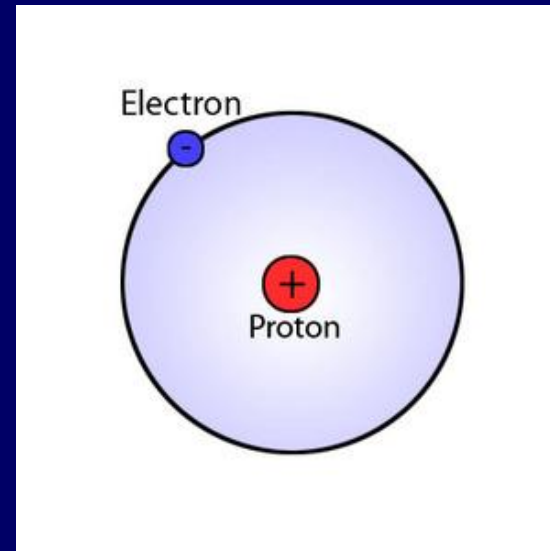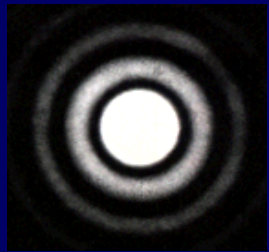# Physics 102: Lecture 24

Heisenberg Uncertainty Principle
& Bohr Model of Atom

Heisenberg Uncertainty Principle

Recall: Quantum Mechanics tells us outcomes of individual measurements are uncertain

$$\Delta p_y \, \Delta y \geq \frac{\hbar}{2}$$
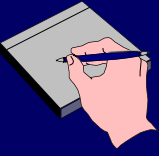
$$\hbar = h/2\pi$$

Uncertainty in momentum (along y)

Uncertainty in position (along y)

Rough idea: if we know momentum very precisely, we lose knowledge of location, and vice versa.
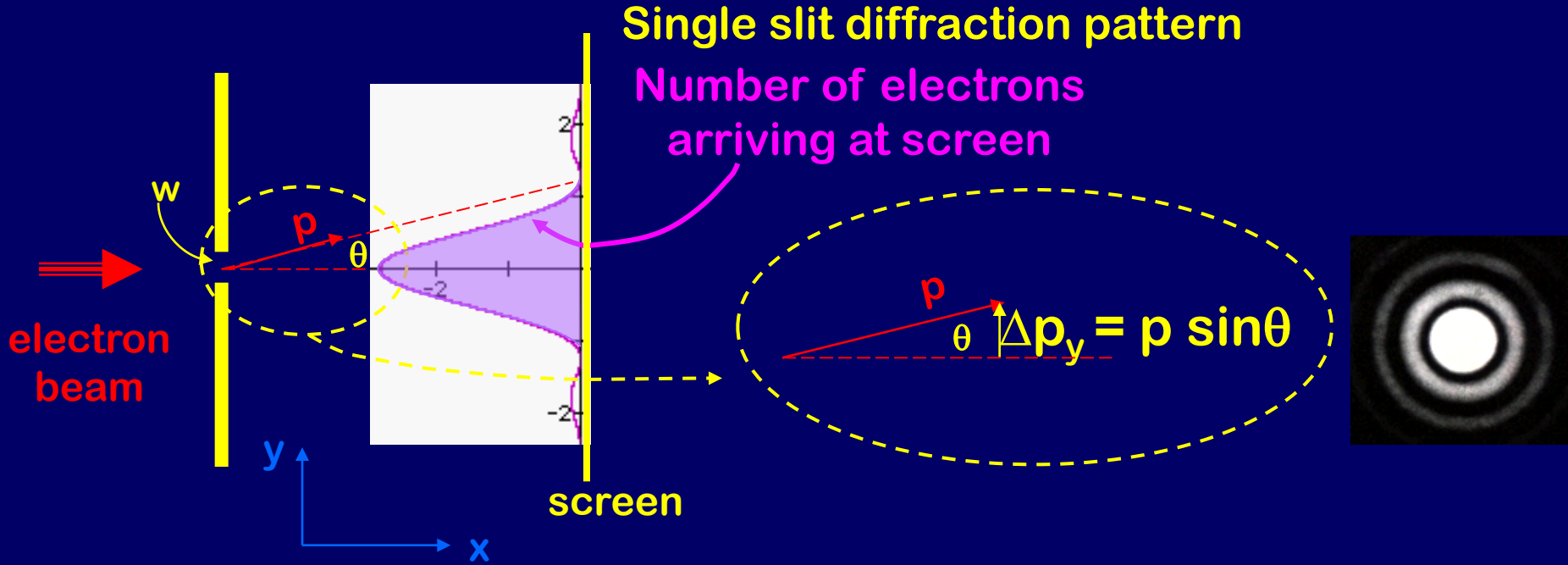
This "uncertainty" is fundamental: it arises because quantum particles behave like waves!
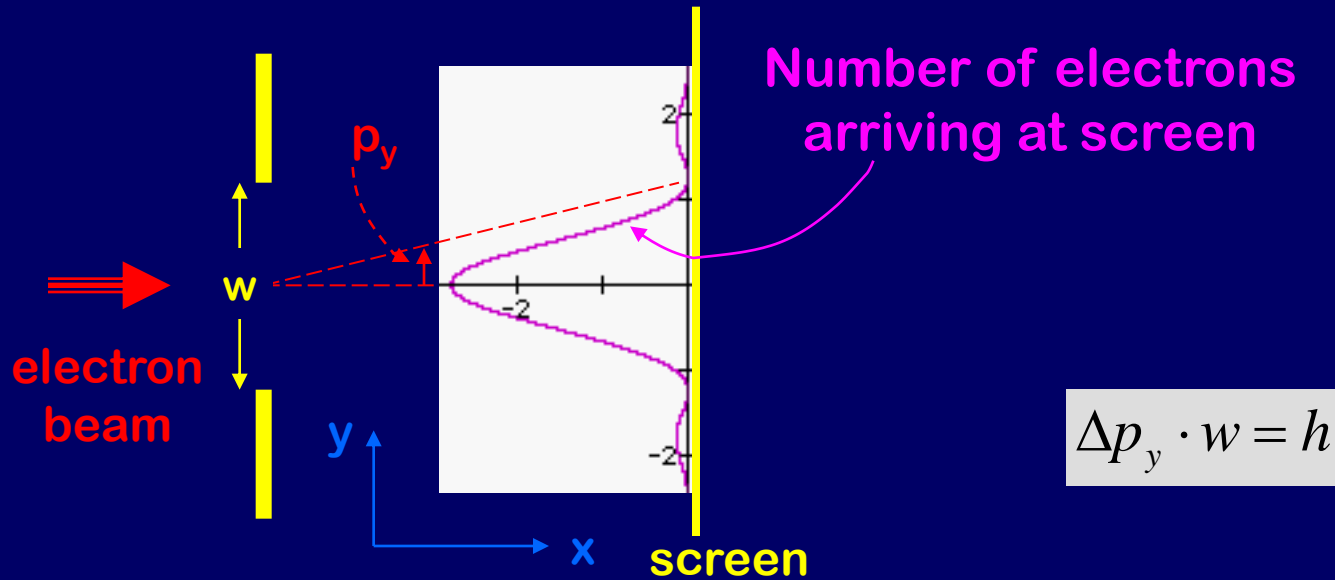
Electron diffraction

Electron beam traveling through slit will diffract

Single slit diffraction pattern

Number of electrons arriving at screen

w

p

θ

electron beam

y

x

screen

p

θ $\Delta p_y = p \sin\theta$

Recall single-slit diffraction 1st minimum:

$\sin\theta = \lambda/w$ $w = \lambda/\sin\theta$ $= \Delta y$

$$\Delta p_y \Delta y = p \sin\theta \frac{\lambda}{\sin\theta} = \lambda p = h$$

Using de Broglie λ

Number of electrons arriving at screen

p_y

w

electron beam

y
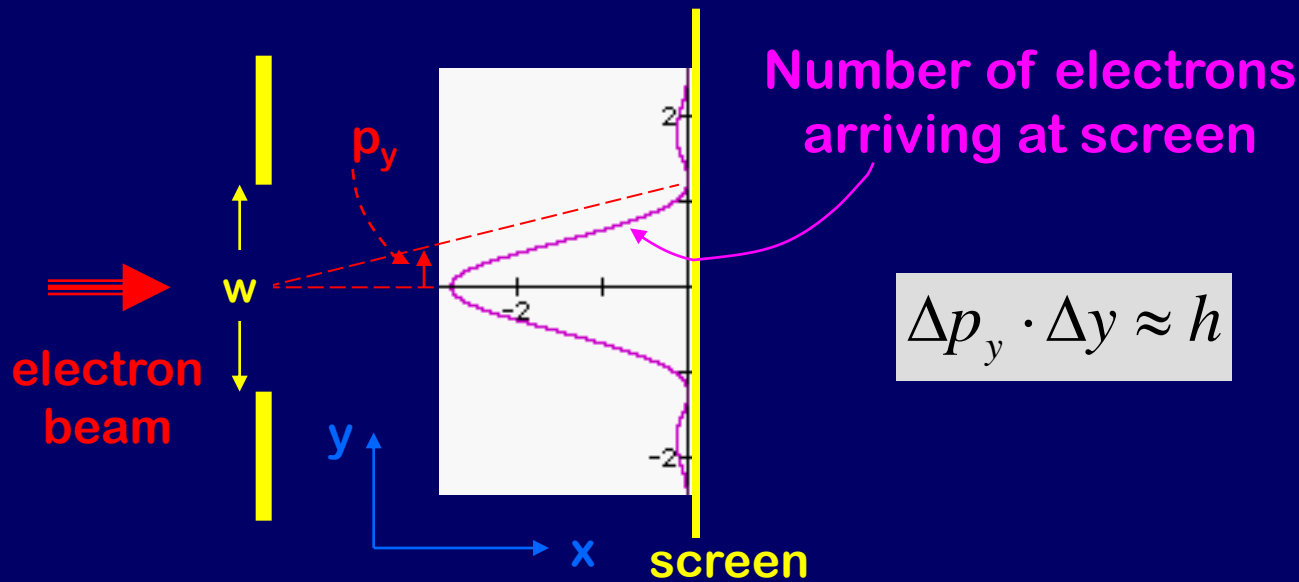
x   **screen**
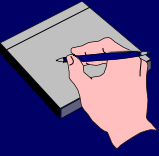
$$\Delta p_y \cdot w = h$$

Electron entered slit with momentum along x direction and no momentum in the y direction. When it is diffracted it acquires a p_y which can be as big as h/w.

The "Uncertainty in p_y" is $\Delta p_y \approx$ h/w.

An electron passed through the slit somewhere along the y direction. The "Uncertainty in y" is $\Delta y \approx$ w.

$$\Delta p_y \cdot \Delta y \approx h$$

Number of electrons arriving at screen

p_y

w

electron beam

y

x

screen

$$\Delta p_y \cdot \Delta y \approx h$$

If we make the slit narrower (decrease w =Δy) the diffraction peak gets broader (Δp$_y$ increases).

$$\Delta p_y \approx h/\Delta y$$

"If we know location very precisely, we lose knowledge of momentum, and vice versa."

to be precise...

$$\Delta p_y \Delta y \geq \frac{\hbar}{2}$$

Of course if we try to locate the position of the particle along the x axis to Δx we will not know its x component of momentum better than Δp$_x$, where

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

and the same for z.

Checkpoint 1

According to the H.U.P., if we know the x-position of a particle, we cannot know its:

(1)   **y-position**   (2)   **x-momentum**
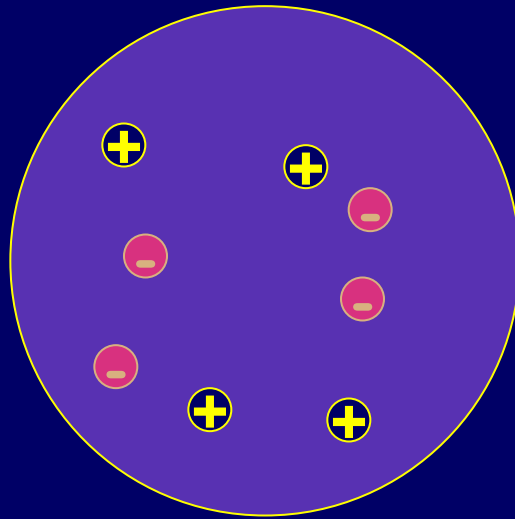
(3)   **y-momentum**   (4)   **Energy**

Atoms

- Evidence for the nuclear atom
- Bohr model of the atom

} Today

- Spectroscopy of atoms
- Quantum atom

} Next lecture

Early Model for Atom

- **Plum Pudding**

  – **positive and negative charges uniformly distributed throughout the atom like plums in pudding**

But how can you look inside an atom 10^{-10} m across?

|	
Light (visible)	$\lambda = 10^{-7}$ m
Electron (1 eV)	$\lambda = 10^{-9}$ m
Helium atom	$\lambda = 10^{-11}$ m

Rutherford Scattering

1911: Scattering He^{++} (an "alpha particle") atoms off of gold.
Mostly go through, some scattered back!

Plum pudding theory:
+ and − charges uniformly distributed → electric field felt by alpha never gets too large

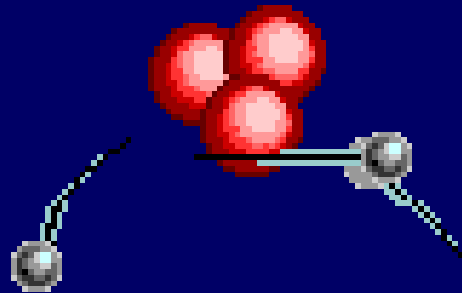To scatter at large angles, need positive charge concentrated in small region (the nucleus)

$10^{-10}\ m$ $10^{-15}\ m$
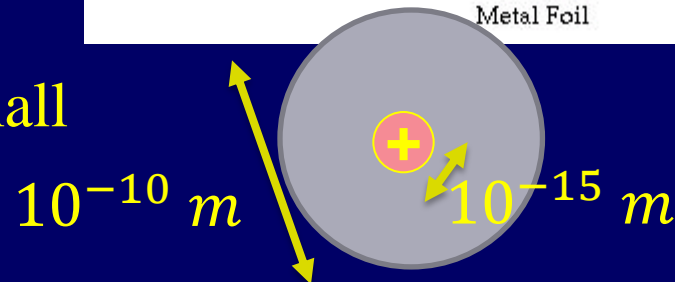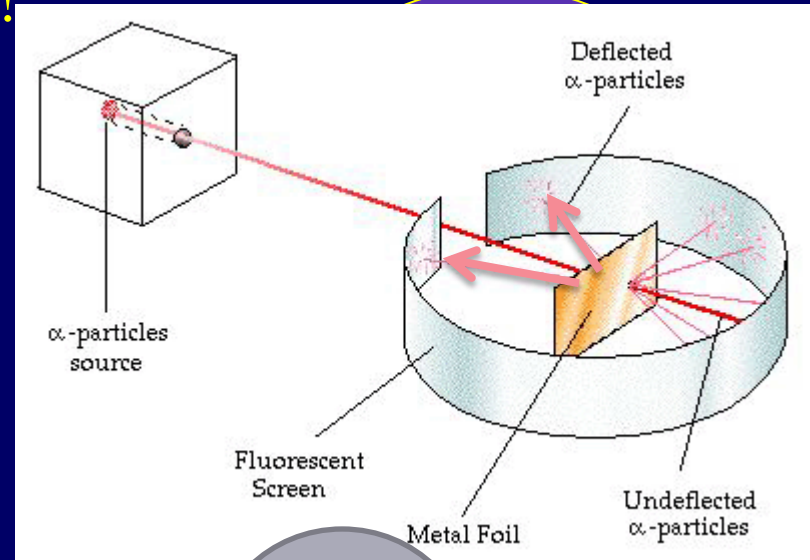
Atom is mostly empty space with a small (r = 10^{-15} m) positively charged nucleus surrounded by "cloud" of electrons (r = 10^{-10} m)

Nuclear Atom (Rutherford)

Large angle scattering ➡ **Nuclear atom**

Classic nuclear atom is not stable!

Electrons will radiate and spiral into nucleus

} **Need quantum theory**

Early "quantum" model: Bohr

Bohr Model is Science fiction

The Bohr model is complete nonsense.

Electrons do not circle the nucleus in little planet-like orbits.
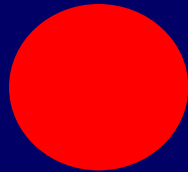
The assumptions injected into the Bohr model have no basis in physical reality.

BUT the model does get some of the numbers right for SIMPLE atoms…
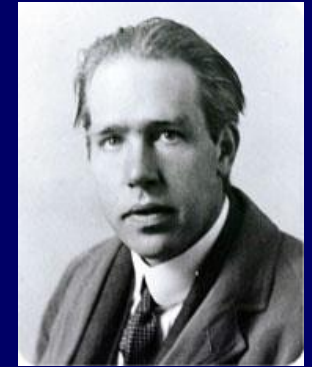
Hydrogen-Like Atoms

single electron with charge -e

nucleus with charge +Ze
(Z protons)

$$e = 1.6 \times 10^{-19} \text{ C}$$
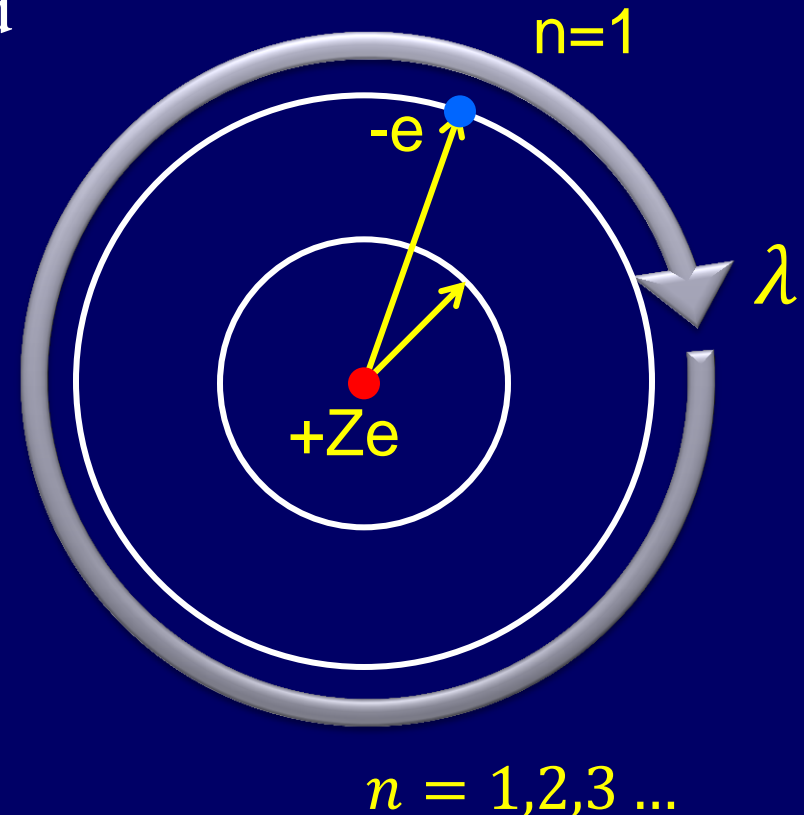
Ex: H (Z=1), He$^+$ (Z=2), Li^{++} (Z=3), etc
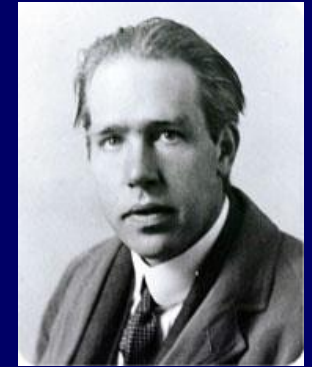
The Bohr Model

Electrons circle the nucleus in orbits

Only certain orbits are allowed

$$2\pi r = n\lambda$$
$$= nh/p$$

n=1

-e

λ

+Ze

$n = 1,2,3 \ldots$
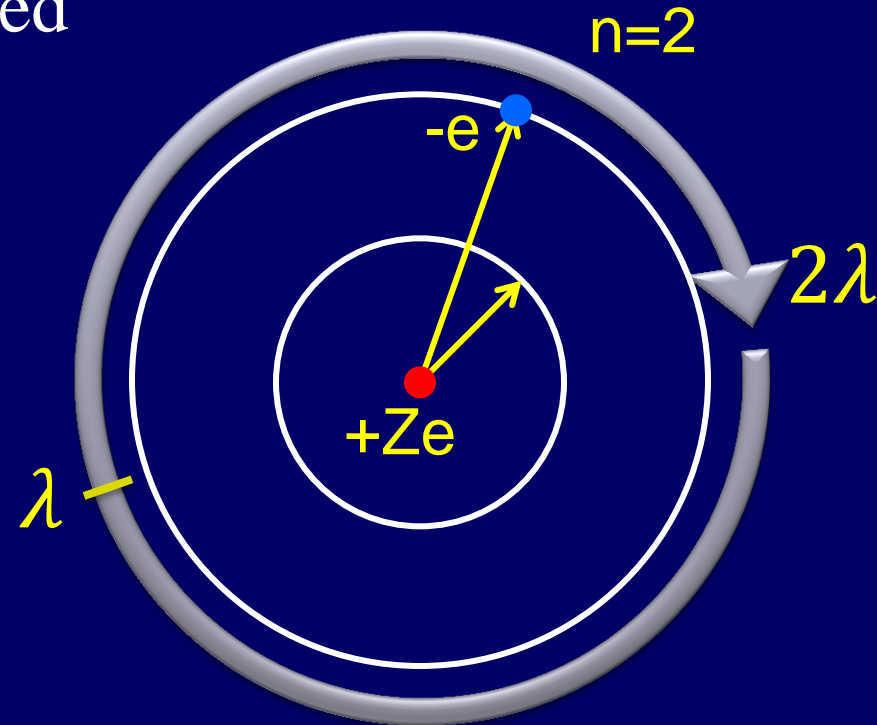
The Bohr Model

Electrons circle the nucleus in orbits

Only certain orbits are allowed

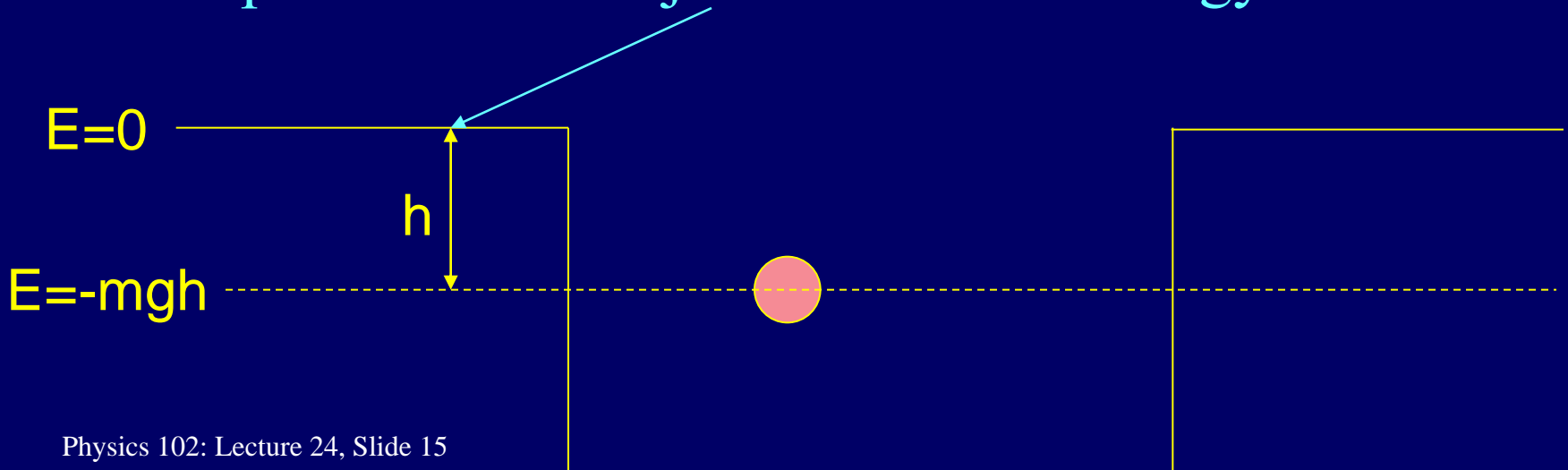$$2\pi r = n\lambda$$
$$= nh/p$$

$$L = pr = nh/2\pi = n\hbar$$

Angular momentum is quantized

Energy is quantized: $E = -13.6 \; eV \; Z^2/n^2$ $n = 1,2,3 \ldots$

v is also quantized in the Bohr model!

n=2

-e

2λ

λ

+Ze

An analogy: Particle in Hole

- The particle is trapped in the hole
- To free the particle, need to provide energy mgh
- Relative to the surface, energy = -mgh
  - a particle that is "just free" has 0 energy

E=0

h

E=-mgh

An analogy: Particle in Hole

- Quantized: only fixed discrete heights of particle allowed

- Lowest energy (deepest hole) state is called the "ground state"

E=0

h

$$E = -13.6 \; eV \; Z^2$$

ground state
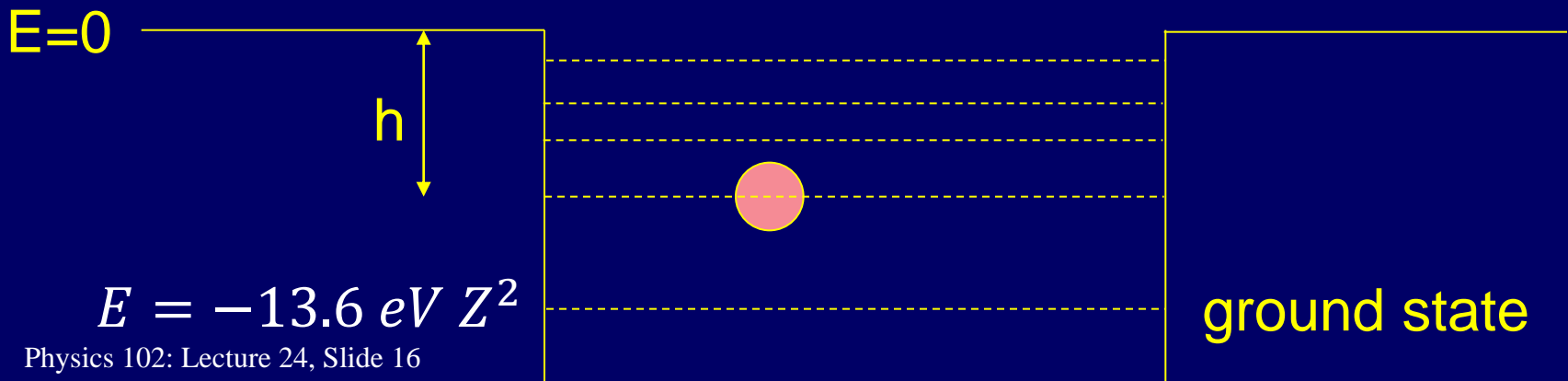
Some (more) numerology

- 1 eV = kinetic energy of an electron that has been accelerated through a potential difference of 1 V

  1 eV = qΔV = 1.6 x 10^{-19} J

- h (Planck's constant) = 6.63 x 10^{-34} J·s

  hc = 1240 eV·nm

- m = mass of electron = 9.1 x 10^{-31} kg

  mc^2 = 511,000 eV

- U = ke^2/r, so ke^2 has units eV·nm (like hc)

  $2\pi ke^2/(hc)$ = 1/137   (dimensionless)

      "fine structure constant"

For Hydrogen-like atoms:

Energy levels (relative to a "just free" E=0 electron):

$$E_n = -\frac{mk^2e^4}{2\hbar^2}\frac{Z^2}{n^2} \approx -\frac{13.6 \cdot Z^2}{n^2} \ eV \ (where \ \hbar \equiv h/2\pi)$$

Radius of orbit:

$$r_n = \left(\frac{h}{2\pi}\right)^2 \frac{1}{mke^2}\frac{n^2}{Z} = (0.0529 \ nm)\frac{n^2}{Z}$$

Checkpoint 2

If the electron in the hydrogen atom was 207 times heavier (a muon), the Bohr radius would be

1) 207 Times Larger

2) Same Size

3) 207 Times Smaller

ACT/Checkpoint 3

A single electron is orbiting around a nucleus with charge +3. What is its ground state (n=1) energy? (Recall for charge +1, E= -13.6 eV)

1) E = 9 (-13.6 eV)

2) E = 3 (-13.6 eV)

3) E = 1 (-13.6 eV)

ACT: What about the radius?

Z=3, n=1

1. larger than H atom
2. same as H atom
3. smaller than H atom

Summary

- Bohr's Model gives accurate values for electron energy levels...

- But Quantum Mechanics is needed to describe electrons in atom.

- Next time: electrons jump between states by emitting or absorbing photons of the appropriate energy.