicker Q

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

  $F = -k\,x$ Where $x$ is the displacement from the relaxed position and $k$ is the constant of proportionality.

What is the sign of the force of the spring when it is stretched as shown below.

A) $F > 0$ B) $F = 0$ C) $F < 0$

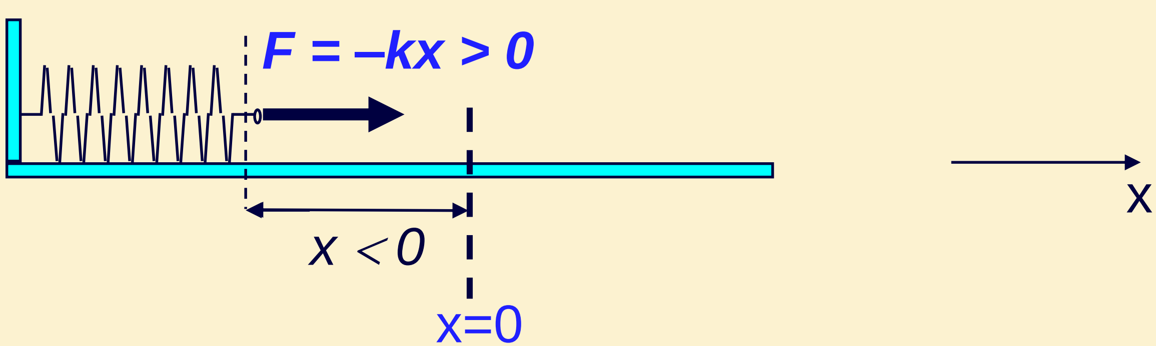


# Springs

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

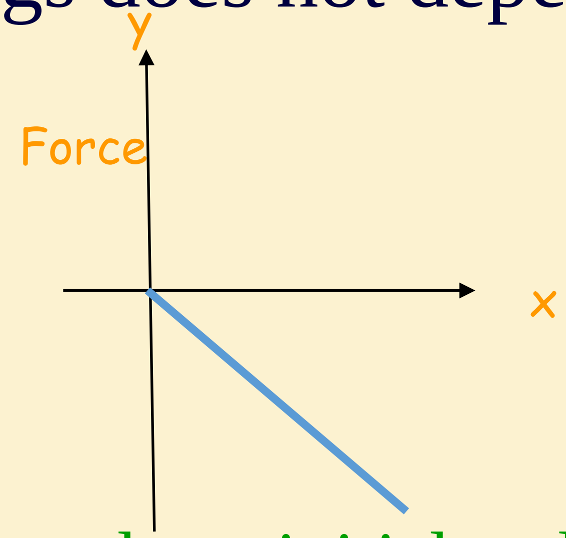
$F = -kx$ Where $x$ is the displacement from the relaxed position and $k$ is the constant of proportionality.

If compressed, force points in positive x direction



# Potential Energy in Spring

- Hooke's Law force is Conservative: Work done by springs does not depend on path
  - ➔ $F = -k\,x$
  - ➔ $W = -\frac{1}{2}\,k\,x^2$




  - ➔ Work done only depends on initial and final position
  - ➔ Define Potential Energy $U_{spring} = \frac{1}{2}\,k\,x^2$

# Simple Harmonic Motion

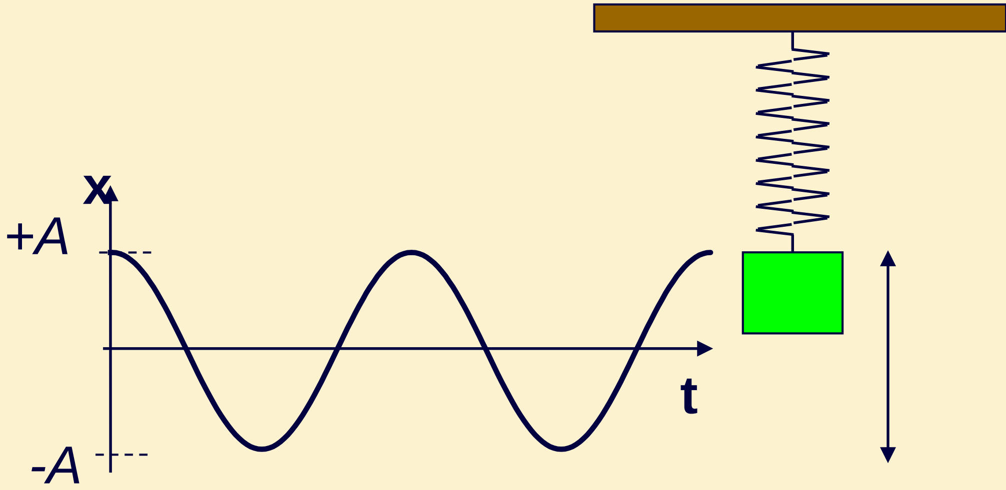
- Vibrations
  - Vocal cords when singing/speaking
  - String/rubber band
- Simple Harmonic Motion
  - Restoring force proportional to displacement
  - Springs  $F = -kx$

# Clicker Q

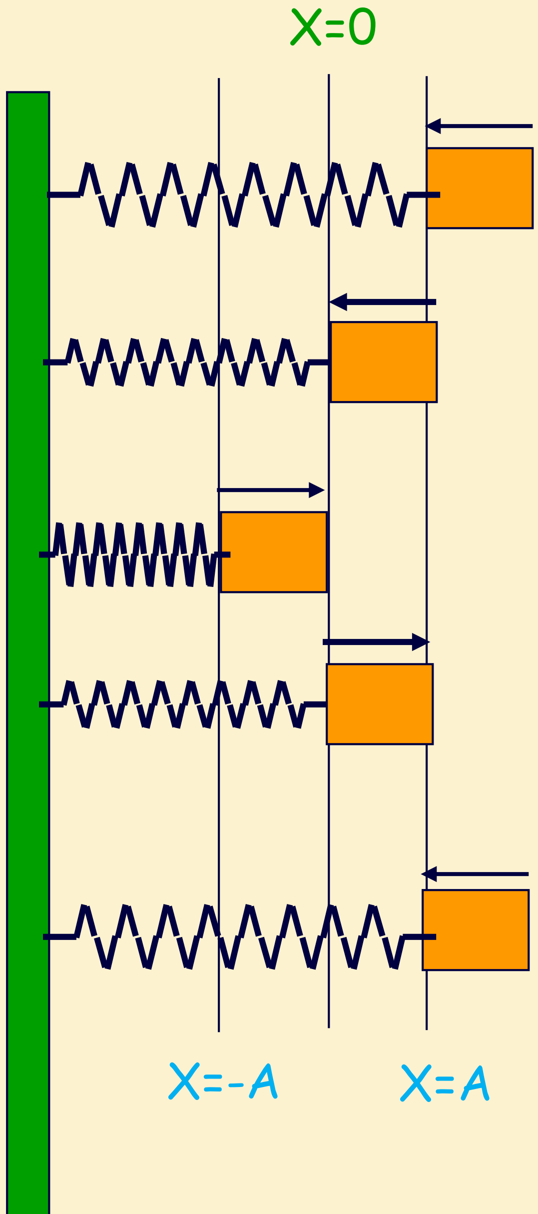
A mass on a spring oscillates back & forth with simple harmonic motion of amplitude A. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the magnitude of the acceleration of the block biggest?

1. When x = +A or -A (i.e. maximum displacement)
2. When x = 0 (i.e. zero displacement)
3. The acceleration of the mass is constant

F=ma



# Springs and Simple Harmonic Motion




$X=A$; $v=0$; $a=-a_{max}$

$X=0$; $v=-v_{max}$; $a=0$

$X=-A$; $v=0$; $a=a_{max}$

$X=0$; $v=v_{max}$; $a=0$

$X=A$; $v=0$; $a=-a_{max}$

# Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$

$v(t) = -[A\omega]\sin(\omega t)$

$a(t) = -[A\omega^2]\cos(\omega t)$

OR

$x(t) = [A]\sin(\omega t)$

$v(t) = [A\omega]\cos(\omega t)$

$a(t) = -[A\omega^2]\sin(\omega t)$

$x_{max} = A$

$v_{max} = A\omega$

$a_{max} = A\omega^2$

Period = T (seconds per cycle)

Frequency = f = 1/T (cycles per second)

Angular frequency = $\omega = 2\pi f = 2\pi/T$

For spring: $\omega^2 = k/m$, $T = 2\pi\sqrt{m/k}$

# Energy

- A mass is attached to a spring and set to motion. The maximum displacement is $x=A$
  - ➔ Apply Work-Kinetic Energy Thm: $W_{nc} = \Delta E = \Delta(K + U)$
  - ➔ $0 = \Delta(K + U)$ or Energy, U+K, is constant!

    Energy $= \frac{1}{2} m v^2 + \frac{1}{2} k x^2$
  - ➔ At maximum displacement $x=A$, $v = 0$

    Energy $= \frac{1}{2} k A^2 + 0$
  - ➔ At zero displacement $x = 0$

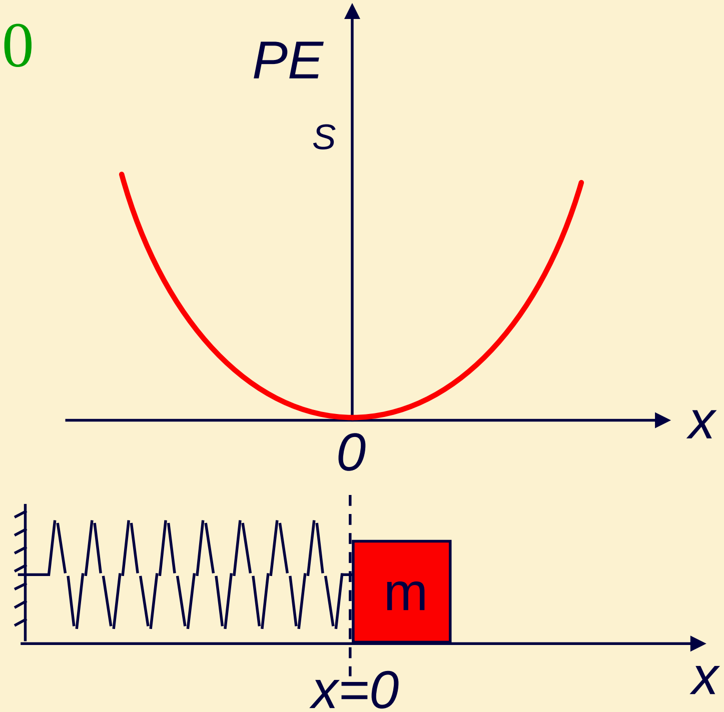
    Energy $= 0 + \frac{1}{2} m v_{max}^2$

    Since Total Energy is same

    $\frac{1}{2} k A^2 = \frac{1}{2} m v_{max}^2$

    $v_{max} = A \,\mathrm{sqrt}(k/m) = A\omega$

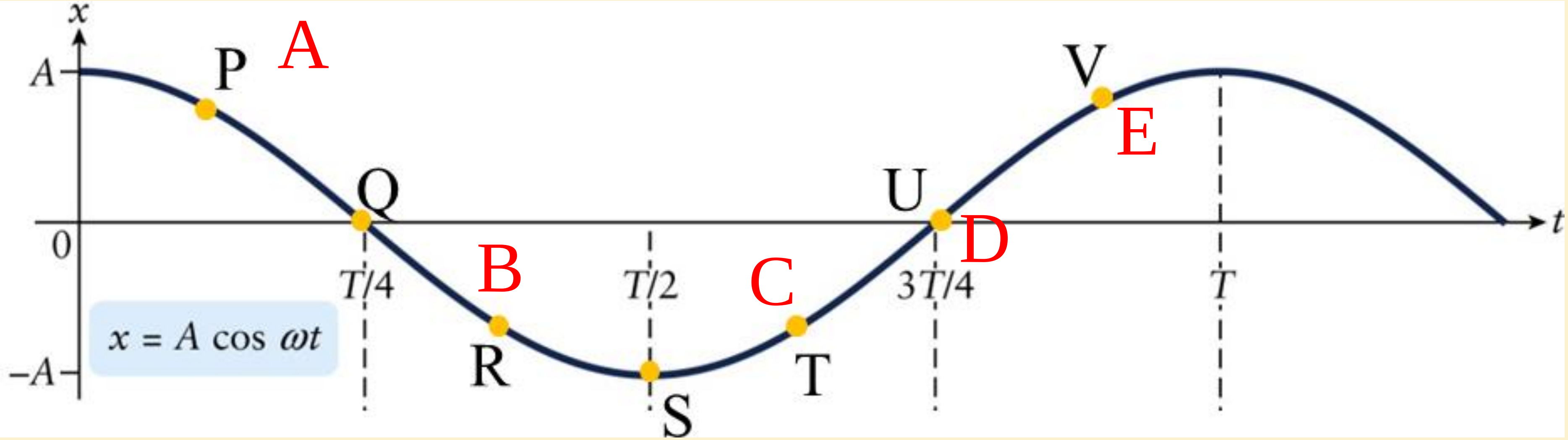
    Same as in last slide



# Checkpoint 1

A block on a spring slides on a horizontal frictionless surface with simple harmonic motion. The corresponding displacement graph of the block is shown below. At which point is the velocity of the block positive and the acceleration negative?

When is U greatest?  When is K greatest?



# Checkpoint 2a

A small object is attached to a horizontal spring, pushed to position $x = -A$, and released. The object in the resulting simple harmonic motion oscillates with period T. How long does it take for the object to travel a total distance of 3A?

A) T/4 B) T/2 C) 3T/4 D) T E) 5T/4

# Checkpoint 2b

How would your answer to the previous question change if the object was initially pushed to position x=A/2 and given an initial velocity such that in the resulting simple harmonic motion the object oscillates with amplitude A and period T?

A) The time will increase by T/2

B) The time will increase by T/4

C) The time will decrease by T/2

D) The time will decrease by T/4

E) No change in the answer

# Checkpoint 3

A block attached at the end of a spring undergoes simple harmonic motion with frequency of oscillation $\omega$. Which of the following modifications will make the block oscillate with a greater frequency?

A) Increase the amplitude of the simple harmonic motion

B) Increase the spring constant

C) Increase the mass of the block

# Clicker Q

A 3 kg mass is attached to a spring (k=12 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

Which equation describes the position as a function of time x(t) =

A) 0.05 sin(4t) B) 0.05 cos(2t) C) 24 sin(4t)

D) 24 cos(2t) E) -24 cos(2t)

# Clicker Q continued

A 3 kg mass is attached to a spring (k=12 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

What is the total energy of the block spring system?

A) 0.015 J B) 0.025 J C) 0.04 J

# Clicker Q continued

A 3 kg mass is attached to a spring (k=12 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

What is the maximum speed of the block?

A) 0.018 m/s B) 0.23 m/s C) 0.1 m/s

# Clicker Q last one

A 3 kg mass is attached to a spring ($k=12$ N/m). It is stretched 5 cm. At time $t=0$ it is released and oscillates.

How long does it take for the block to return to $x=+5$cm?

A) 1.4 s B) 3.1 s C) 3.5 s

# Summary

- Springs
  - ➔ $F = -kx$
  - ➔ $U = \frac{1}{2} k x^2$
  - ➔ $\omega = \mathrm{sqrt}(k/m)$, $\omega = 2\pi f = 2\pi/T$, $f = 1/T$

- Simple Harmonic Motion
  - ➔ Occurs when have linear restoring force $F= -kx$
  - ➔ $x(t) = [A] \cos(\omega t)$ or $[A] \sin(\omega t)$
  - ➔ $v(t) = -[A\omega] \sin(\omega t)$ or $[A\omega] \cos(\omega t)$
  - ➔ $a(t) = -[A\omega^2] \cos(\omega t)$ or $-[A\omega^2] \sin(\omega t)$