

# Physics 101: Lecture 11

## Momentum and Impulse (vectors)

# Key Ideas

- Previous lectures: **Work-Energy**

- $F_{\text{Net}} = m a$       multiply both sides by  $d$

- $F_{\text{Net}} d = m a d$       (note:  $ad = \frac{1}{2} \Delta v^2$  from  $v^2 = v_o^2 + 2ad$ )

- $F_{\text{Net}} d = \frac{1}{2} m \Delta v^2$

- **Work-Kinetic Energy Thm:**

- »  $W_{\text{Net}} = \Delta K$       OR       $W_{\text{non-cons}} = \Delta E = \Delta(K + U)$

- This Time: **Impulse-Momentum**

- $F_{\text{Net}} = m a$

- $F_{\text{Net}} = m \Delta v / \Delta t$       (note:  $a = \Delta v / \Delta t$  )

- $F_{\text{Net}} = \Delta (mv) / \Delta t = \Delta p / \Delta t$

- $F_{\text{Net}} \Delta t = I_{\text{Net}} = \Delta p$       Define Impulse and Momentum

# Momentum and Impulse

→ Momentum  $\mathbf{p} = m\mathbf{v}$

» Momentum is a **vector**

→ Impulse-Momentum Thm:  $\mathbf{I} = \mathbf{F}\Delta t = \Delta m\mathbf{v}$

» Impulse is = change in momentum:  $\mathbf{I} = \Delta\mathbf{p}$

» Impulse is a vector because  $\mathbf{F}$  is a vector

» If there is no impulse, momentum does not change (i.e., it is conserved)

❖ When is momentum conserved?

When no **external** forces cause an Impulse.

# Clicker Q (w/ demo)

Two identical balls are dropped from the same height onto the floor. In each case they have velocity  $v$  downward just before hitting the floor. In **case 1** the ball bounces back up, and in **case 2** the ball sticks to the floor without bouncing. In which case is the magnitude of the impulse given to the ball by the floor the biggest?

- A. Case 1
- B. Case 2
- C. The same

# Clicker Q

In both cases of the above question, the direction of the impulse given to the ball by the floor is the same. What is this direction?

A. Upward

B. Downward

# Clicker Qs

You drop an egg onto 1) the floor 2) a thick piece of foam rubber.  
In both cases, the egg does not bounce (demo).

In which case is the impulse greater?

- A) Floor
- B) Foam
- C) the same

In which case is the average force greater

- A) Floor
- B) Foam
- C) the same

# Clicker Q: Pushing Off...

Fred (75 kg) and Jane (50 kg) are at rest on skates facing each other. Jane then pushes Fred w/ a constant force  $F = 45 \text{ N}$  for a time  $\Delta t = 3$  seconds. Who will be moving fastest at the end of the push?

A) Fred

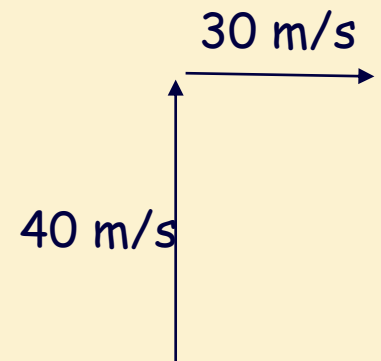
B) Same

C) Jane

# Clicker Q

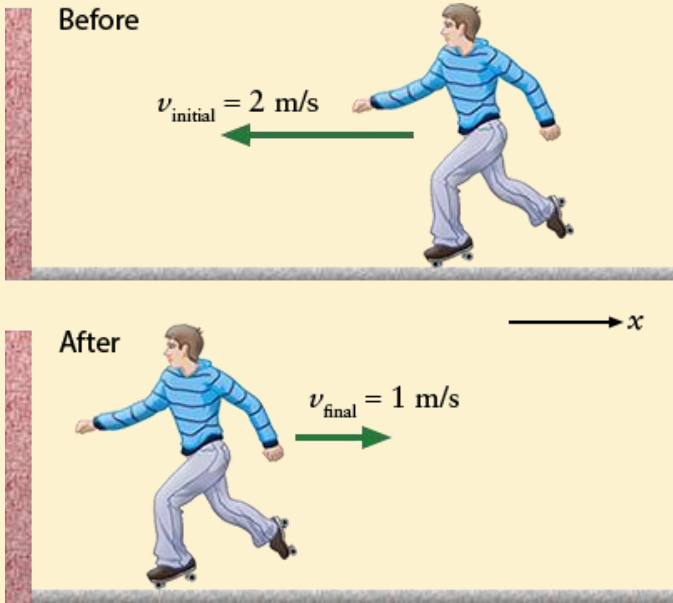
A car w/ mass 1200 kg is driving north at 40 m/s, and turns east driving 30 m/s. What is the magnitude of the car's change in momentum?

- A) 0    B) 12,000    C) 36,000    D) 48,000    E) 60,000





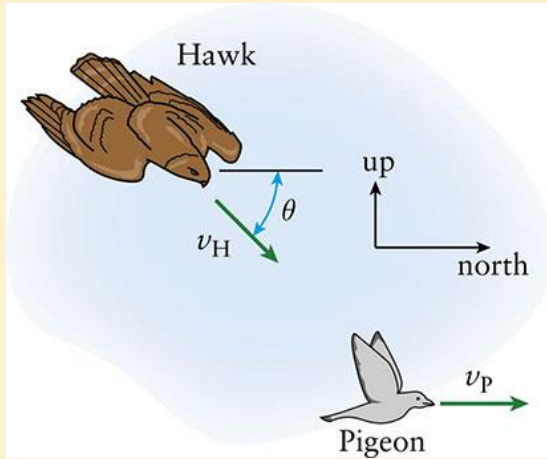
# Checkpoint 1



A boy roller skating at constant velocity hits a wall and bounces back at constant velocity. The boy's change in momentum is:

- A) zero
- B) in the  $+x$ -direction
- C) in the  $-x$ -direction

# Checkpoint 2

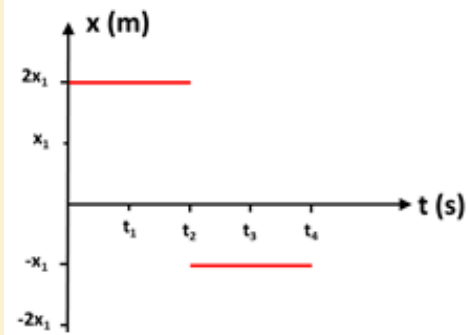
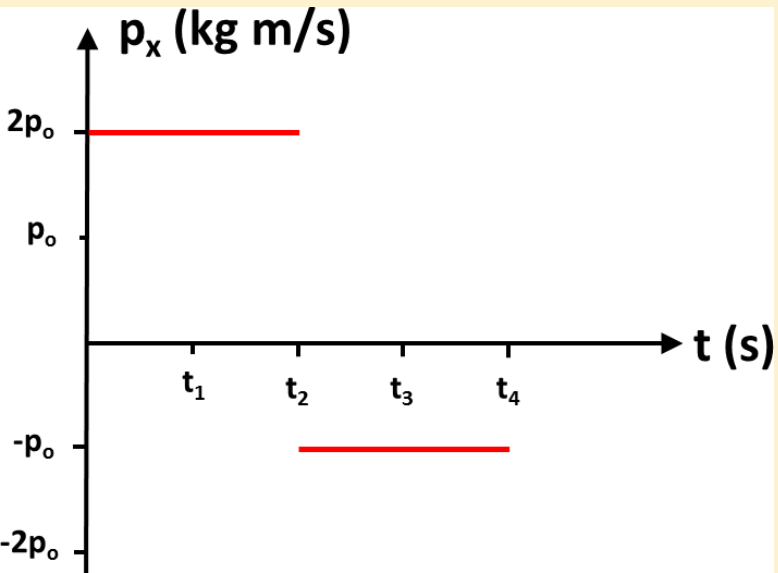


The mass of a pigeon hawk is twice that of the pigeon it hunts. A pigeon is gliding north at a speed of  $v_p = 23.0$  m/s when a hawk swoops down, grabs the pigeon, and flies off. The hawk was flying north at a speed of  $v_H = 35.0$  m/s, at an angle of  $45^\circ$  below the horizontal, at the instant of the attack. We can use conservation of momentum to find the final velocity of the pigeon–hawk system just after the hawk grabs the pigeon because

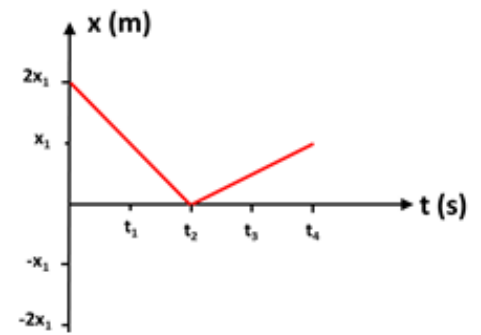
- A) The force of gravity cancels the drag due to air
- B) While the hawk latches on to the pigeon, the forces between the birds are stronger than any other external force acting on the birds
- C) The hawk gently latches on to the pigeon

# Checkpoint 3

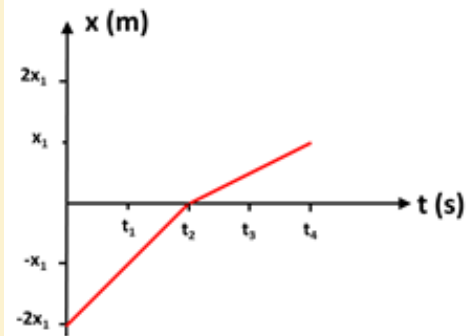
A 170-g cue ball has the momentum vs time graph  
What is a possible corresponding position vs time graph  
for the cue ball?



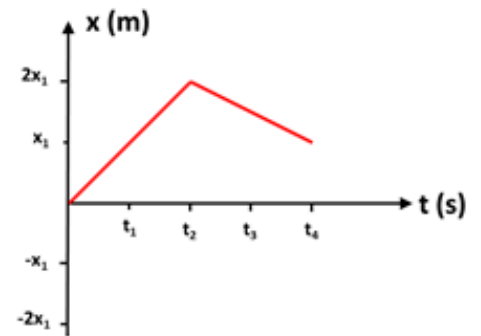
A



B



C



D

# Momentum is Conserved!

- Momentum is “Conserved” when there is **no external impulse**, meaning it cannot be created or destroyed
  - ➔ Momentum can be transferred but if it is conserved, then  $P_{\text{tot},i} = P_{\text{tot},f}$ . Thus  $P_{\text{tot}}$  does not change with time *absent external forces*.
- Recall that Mech. Energy,  $E=K+U$ , is conserved when there is **no external work** done on system.

These are 2 BIG IDEAS in physics

# Impulse and Momentum Summary

$$F_{\text{Net}}\Delta t = \Delta p$$

- For single object....
  - ➔  $F_{\text{Net}} = 0 \Rightarrow$  momentum conserved ( $\Delta p = 0$ )
- For collection of objects ...
  - ➔  $F_{\text{Net}} = 0 \Rightarrow$  total momentum conserved ( $\Delta P_{\text{tot}} = 0$ )
  - ➔  $F_{\text{Net}} \neq 0 \Rightarrow$  impulse  $\Rightarrow$   
momentum is NOT conserved ( $\Delta P_{\text{tot}} \neq 0$ )

# Clicker Q

Movies often show someone firing a gun loaded with blanks. In a blank cartridge the lead bullet is removed and the end of the shell casing is crimped shut to prevent the gunpowder from spilling out. When a gun fires a blank, is the recoil greater than, the same as, or less than when the gun fires a standard bullet?

A. greater than

B. same as

C. less than

# Example

A mother and a daughter are ice skating. The mother (mass  $M=70$  kg) is skating at  $5$  m/s toward her stationary daughter (mass  $m=40$  kg). When she reaches her daughter she bear-hugs her daughter and both slide off together. What is the common speed of the mother and daughter right after the collision?

**Big Idea:** Conservation of momentum

**Justification:** Force between mother-daughter is an internal force. Thus no external impulse so momentum is conserved.

**Plan:** 1) Conserve momentum by setting  $P_{\text{tot},i}$  equal to  $P_{\text{tot},f}$   
2) Find the common speed of both after collision

Execution of plan:

$$1) \quad P_{i,\text{tot}} = P_{f,\text{tot}}$$

$$2) \quad MV + m(0) = (M+m)V_{\text{final}}$$

$$(70 \text{ kg})(5 \text{ m/s}) + 0 = (70 \text{ kg} + 40 \text{ kg}) V_{\text{final}}$$

Solve for  $V_{\text{final}}$ :

$$V_{\text{final}} = 3.18 \text{ m/s}$$

What would change if the daughter had been initially moving toward mom?

# Example

A car ( $M=1500$  kg) headed north at 30 mph collides with a car ( $m=1200$  kg) headed east at 45 mph. The cars lock together after the collision. What is the final speed and direction of the cars?

**Big Idea:** Conservation of momentum

**Justification:** Force between the cars is an internal force. Thus no external impulse, so momentum is conserved.

**Plan:** 1) Conserve momentum by setting  $\mathbf{P}_{\text{tot},i}$  equal to  $\mathbf{P}_{\text{tot},f}$

2) Find the common speed of both after collision, and the angle.

Execution of plan (x axis goes east-west and y-axis goes north-south):

$$1) \quad \mathbf{P}_{i,\text{tot}} = \mathbf{P}_{i,M} + \mathbf{P}_{i,m} = \mathbf{P}_{f,\text{tot}}$$

$$\mathbf{P}_{i,m} = 54,000 \text{ kg-mph}$$

$$\mathbf{P}_{i,M} = 45,000 \text{ kg-mph} \quad \mathbf{P}_{i,\text{tot}} = 70,292 \text{ kg-mph}$$

$$2) \quad \mathbf{P}_{f,\text{tot}} = 70,292 \text{ kg-mph}$$

$$\begin{aligned} \mathbf{V}_{\text{final}} &= \mathbf{P}_{f,\text{tot}} / \mathbf{M}_{\text{tot}} = 70,292 \text{ kg-mph} / 2700 \text{ kg} \\ &= 26 \text{ mph} \end{aligned}$$

$$\theta = \arctan(54,000/45,000) = 50.2^\circ \text{ east of north}$$



# Summary

→ Impulse  $I = F\Delta t$

» Gives change in momentum  $I = \Delta p$

→ Momentum  $p = mv$

» Momentum is VECTOR

» Momentum is conserved if  $F_{\text{Net}} = 0$

$$\diamond \sum m v_{\text{initial}} = \sum m v_{\text{final}}$$