Physics 101: Lecture 21

Oscillations 2: More mass-at-end-of-spring oscillations & Pendula

Review Energy in Simple Harmonic Motion

- A mass is attached to a spring and set to motion. The maximum displacement is $x=A$
  - ➔ Energy = U + K = constant!

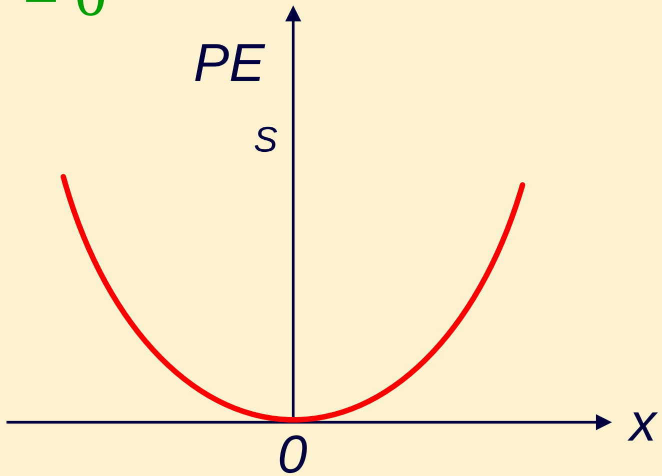
    $= \frac{1}{2} k x^2 + \frac{1}{2} m v^2$
  - ➔ At maximum displacement $x=A$, $v = 0$

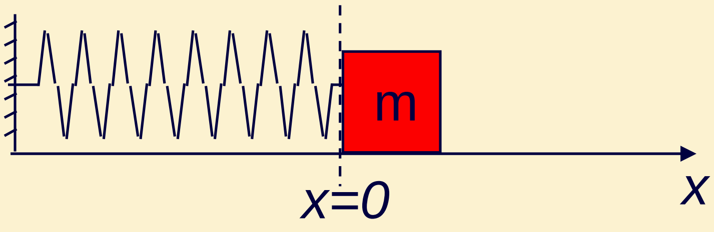
    Energy $= \frac{1}{2} k A^2 + 0$
  - ➔ At zero displacement $x = 0$

    Energy $= 0 + \frac{1}{2} m v_{max}^2$

    $\frac{1}{2} k A^2 = \frac{1}{2} m v_m^2$

    $v_{max} = A\,\mathrm{sqrt}(k/m)$
  - ➔ Analogy with gravity/ball

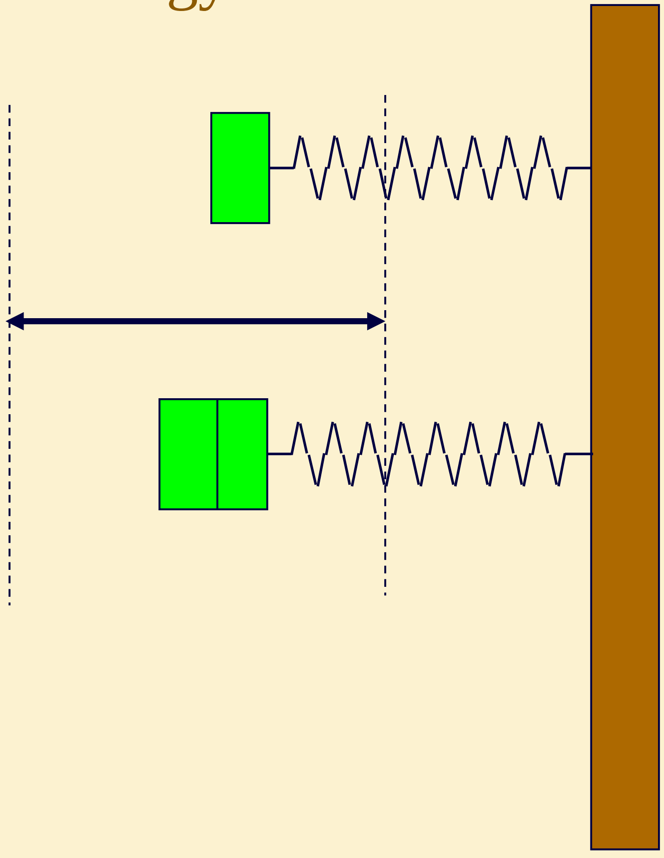




Potential Energy Clicker Q

In Case 1 a mass on a spring oscillates back and forth. In Case 2, the mass is doubled but the spring and the amplitude of the oscillation is the same as in Case 1.

In which case is the maximum potential energy of the mass and spring the biggest?

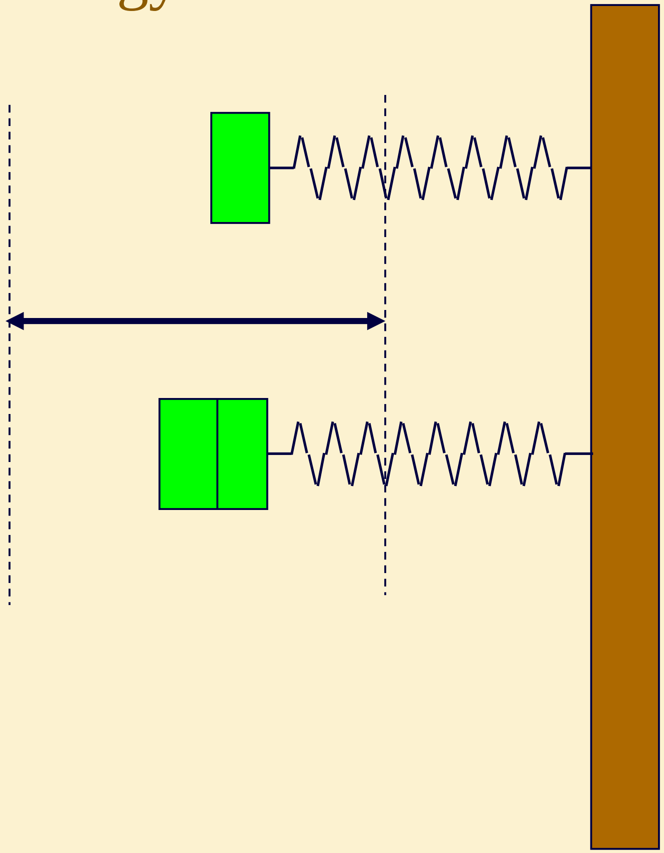


A. Case 1
B. Case 2
C. Same

Kinetic Energy Clicker Q

In Case 1 a mass on a spring oscillates back and forth. In Case 2, the mass is doubled but the spring and the amplitude of the oscillation is the same as in Case 1.

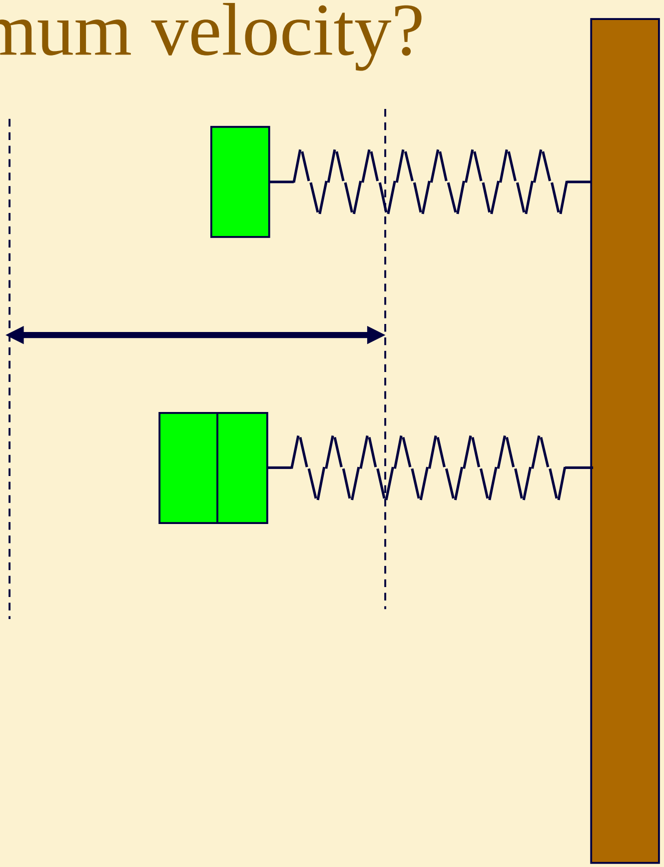
In which case is the maximum kinetic energy of the mass the biggest?



A. Case 1
B. Case 2
C. Same

Velocity Clicker Q

In Case 1 a mass on a spring oscillates back and forth. In Case 2, the mass is doubled but the spring and the amplitude of the oscillation is the same as in Case 1. Which case has the largest maximum velocity?

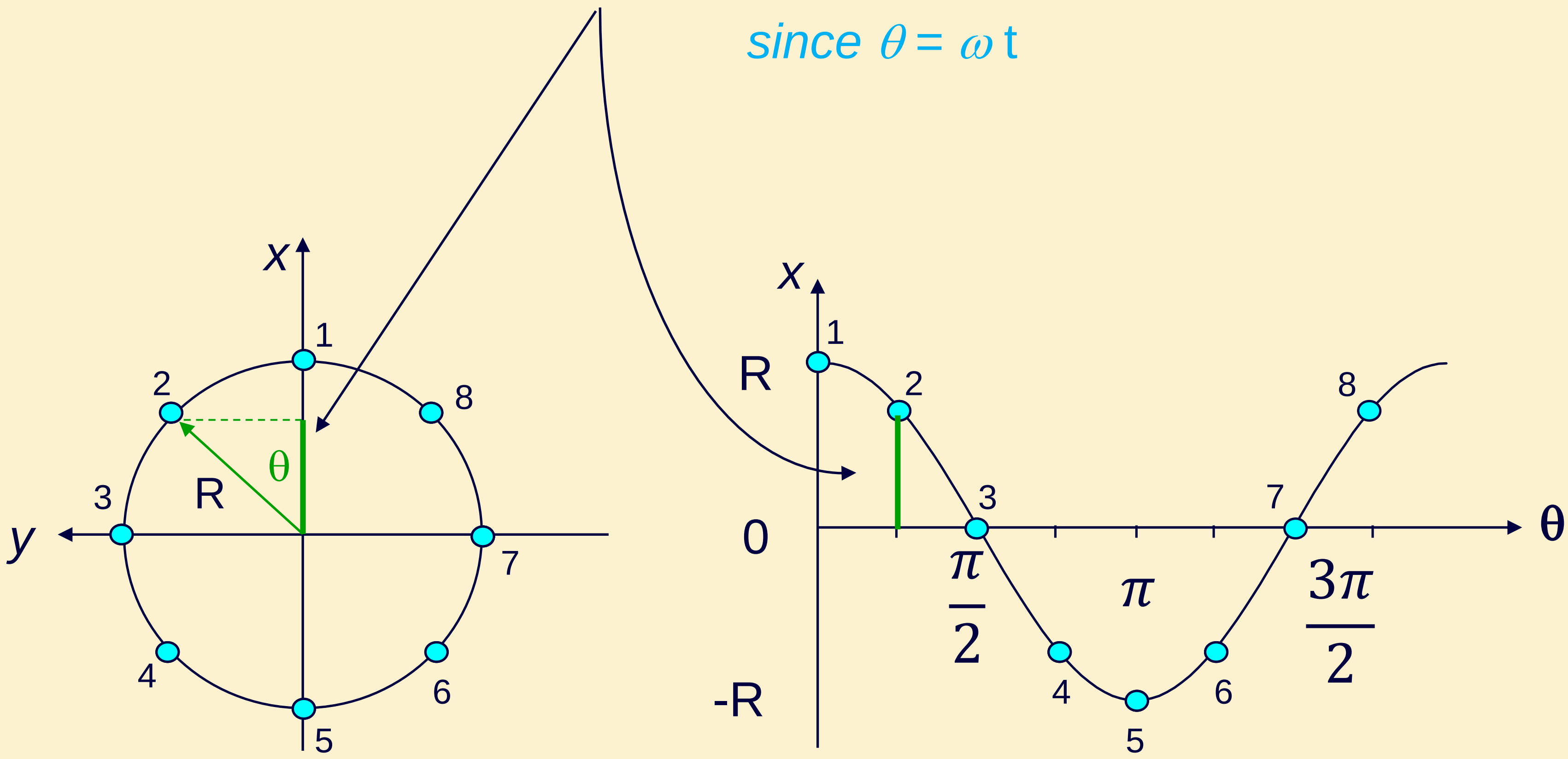


1. Case 1
2. Case 2
3. Same

What does *moving in a circle* have to do with moving back & forth *in a straight line* ?

$$x = R\cos\theta = R\cos(\omega t)$$

since $\theta = \omega t$



Spring Oscillations

- Simple Harmonic Oscillator
  - ➔ $\omega = 2 \pi f = 2 \pi / T$
  - ➔ $x(t) = [A] \cos(\omega t)$
  - ➔ $v(t) = -[A\omega] \sin(\omega t)$
  - ➔ $a(t) = -[A\omega^2] \cos(\omega t) = -\omega^2 x(t)$

- Draw FBD write F=ma

$-k x = m a$

$k A = m a_{max}$

$A m \omega^2 = m a_{max}$

$a_{max} = A\omega^2$

Demos:

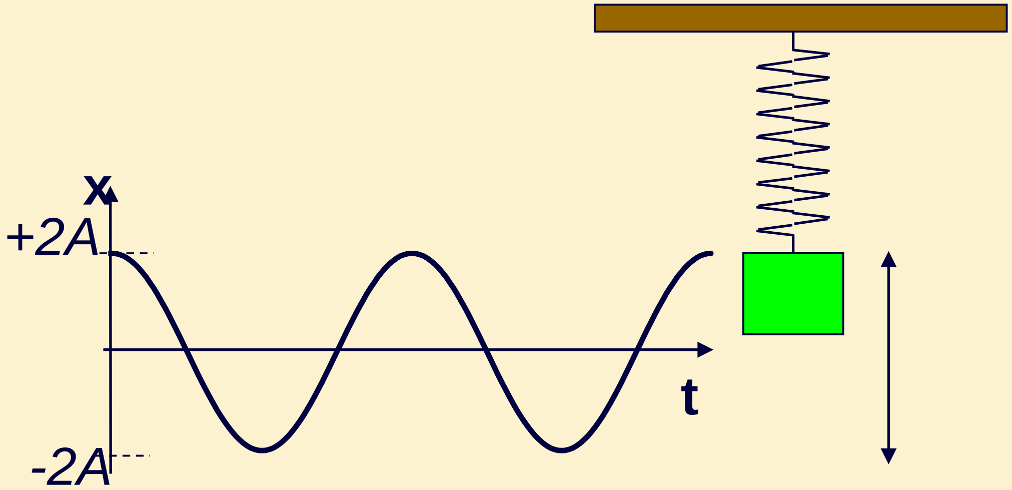
A,m,k dependence

$$\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Period Clicker Q

If the amplitude of the oscillation (same block and same spring) is doubled, how would the period of the oscillation change? (The period is the time it takes to make one complete oscillation)

A. The period of the oscillation would double.
B. The period of the oscillation would be halved
C. The period of the oscillation would stay the same



Vertical Mass and Spring

- If we include gravity, there are two forces acting on mass. With mass hanging on the spring, new equilibrium position has spring stretched d

➔ $F_{Net,\,y} = 0$

$kd – mg = 0$

$d = mg/k$

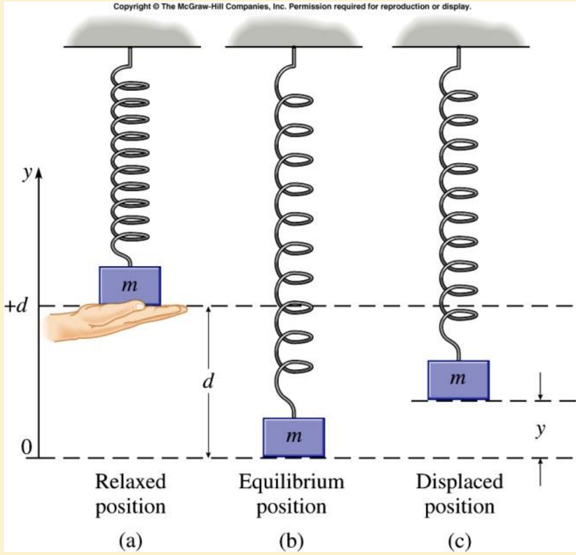
- Now what happens when we compress spring a distance y?

➔ $F_{net} = k(d-y) – mg$

$= kd – ky – mg = -ky$

➔ Still Hooke's Law! SHO

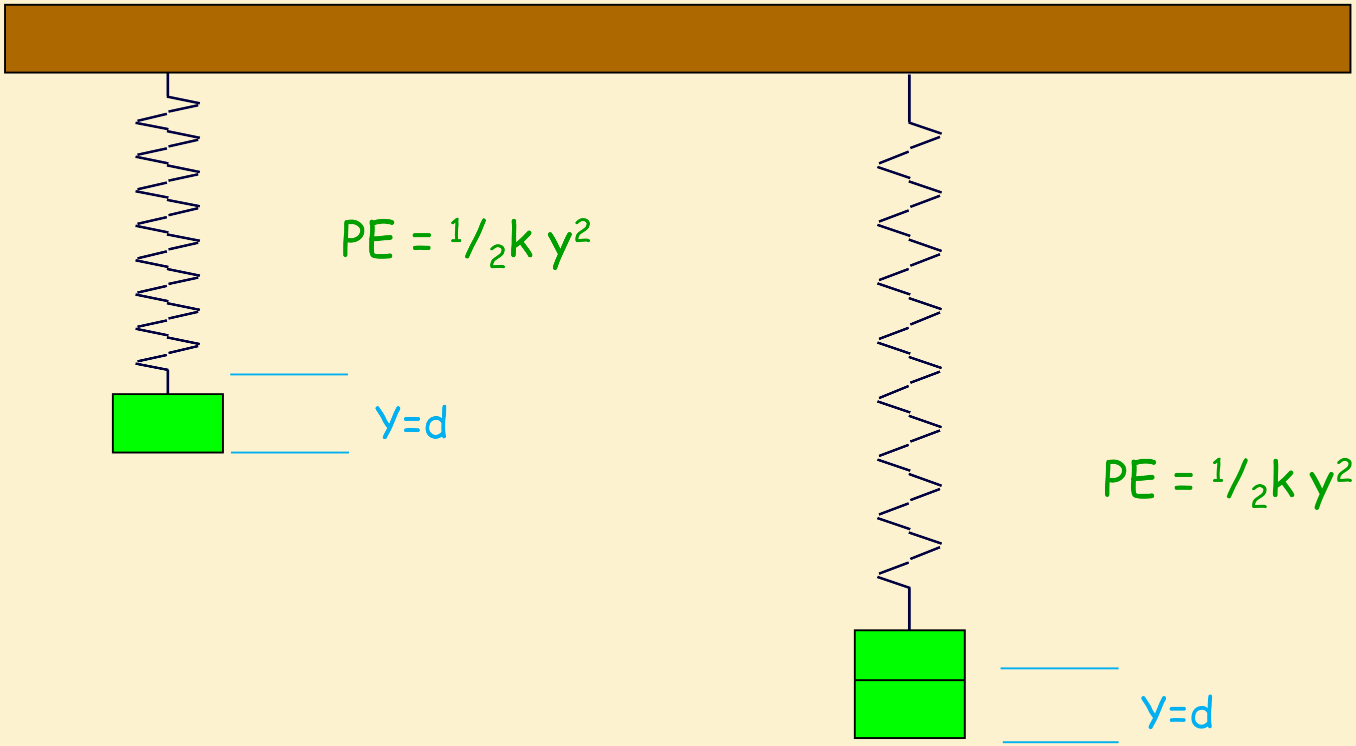
➔ New equilibrium position



Vertical Spring Clicker Q

If the springs were vertical, and stretched the same distance d from their equilibrium position and then released, which would have the largest maximum kinetic energy?

1) M   2) 2M   3) Same



Pendulum Motion

- For *small angles*
  - ➔ $T \cong mg$ (T is *approximately* equal to mg)
  - ➔ $F_{net,x} = T_x = -T \sin \theta$

    $= -mg\,(x/L) = -(mg/L)\,x$ Hooke's Law!
  - ➔ $F_{net,\,x} = m\,a_x$

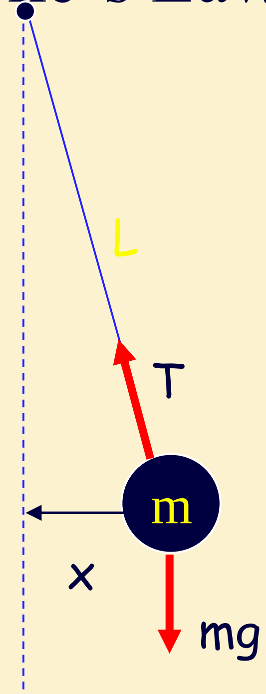
    $-mg\,(x/L) = m\,a_x$

    $a_x = -(g/L)\,x$
  - ➔ Recall for SHO $a_x = -\omega^2\,x$

    $\omega = \mathrm{sqrt}(g/L)$

    $T = 2\,\pi\,\mathrm{sqrt}(L/g)$

  Period does not depend on A, or m!



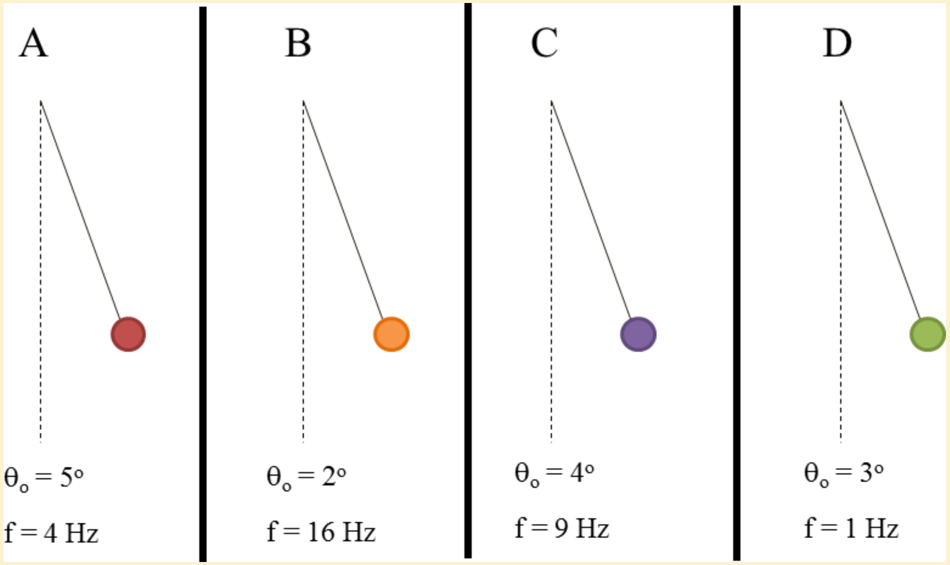
Checkpoint 1

The figure below (not drawn to scale) shows four different pendula, each made of a ball attached to a rope with negligible mass that is hung from a ceiling support. In each case, when the ball is pulled back a small angle θ_o and released, it swings in simple harmonic motion.

Which of the following options represents the correct ranking from smallest to largest based on the mass of the ball?

A) B < D < C < A   B) D < A < C < B   C) B < C < A < D

D) None of the above because period or frequency do not depend on mass of the pendulum



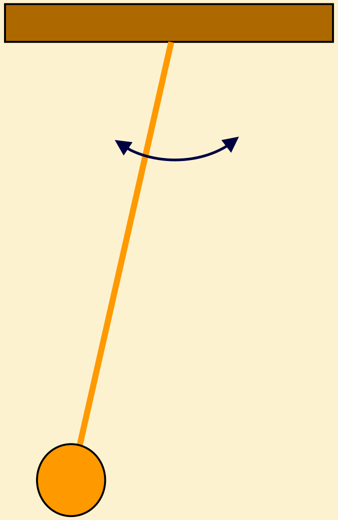
Elevator Clicker Q

A pendulum is hanging vertically from the ceiling of an elevator. Initially the elevator is at rest and the period of the pendulum is T. Now the pendulum accelerates upward. The period of the pendulum will now be…

A. greater than T

B. equal to T

C. less than T



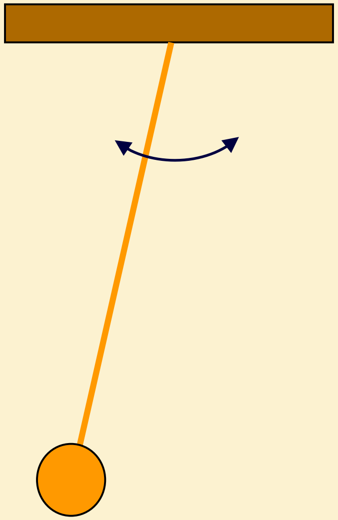
Before doing this one, let's first do another clicker question that will better prepare you to answer this one

Preliminary Clicker Q

A pendulum is hanging vertically from the ceiling of an elevator. Initially the elevator is at rest and the period of the pendulum is T. Now the pendulum accelerates upward. The period of the pendulum will now be…

If you are accelerating upward your weight is the same as if g had

See Hint



1. increased
2. same
3. decreased

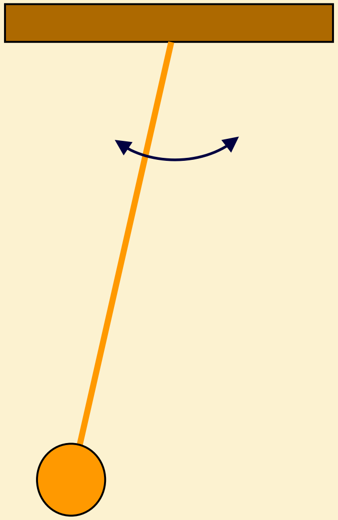
Hint: Do you feel lighter or heavier when the elevator accelerates up?

Elevator Clicker Q

A pendulum is hanging vertically from the ceiling of an elevator. Initially the elevator is at rest and the period of the pendulum is T. Now the pendulum accelerates upward. The period of the pendulum will now be

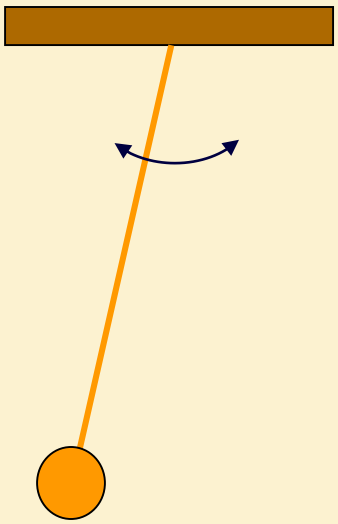
1. greater than T
2. equal to T
3. less than T

Recall: $T = 2\pi\sqrt{\frac{L}{g}}$



McGyver

Imagine you have been kidnapped by space invaders and are being held prisoner in a room with no windows. All you have is a cheap digital wristwatch and a pair of shoes (including shoelaces of known length). Explain how you might figure out whether this room is on the earth or on the moon



$$\omega = \sqrt{\frac{g}{L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

$$g = (2\pi)^2\frac{L}{T^2}$$

make a pendulum with the shoelace and shoes and use the wristwatch to determine the length of each period.

Summary

- Simple Harmonic Motion
  - ➔ Occurs when have linear restoring force $F = -kx$
  - ➔ $x(t) = [A] \cos(\omega t)$ (if $x(t=0) = A$)
  - ➔ $v(t) = -[A\omega] \sin(\omega t)$
  - ➔ $a(t) = -[A\omega^2] \cos(\omega t)$
- Springs
  - ➔ $F = -kx$
  - ➔ $U = \frac{1}{2} k x^2$
  - ➔ $\omega = \mathrm{sqrt}(k/m)$
- Pendulum (Small oscillations)
  - ➔ $\omega = \mathrm{sqrt}(L/g)$