Kinetic Theory

The relationship between energy and temperature (for monatomic ideal gas)

\[ \Delta p_x = 2mv_x \]

\[ \Delta t = 2 \frac{L}{v_x} \]

\[ F_{avg} = \frac{\Delta p_x}{\Delta t} = \frac{mv_x^2}{L} \]

For N molecules, multiply by N

\[ P = \frac{F}{A} = \frac{Nm v_x^2}{V} \]

Note \( K_{tr} = \frac{1}{2} m v^2 = 3/2 m v_x^2 \)

\[ P = \frac{2}{3} \frac{N}{V} \langle K_{tr} \rangle \]

Using \( PV = NkT \)

\[ \langle K_{tr} \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT \]

\( \langle \rangle \) means average. 

kT/2 energy per degree of freedom = equipartition theorem
Example

- What is the rms (root mean squared) speed of a nitrogen (N$_2$) molecule in this lecture hall?

$$\langle KE \rangle = \frac{3}{2} k_B T$$

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$\langle v^2 \rangle = \frac{3k_B T}{m}$$

\[
\langle v^2 \rangle = \frac{3 \left(1.38 \times 10^{-23} \text{J/K}\right) \left(273 + 20\right) \text{K}}{(28 \text{ u}) \times \left(1.66 \times 10^{-27} \text{kg/u}\right)}
\]

$$v = 511 \text{ m/s}$$

$$= 1143 \text{ mph!}$$

(Speed of sound is 767 mph)
Internal Energy

- Energy of all molecules, including:
  - Random motion of individual molecules
    - $<K_{tr}> = (3/2)kT$ for ideal gas
    - Vibrational energy of molecules and atoms
  - Chemical energy in bonds and interactions

- DOES NOT INCLUDE:
  - Macroscopic motion of object
  - Potential energy due to interactions w/ other objects
  - $E_{tot} = K + U + U_{internal}$
Heat

Definition: Heat is the **FLOW** of energy between two objects due to difference in temperature

- Changes internal energy
- Note: similar to **WORK**
- Object does not “have” heat (it has energy)

Units: **Joules** or **calories**

- calorie: Amount of heat needed to raise 1g of water 1°C
- 1 Calorie = 1000 calories = 1 kcal = 4186 Joules
Specific Heat (for solids and liquids)

- Heat adds energy to object/system
- IF system does NO work then:
  - Heat increases internal energy. $Q = \Delta U$
  - Heat increases temperature!
- $Q = c \, m \, \Delta T$
  - Specific heat $c$, units = J/kg°C
  - Heat required to increase Temp depends on amount of material $(m)$ and type of material $(c)$

Demo: water in paper cup takes up heat from flame
After a grueling work out, you drink a liter (1kg) of cold water (0 C). How many Calories does it take for your body to raise the water up to body temperature of 37 C? (Specific Heat of water is 1 calorie/(gram °C) ; 1 liter = 1000 g)

1) 37  2) 370  3) 3,700  4) 37,000
A potato and raisins salad has been warmed up to a temperature of 80°C and let stand for three minutes. Then one tries a bite. Would the potatoes and raisins be equally warm? Potatoes have a specific heat of 3430 \( J/(kg \cdot ^\circ C) \). Raisins have a specific heat of 1630 \( J/(kg \cdot ^\circ C) \). [Consider the potatoes and raisins on surface of casserole]

A) No. Potatoes will be warmer.

B) No. Raisins will be warmer.

C) Yes.
Latent Heat $L$

- As you add heat to water, the temperature increases to the boiling point, then it remains constant despite the additional heat!

- Latent Heat $L$ [J/kg] is heat which must be added (or removed) for material to change phase (liquid-gas).

- Latent Heat of Fusion ($L_f$) and Latent Heat of Vaporization ($L_v$)

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Demo: evaporative cooling with alcohol

<table>
<thead>
<tr>
<th>Substance</th>
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<th>$L_v$ (J/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>$33.5 \times 10^4$</td>
<td>$22.6 \times 10^5$</td>
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</table>
Which will do a better job cooling your soda, a “cooler” filled with water at 0°C, or a cooler filled with ice at 0°C.

A) Water  B) About Same  C) Ice

Latent Heat $L$ [J/kg] is heat which must be added (or removed) for material to change phase (liquid-gas, or solid-liquid).

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The temperature of a 100 g piece of ice is risen steadily so that the ice converts first into liquid water and then evaporates completely, all at the same pressure of 1 atm. Which of the following qualitative plots of temperature versus energy may represent best the process?

\[ L_f = 33.5 \times 10^4 \text{ J/kg} \]
\[ L_v = 226 \times 10^4 \text{ J/kg} \]
During a tough workout, your body sweats (and evaporates) 1 liter of water to keep cool (37 °C). How much water would you need to drink (at 2 °C) to achieve the same thermal cooling? (recall \( c = 4.2 \text{ J/g C} \) for water, \( L_v = 2.2 \times 10^3 \text{ J/g} \))

A) 0.15 liters   B) 1.0 liters   C) 15 liters   D) 150 liters
Example

How much ice (at 0 °C) do you need to add to 0.5 liters of water at 25 °C, to cool it down to 10 °C?

\( (L_f = 80 \text{ cal/g, } c = 1 \text{ cal/g °C}) \)

Key ideas: conserve energy by accounting for all the heat flow

1) \( Q \) leaving water goes into heating ice.
2) Final temps are same

Advice: compute the heat leaving one object (keep it positive) and set it to heat entering the other (keep it positive)

\[
m_{\text{water}}c(T_f - T_0) = -m_{\text{ice}}L_f - m_{\text{ice}}c(T_f - 0)
\]

\[
\begin{array}{ccc}
\text{Cool Water} & \text{Melt} & \text{Warm water that was ice} \\
\end{array}
\]

\[
m_{\text{water}}c(T_f - T_0) = -m_{\text{ice}}(L_f + cT_f)
\]

\[
m_{\text{ice}} = m_{\text{water}}c(T_0 - T_f)/(L_f + cT_f)
\]

\[
= 83.3 \text{ g}
\]

Demo: Hot brass into water, What’s the final Temperature?
Example

Summers in Phoenix Arizona are very hot (125 F is not uncommon), and very dry. If you hop into an outdoor swimming pool on a summer day in Phoenix, you will probably find that the water is too warm to be very refreshing. However, when you get out of the pool and let the sun dry you off, you find that you are quite cold for a few minutes (yes...you will have goose-bumps on a day when the air temperature is over 120 degrees).

How can you explain this?

When you leave the pool, the water on your skin starts to evaporate. This evaporation takes energy from the surface of your skin, which is why you feel cool when you get out of the pool. This is similar to sweating.

-- explanation from a previous Phys 101 student
Phase Diagrams

Demo: Dry ice--Sometimes can go directly from solid to gas, called sublimation.

Phase changes across this boundary are fusion (solid to liquid) and freezing (liquid to solid).

Phase changes across this boundary are vaporization (liquid to vapor) and condensation (vapor to liquid).

Phase changes across this boundary are sublimation (solid to vapor) and deposition (vapor to solid).
Summary

- **Heat is FLOW of energy**
  - Flow of energy may increase temperature

- **Specific Heat**
  - \( \Delta t = \frac{Q}{c \cdot m} \)
  - Monatomic IDEAL Gas \( C_V = \frac{3}{2} R \)
  - Diatomic IDEAL Gas \( C_V = \frac{5}{2} R \)

- **Latent Heat**
  - heat associated with change in phase