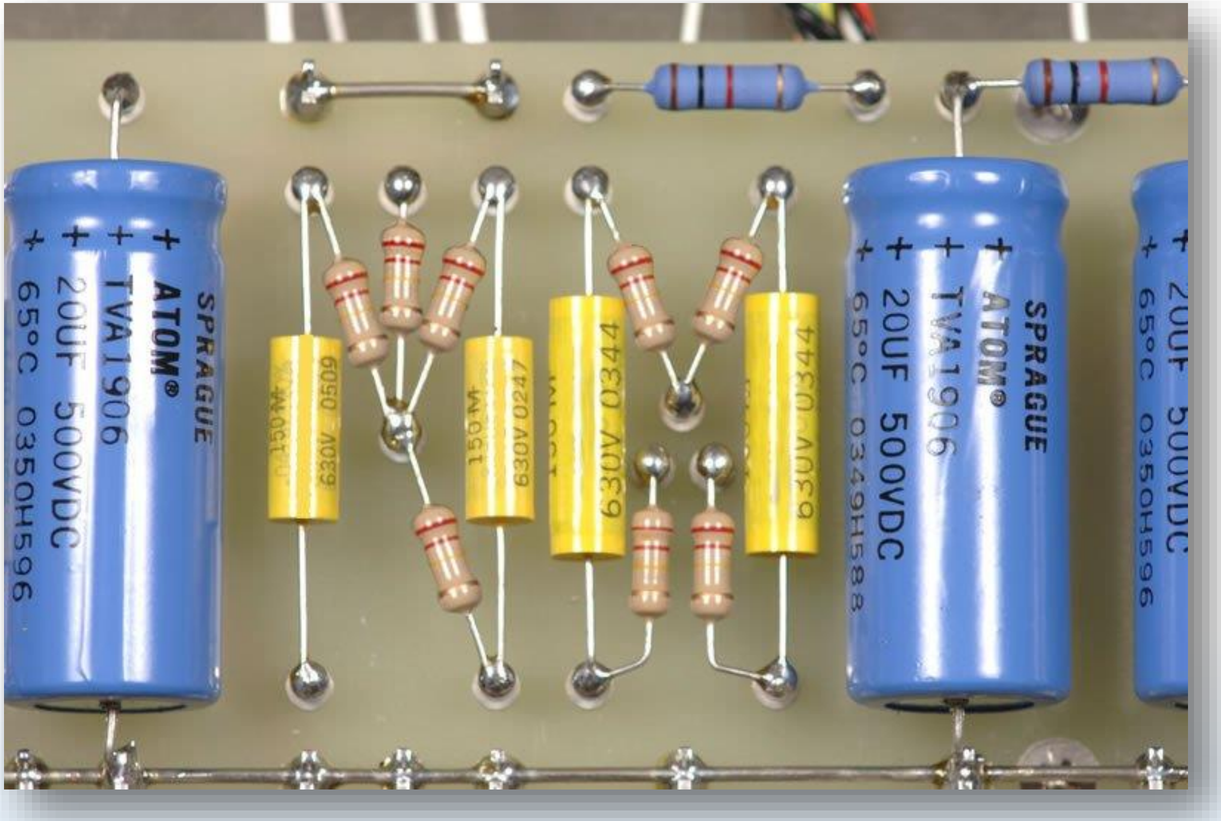


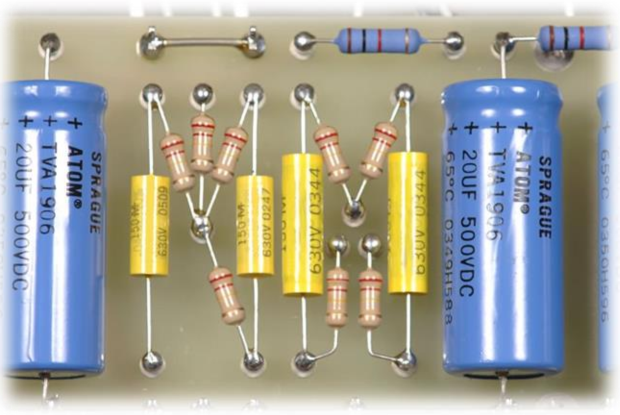
Phys 102 – Lecture 6

Circuit elements: resistors, capacitors, and batteries

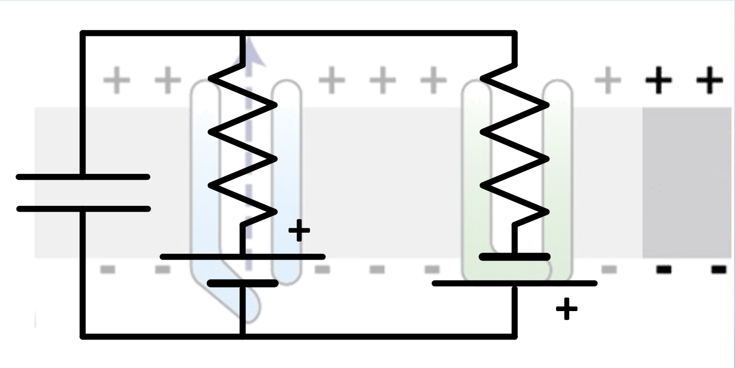
Today we will learn about...

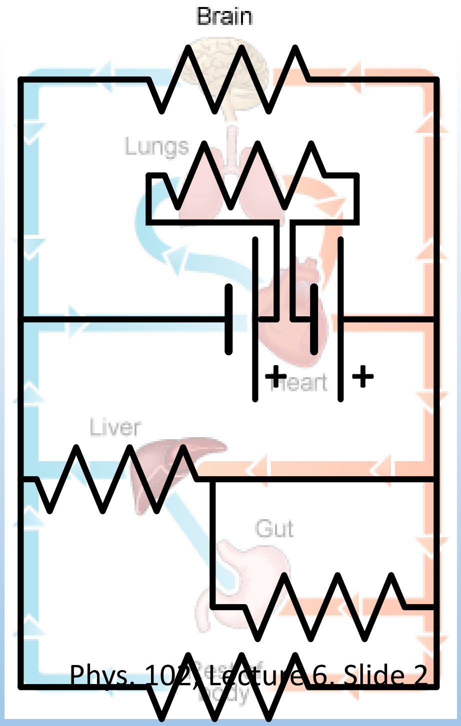
Circuit elements that:

1) Serve as conduits for charge – *wires*
2) Pump charges around – *batteries*
3) Regulate flow of charge – *resistors*
4) Store and release charge – *capacitors*



These elements are idealizations of components in electronic circuits & in nature





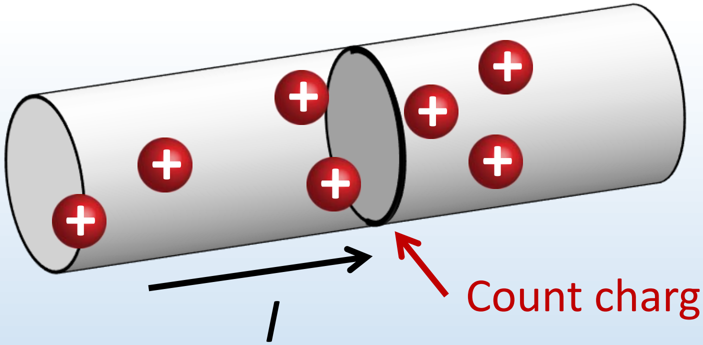
Ex: neurons, circulatory system

Electric current

Current – measure of flow of charge (+ charge, by convention)
Counts total charge Δ*Q* passing through area in a time interval Δ*t*

$$\boxed{I \equiv \frac{\Delta Q}{\Delta t}}$$

Unit: A (“Amp” or “Ampere”)
1A = 1C/s



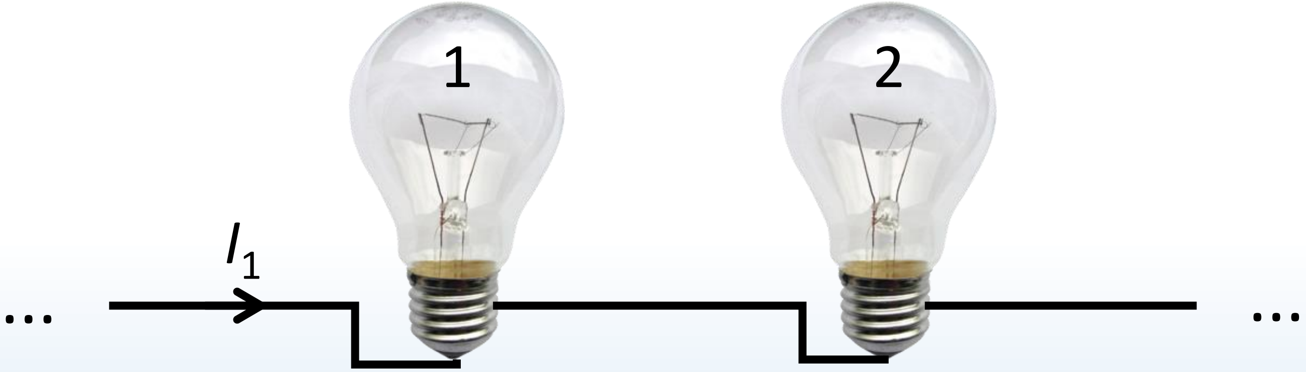
Count charges through this area

In electronic circuits, electrons (–e) carry current, flow *opposite* to current

In liquid or gas, both cations and anions can carry current

ACT: two light bulbs

Two light bulbs 1 and 2 are connected end-to-end by conducting wire. If a current I_1 flows through bulb 1, what is the current I_2 in bulb 2?



A. $I_2 < I_1$

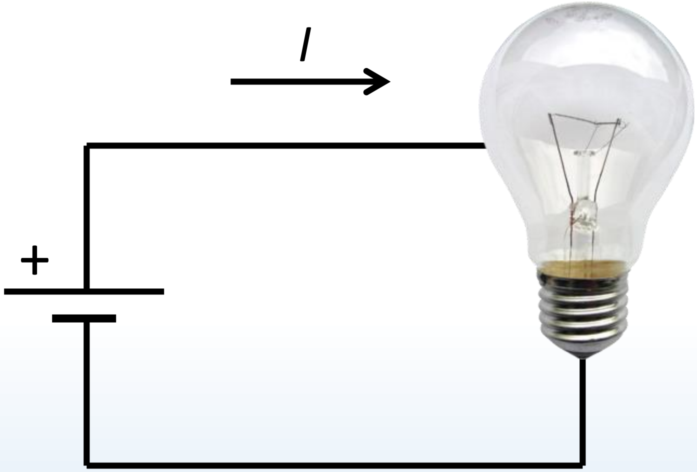
B. $I_2 = I_1$

C. $I_2 > I_1$

Batteries & electromotive force

Battery – maintains a constant electric potential difference (“Electromotive force” – emf ε)

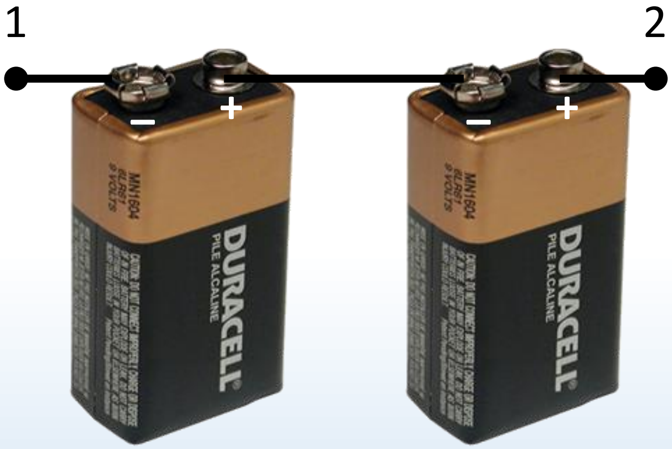
Electric potential is 9 V higher at + end relative to – end. Potential difference across a circuit element is its “voltage”



Electric potential difference drives current around circuit
Battery does NOT determine how much current flows
Battery does NOT generate new charges, it “pushes” charges, like a pump

ACT: Two batteries

Two 9 V batteries are connected end-to-end by conducting wire. What is the electric potential at point 2 relative to point 1?



A. +18 V

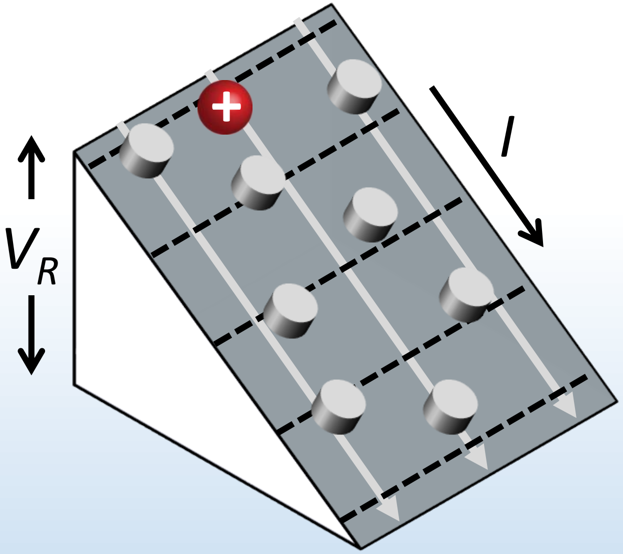
B. +9 V

C. –18 V

D. –9 V

Resistance and Ohm's law

Moving charges collide with each other, ions, defects inside material
Flow rate depends on electric potential difference



$I \propto V_R$ ← Double potential difference, double current

DEMO

Resistance – proportionality constant between current and voltage

Ohm's law: $\boxed{R \equiv \frac{V_R}{I}}$ Units: Ω ("Ohms")

Potential difference causes current to flow ("downhill", by convention)
Resistance regulates the amount of flow

Physical resistance

Resistor – circuit element designed to have resistance

Resistance depends on material parameters and geometry

Resistivity – density of scatterers

$$\boxed{R = \rho \frac{L}{A}}$$

Length – the longer the resistor, the more scattering

Cross sectional area – the wider the resistor, the more charges flow

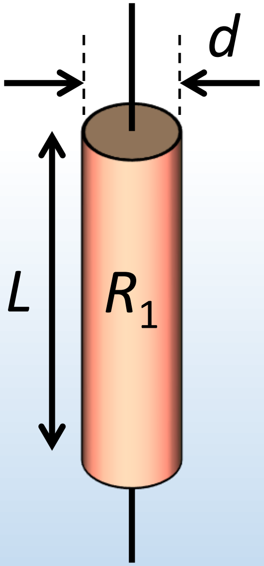
DEMO

Material	ρ ($\Omega \cdot m$)
Copper	1.7×10^{-8}
Iron	9.7×10^{-8}
Sea water	0.22
Muscle	13
Fat	25
Pure water	2.4×10^{5}

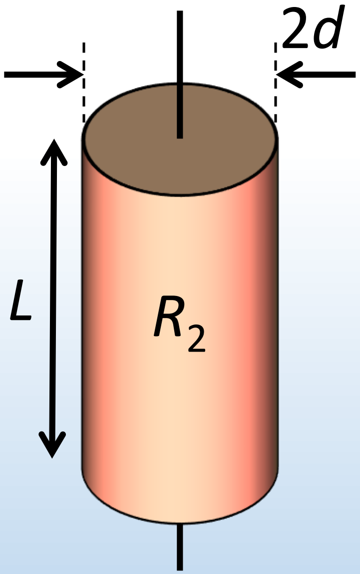
ACT: CheckPoint 1.1

Which of the following three copper resistors has the *lowest* resistance?

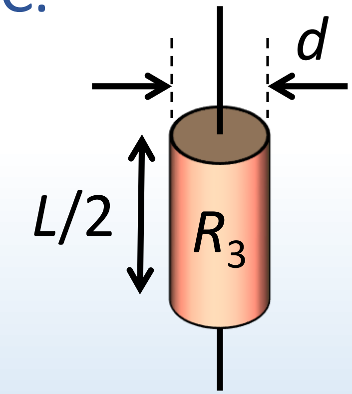
A.



B.



C.



Power generated and dissipated

Battery does work pumping charges through circuit

Ex: a 9 V battery does 9 J of work per 1 C of charge pumped

Power – rate of energy conversion

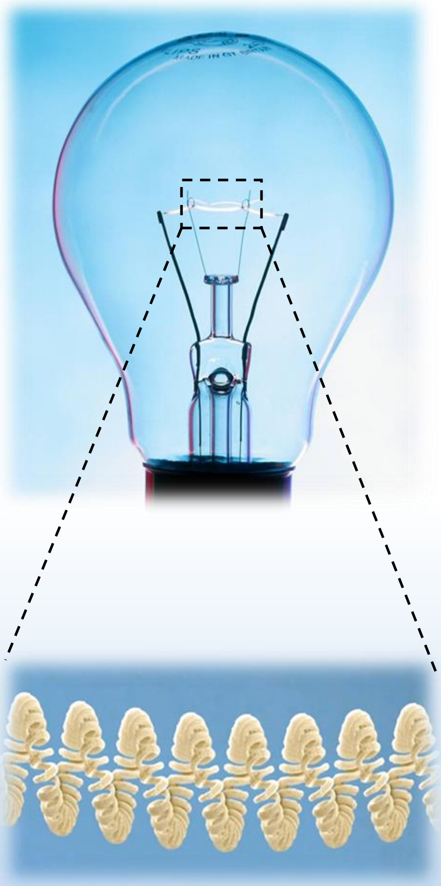
$$P_{batt} = \frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}\varepsilon = I\varepsilon$$

Units: W (“Watts”)
1 W = 1 J/s = 1 V A

Resistor *dissipates* electric potential energy
Charges lose electric potential energy in collisions inside resistor

$$P_{diss} = IV_R = I^2 R = \frac{V_R^2}{R}$$

Calculation: light bulb filament



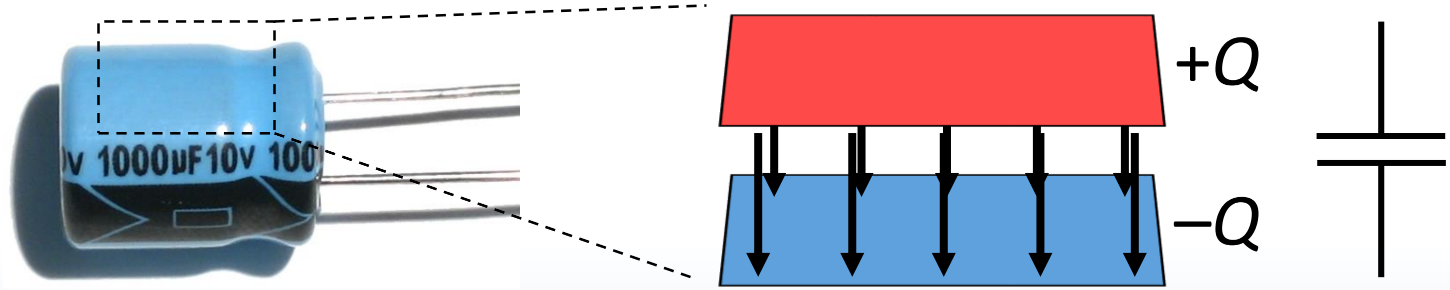
An incandescent light bulb is essentially a resistor that dissipates energy as heat and light. A typical light bulb dissipates 60 W with 120 V from an outlet.

The resistive element is a thin (40-µm diameter) *filament* of tungsten. How long must the filament be?

Capacitance

Capacitor – circuit element that stores separated charge

Consists of two conductors separated by a small gap



Capacitance – measures the ability to store charge *Q* given a voltage V_C applied between the conductors

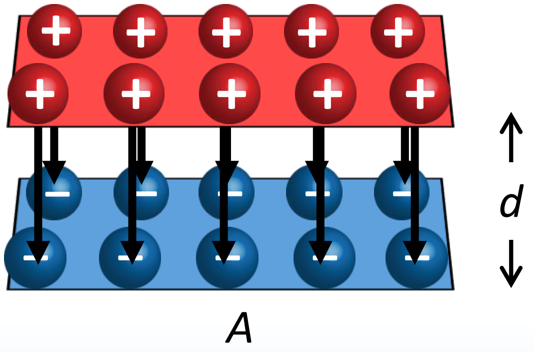
$$\boxed{C \equiv \frac{Q}{V_C}}$$

Units: F ("Farad")
1 F = 1 C/V

Parallel plate capacitor

Capacitor made up of two large conducting plates of area *A* separated by a small gap *d*

Electric field is uniform between plates (Recall Lect. 3)

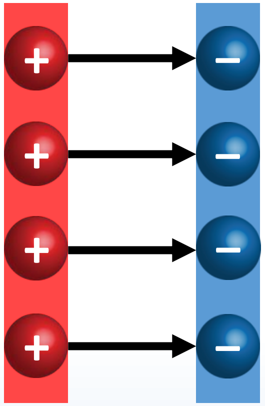


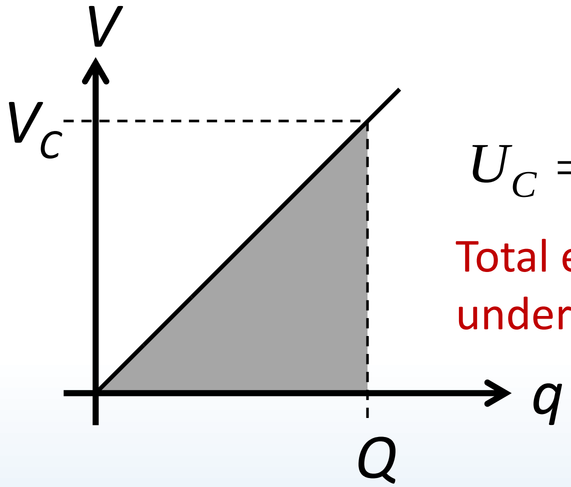
Work to move +*q* charge from + to – plate (Recall Lect. 4)

For a parallel plate capacitor:

Capacitor energy

Separated charges have potential energy (Recall Lect. 4)





$$U_C = \sum \Delta q \Delta V$$

Total energy = area under curve

Imagine transferring + charge from one plate to the other
Each time add Δ*q*, electric field and voltage increase by Δ*E* and Δ*V*

$$U_C = \frac{1}{2}QV_C = \frac{1}{2}CV_C^2 = \frac{1}{2}\frac{Q^2}{C}$$

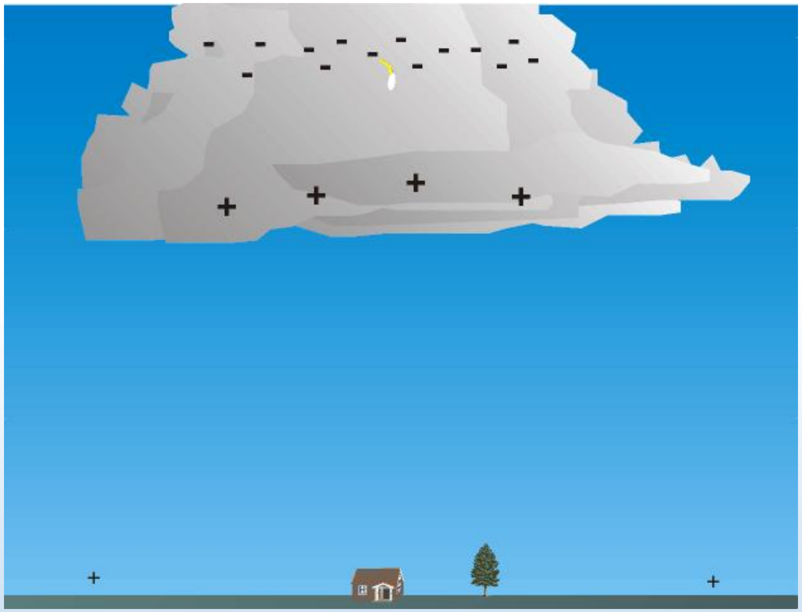
Why separate charge?

A way to store *and* release energy

DEMO

Camera flash

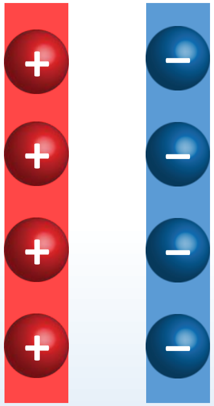
Defibrillator



Lightning strike

ACT: Parallel plates

A parallel plate capacitor carries a charge Q. The plates are then pulled a small distance further apart.



What happens to the charge Q on each plate?

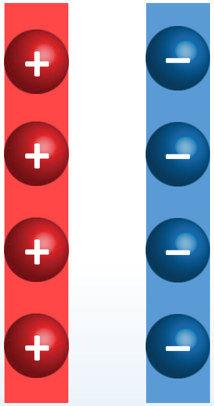
A. Q increases

B. Q stays constant

C. Q decreases

Checkpoint 2

A parallel plate capacitor carries a charge *Q*. The plates are then pulled a small distance further apart.



The capacitance increases True False

The electric field increases True False

The voltage between the plates increases True False

The energy stored increases True False

Dielectrics

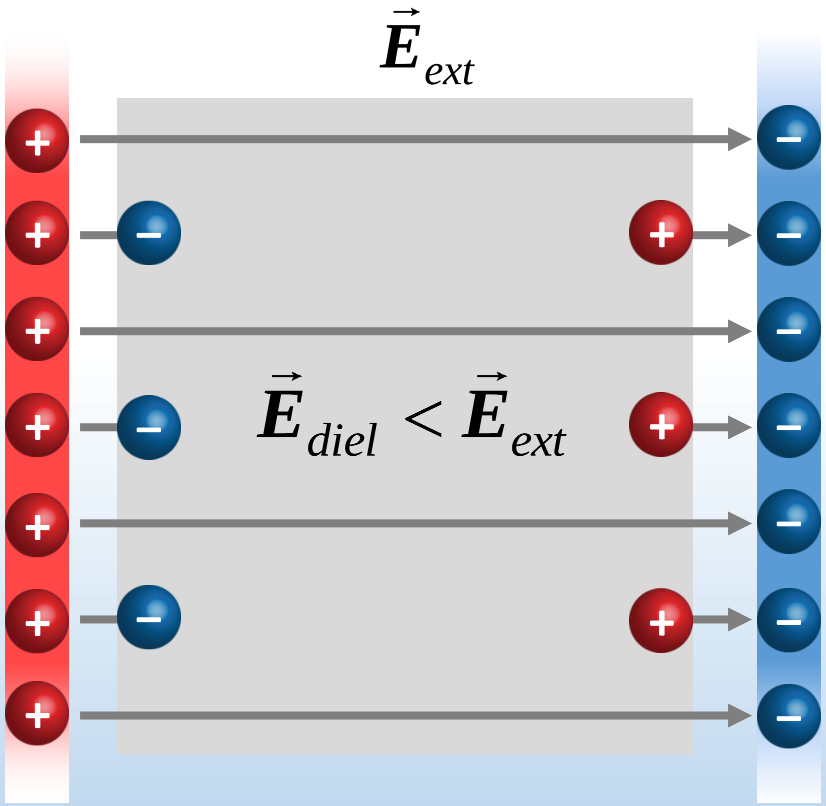
Imagine placing insulating material (dielectric) between plates

External field polarizes dielectric
Excess +*q* and –*q* charges build up on opposite planes

Parallel planes of +*q* and –*q* *create own E field*, cancel out part of external E field

$$\vec{E}_{diel} = \frac{\vec{E}_{ext}}{\kappa}$$

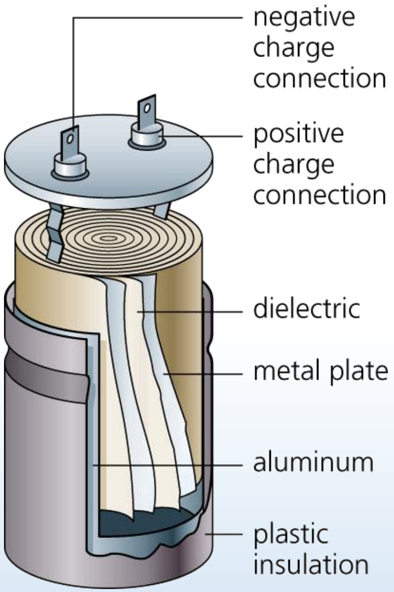
Dielectric constant κ > 1



(Recall Lect. 3 – conductors)

Dielectric constant κ

Dielectric constant κ measures how much a material is polarized by electric field



Since $\vec{E} = \vec{E}_0 / \kappa$, need less *E* (or *V*) to store same *Q*, so *C* = *Q*/*V* increases:

Dielectric constant

Capacitance with dielectric

$$\boxed{C = \kappa C_0}$$

Capacitance without dielectric

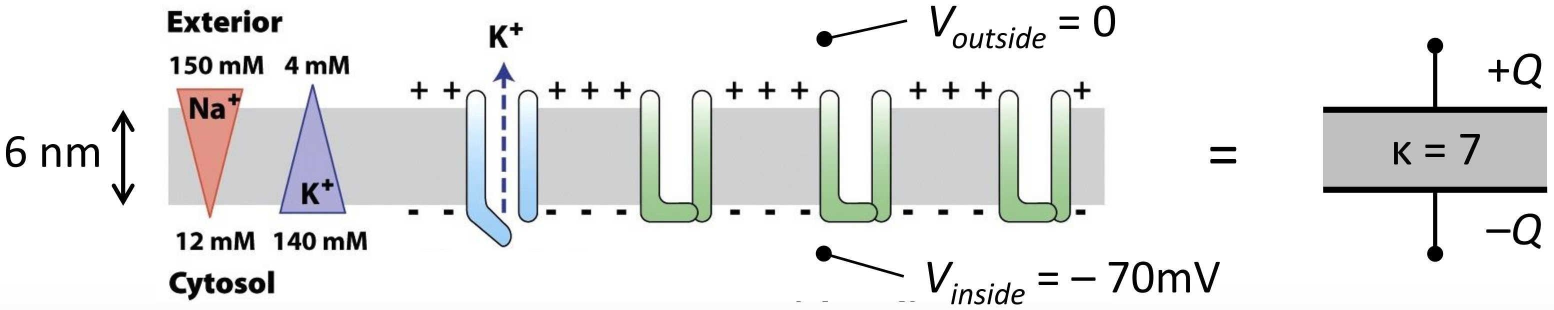
Material	κ (> 1)
Vacuum	1 (exactly)
Air	1.00054
Rubber	3-4
Glass	5
Cell membrane	7-9
Pure water	80

Capacitance depends on material parameters (dielectric) and geometry

Calculation: capacitance of a cell

Channels in a cell’s membrane create a charge imbalance (recall Lect. 5), with + charge outside, – inside. The separated charge gives the cell *capacitance*, with the membrane acting as a dielectric ($\kappa = 7$).

Based on EXAM 1, FA09

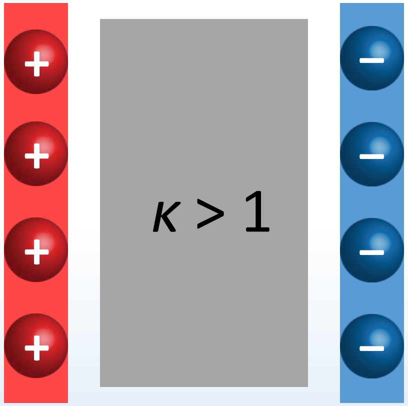


What is the capacitance of a 1-μm^2 flat patch of cell?

At rest, a cell has a –70 mV voltage across it. How much charge is necessary to generate this voltage?

ACT: Capacitor dielectric

A parallel plate capacitor carries a charge Q. A dielectric with $\kappa > 1$ is inserted between the plates.



What happens to energy U_C stored in the capacitor?

A. U_C increases
B. U_C stays constant
C. U_C decreases

Summary of today's lecture

- Batteries generate emf ε, pump charges
- Resistors *dissipate* energy as power: $P = IV$

  Resistance: how difficult it is for charges to get through: $R = \rho L/A$

  Voltage determines *current*: $V = IR$

  Ideal wires have $R = 0$, $V = 0$

- Capacitors *store* energy as separated charge: $U = \frac{1}{2}QV$

  Capacitance: ability to store separated charge: $C = \kappa\varepsilon_0 A/d$

  Voltage determines *charge*: $V = Q/C$

- Don't mix capacitor and resistor equations!