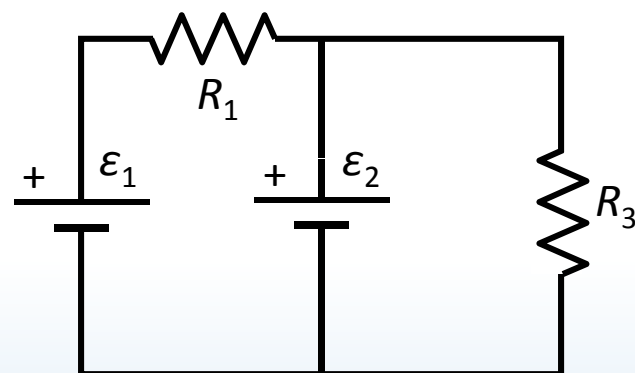
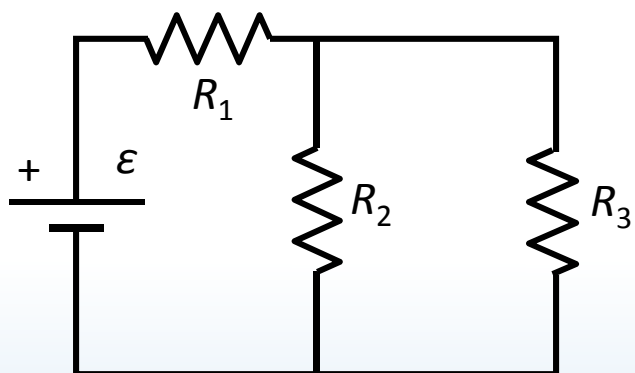


Phys 102 – Lecture 9

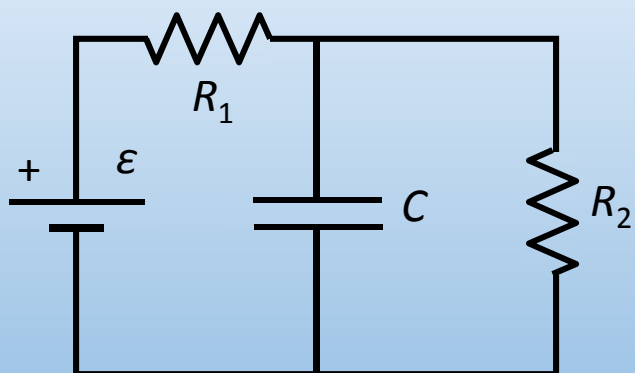
RC circuits

Recall from last time...

We solved various circuits with resistors and batteries (also capacitors and batteries)



What about circuits that combine all three... ...RC circuits



RC circuits

Circuits that store and release energy controllably...



Camera flash



Defibrillator



Nerve cells

Today we will...

- Learn about RC circuits
 - Charge on capacitors cannot change instantly,
so behavior of RC circuit *depends on time*
- Analyze RC circuits under different situations
 - Charging capacitors at short/long times
 - Discharging capacitors at short/long times
 - Time dependence
- Apply these concepts
 - Nerve cells and nerve impulses (action potential)

Charging capacitor

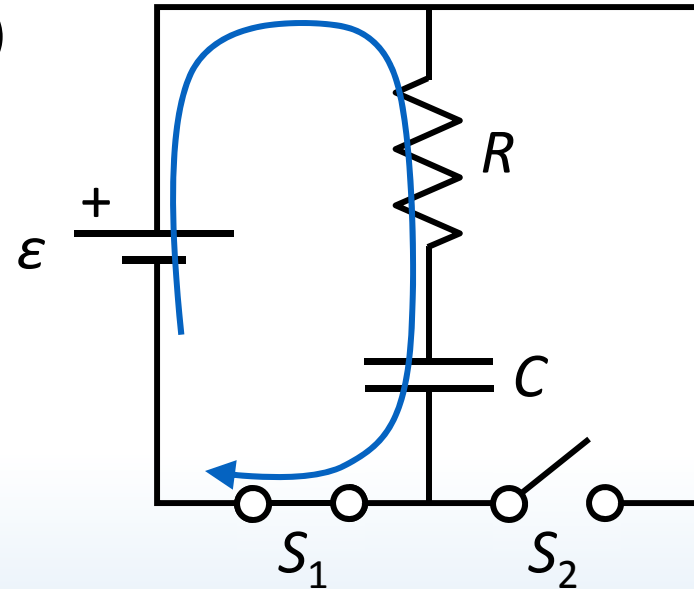
Initially the capacitor is uncharged ($Q_0 = 0$)

At $t = 0$ we close switch S_1 .

Immediately after:

Current I_0 flows around loop, through C

No charge on C ($Q_0 = 0$)



After a long time ($t = \infty$):

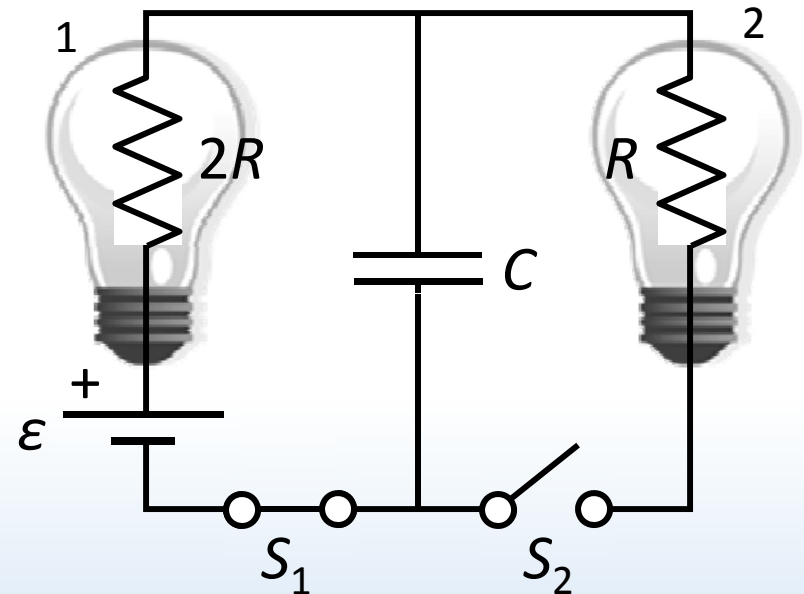
Charge on C builds until $V_C = \varepsilon$. Current decreases to zero ($I_\infty = 0$)



ACT: CheckPoint 1.1

Both switches are initially open and the capacitor is uncharged. What is the current through light bulb 1 right after switch S_1 is closed?

- A. $I_b = 0$
- B. $I_b = \epsilon/2R$
- C. $I_b = \epsilon/3R$
- D. $I_b = \epsilon/R$



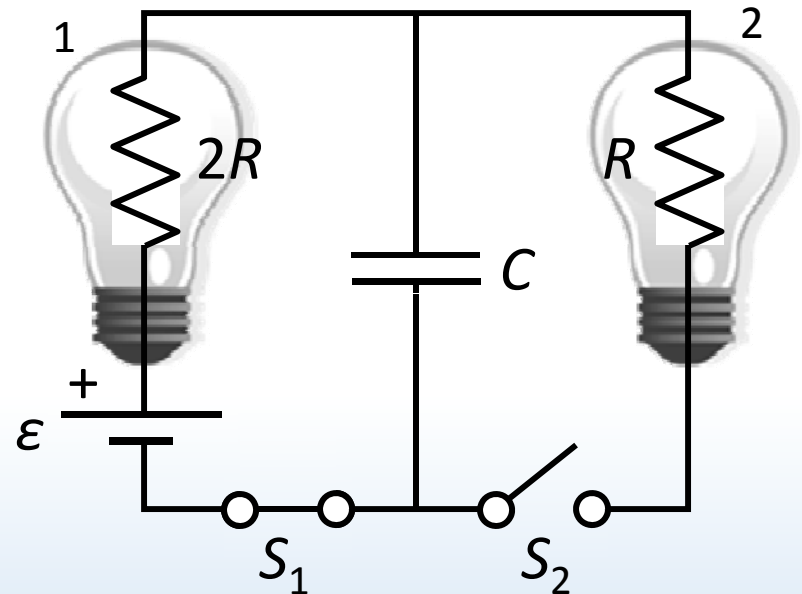
DEMO



ACT: charging

Both switches are initially open and the capacitor is uncharged. What is the voltage across the capacitor at long time after switch S_1 is closed?

- A. $V_C = 0$
- B. $V_C = \epsilon/2$
- C. $V_C = \epsilon$
- D. $V_C = 2\epsilon$



DEMO

Discharging capacitor

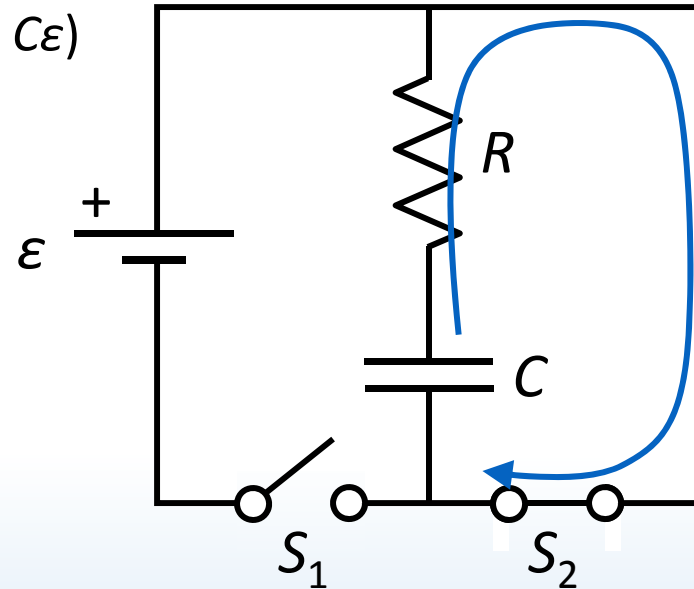
Initially the capacitor is fully charged ($Q_0 = C\varepsilon$)

At $t = 0$ we close switch S_2 .

Immediately after:

Current I_0 driven around loop, through C

Charge on C from before ($Q_0 = C\varepsilon$)



After a long time ($t = \infty$):

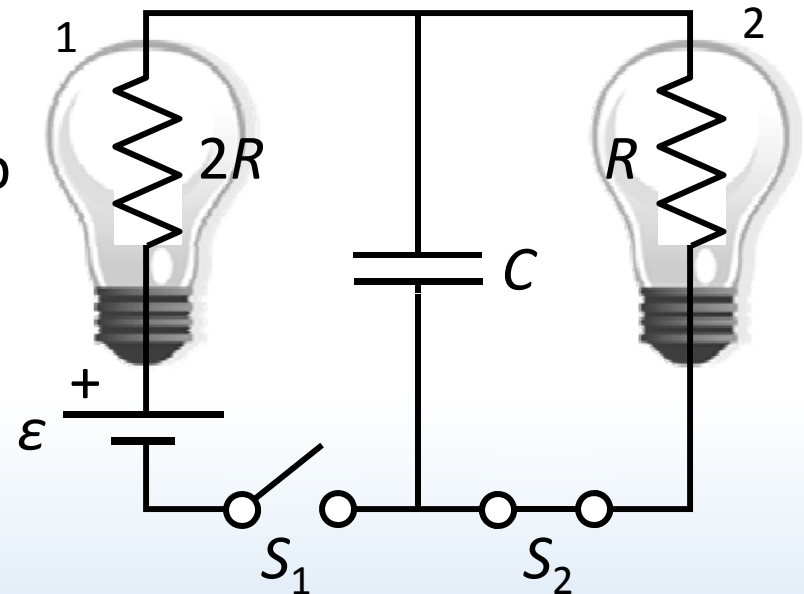
Charge on C dissipates until $V_C = 0$. Current decreases to zero ($I_\infty = 0$)



ACT: CheckPoint 1.5

After S_1 has been closed for a long time, it is opened and S_2 is closed. What is the current through light bulb 2 right after S_2 is closed?

- A. $I_b = 0$
- B. $I_b = \epsilon/2R$
- C. $I_b = \epsilon/3R$
- D. $I_b = \epsilon/R$



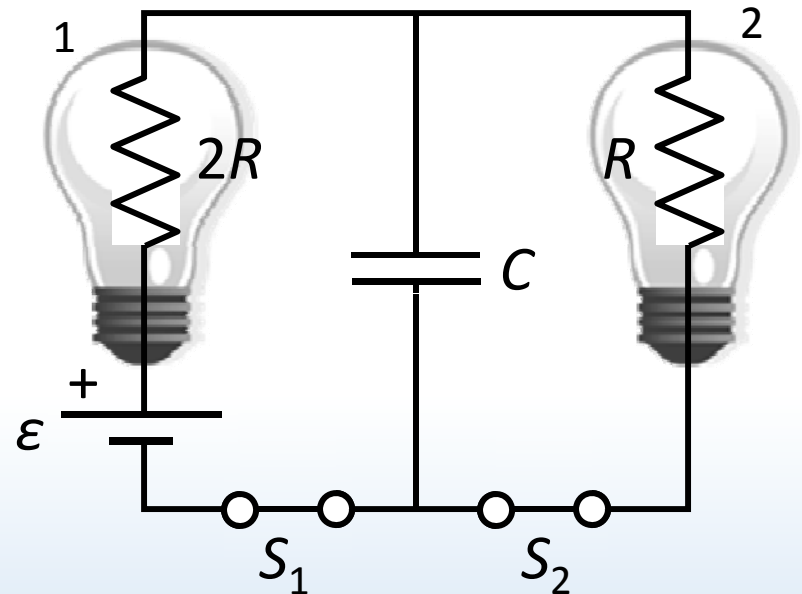
DEMO



ACT: RC circuit practice

Now both S_1 and S_2 are closed. What is the current through light bulb 2 a long time after both switches are closed?

- A. $I_b = 0$
- B. $I_b = \epsilon/2R$
- C. $I_b = \epsilon/3R$
- D. $I_b = \epsilon/R$



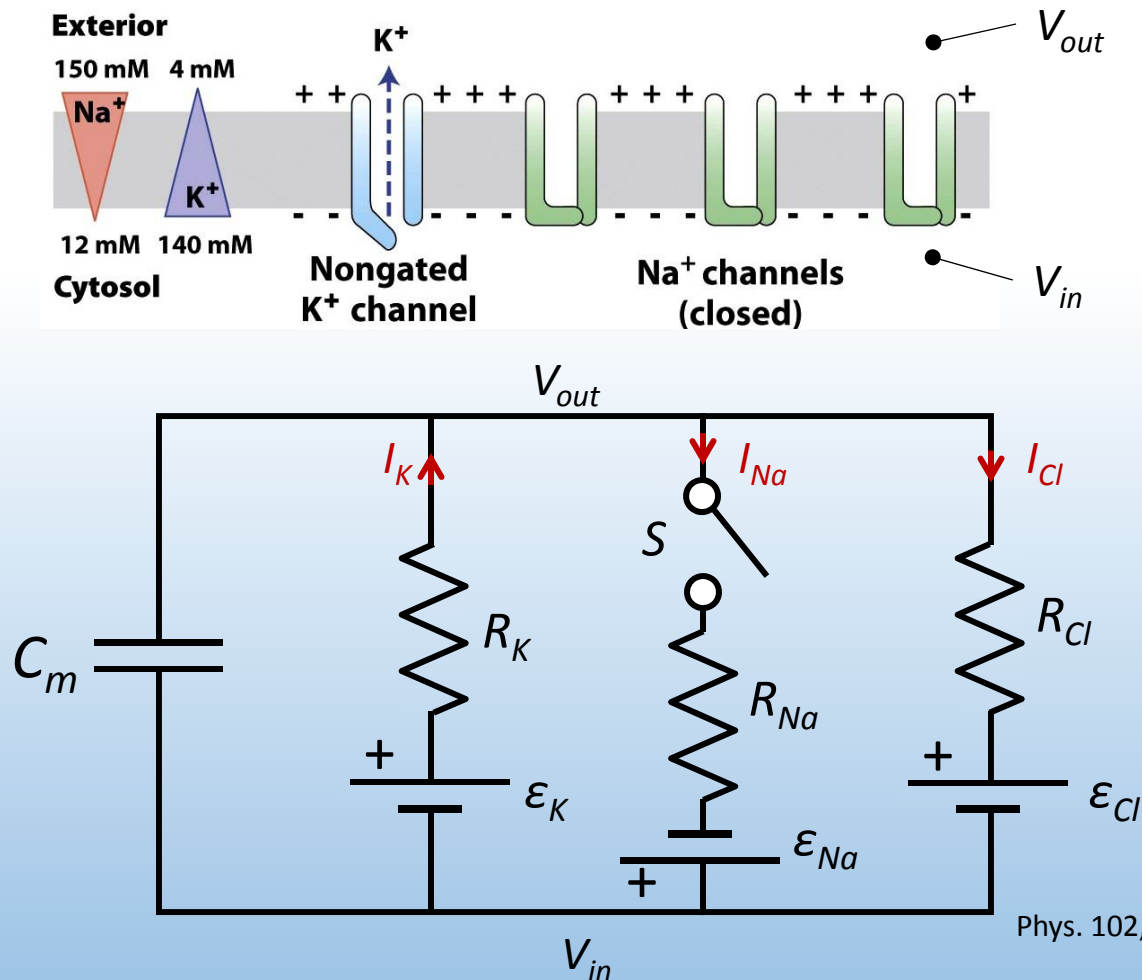
DEMO

Summary: charging & discharging

- Charge (and therefore voltage, since $V_C = Q/C$) on capacitors cannot change instantly
- **Short term** behavior of capacitor:
 - If the capacitor is charging, current I drives charge onto it, and Q increases (acts like a wire)
 - If the capacitor is discharging, current I drives charge off of it, and Q decreases (acts like a battery)
- **Long term** behavior of capacitor:
 - If the capacitor is fully charged, $I = 0$ and Q is maximum (acts like an open circuit)
 - If the capacitor is fully discharged, $I = 0$ and Q is minimum (acts like an open circuit)

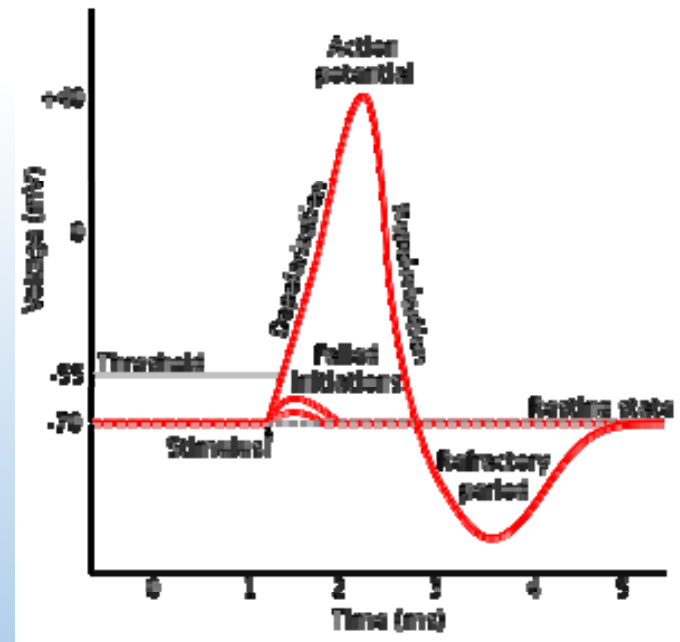
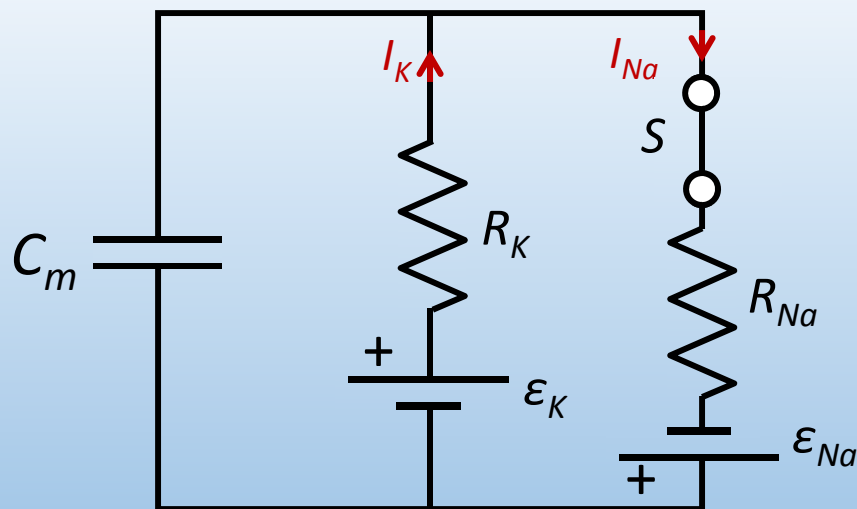
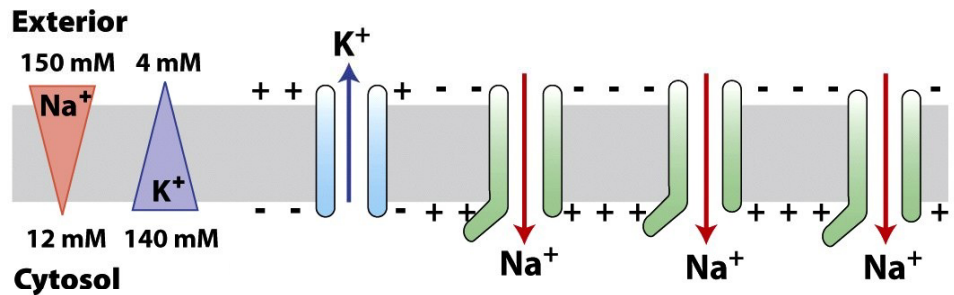
Nerve cell equivalent circuit

Neurons have ion channels (K^+ , Na^+ , and Cl^-) that pump current into and out of cell (it is *polarized*). Cell membrane also has capacitance



Action potential

At rest, Na^+ channels in cell are closed. When stimulated, the cell's voltage increases (*depolarization*). If a threshold is exceeded, the Na^+ channels open & trigger a nerve impulse (*action potential*)

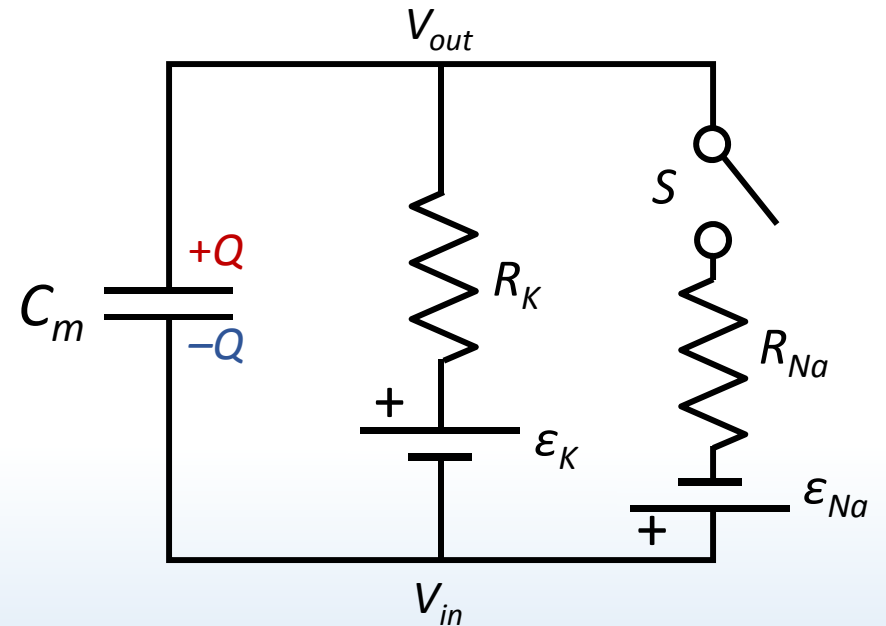




ACT: Resting state of neuron

The neuron has been in resting state for a long time. What is the voltage across the membrane capacitance?

- A. $V_C > \epsilon_K$
- B. $V_C = \epsilon_K$
- C. $V_C < \epsilon_K$



Calculation: action potential I

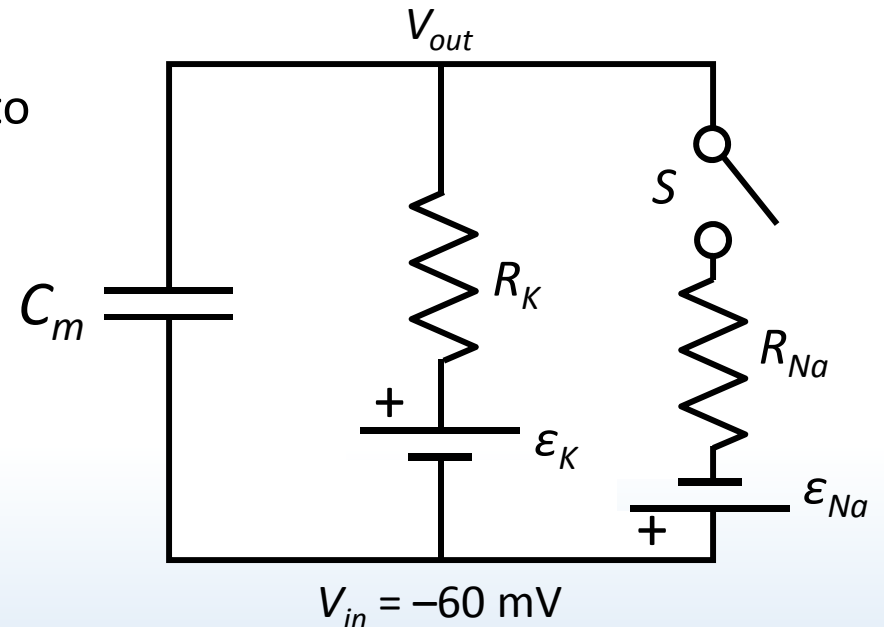
Some time ago, the cell was stimulated and depolarized to -60 mV, less than threshold to open Na^+ channels. What happens next?

Immediately after:

No current through Na^+ channel

Current I_K driven by K^+ channel

Charge Q_0 on C_m from $V_C = 60$ mV



$$\epsilon_K = 70 \text{ mV}, \epsilon_{Na} = 60 \text{ mV},$$

$$R_K = 2 \text{ M}\Omega, R_{Na} = 0.4 \text{ M}\Omega, C_m = 300 \text{ pF}$$

After a long time:

Current I_K decays to 0

Charge on C_m returns to rest value

RC circuit time dependence

Charging:

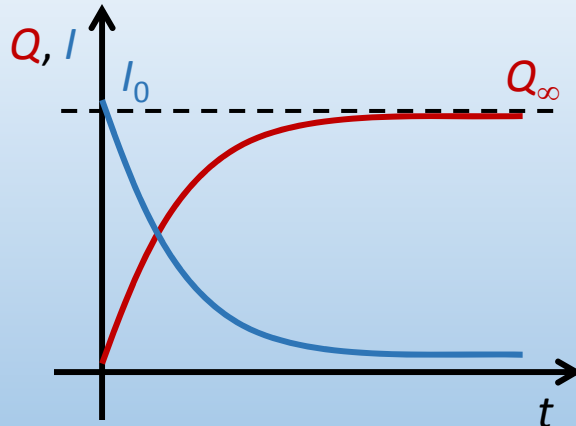
$$\varepsilon - I(t)R - \frac{Q(t)}{C} = 0$$

Charge builds up:

$$Q(t) = Q_{\infty}(1 - e^{-t/RC})$$

Current decays:

$$I(t) = I_0 e^{-t/RC}$$



Discharging:

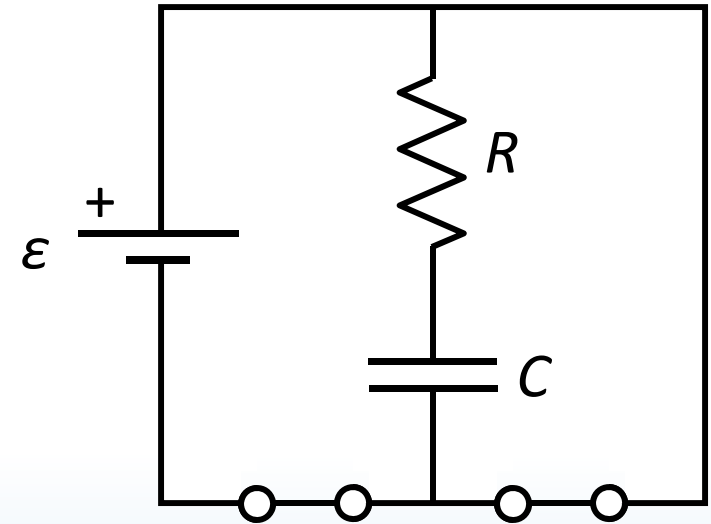
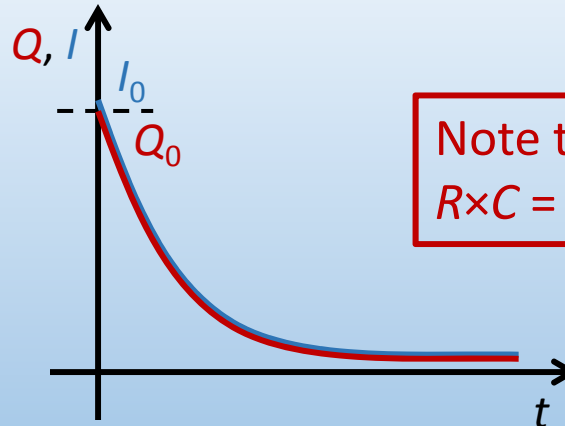
$$\frac{Q(t)}{C} - I(t)R = 0$$

Charge decays:

$$Q(t) = Q_0 e^{-t/RC}$$

Current decays:

$$I(t) = I_0 e^{-t/RC}$$

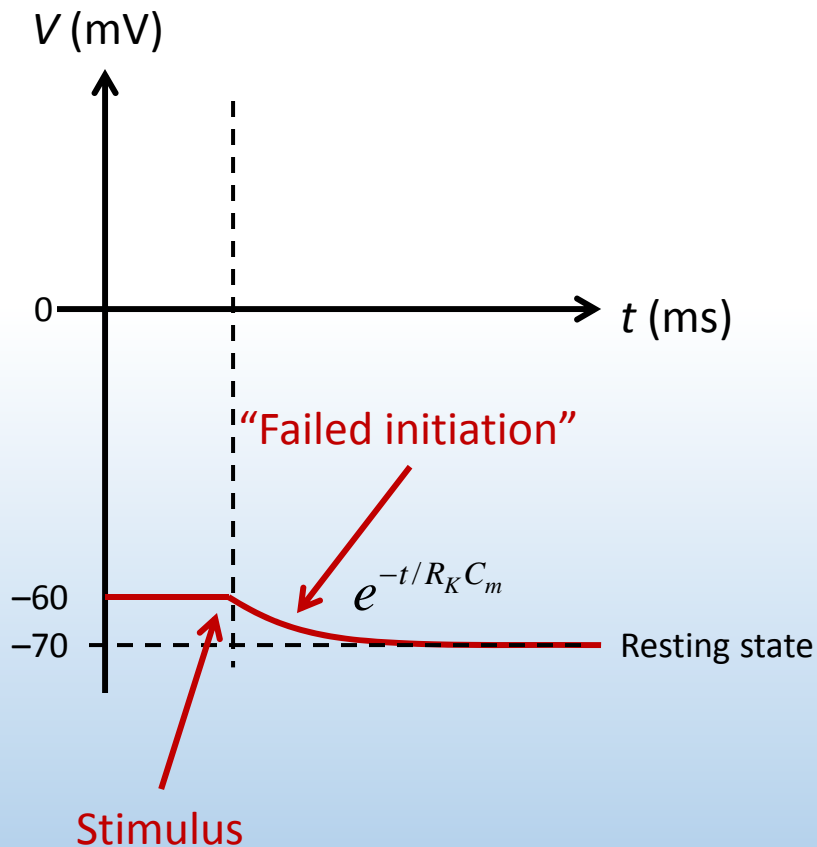


DEMO

Note that RC has units of time!
 $R \times C = [V]/[I] \times [Q]/[V] = [Q]/[I] = [t]$

Calculation: action potential I

How long does the cell take to return to 90% of its resting voltage?



Cell voltage $V_{in} - V_{out} = -V_C = -Q/C$:

$$\Delta V(t) = 10e^{-t/R_K C_m}$$

$$\Delta V(t_{90}) = 10(1 - 0.9) = 10e^{-t_{90}/R_K C_m}$$

Take natural log of both sides:

$$\begin{aligned} t_{90} &= -R_K C_m \ln(0.1) \\ &= -(2 \times 10^9)(300 \times 10^{-12})(-2.3) \\ &\approx 1.4 \text{ ms} \end{aligned}$$

Calculation: action potential II

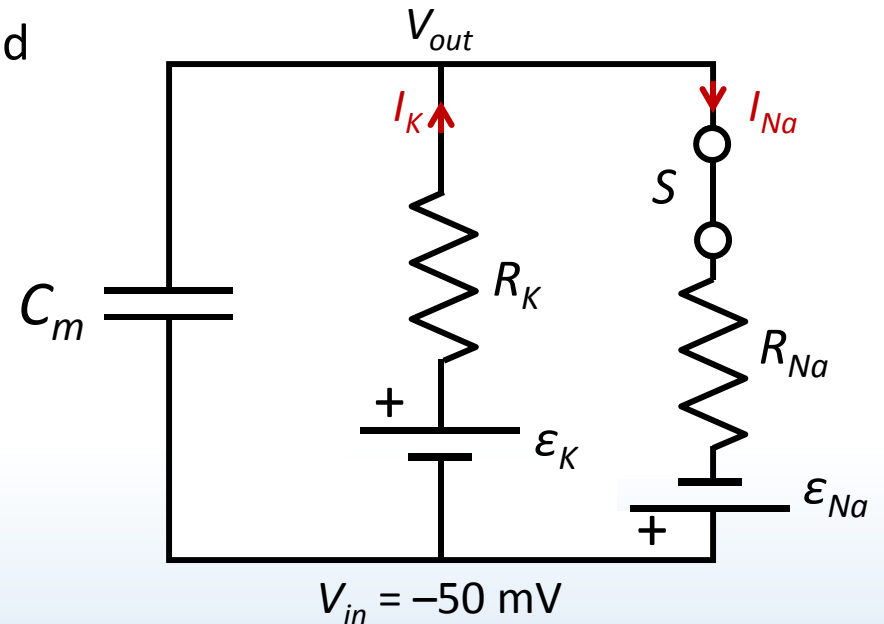
Now, the cell was stimulated and depolarized to -50 mV, over the threshold to open Na^+ channels. What happens next?

Immediately after:

Current I_{Na} through Na^+ channel

Current I_{K} driven by K^+ channel

Charge Q_0 on C_m from before



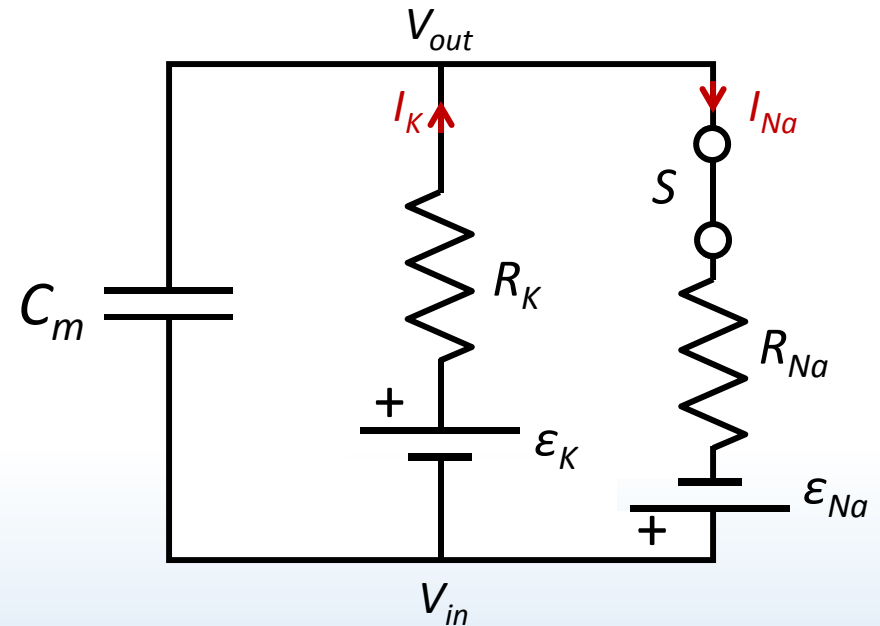
$$\begin{aligned}\epsilon_K &= 70 \text{ mV}, \epsilon_{Na} = 60 \text{ mV}, \\ R_K &= 2 \text{ M}\Omega, R_{Na} = 0.4 \text{ M}\Omega, C_m = 300 \text{ pF}\end{aligned}$$



ACT: action potential II

A long time after stimulating the cell, which statement below holds TRUE?

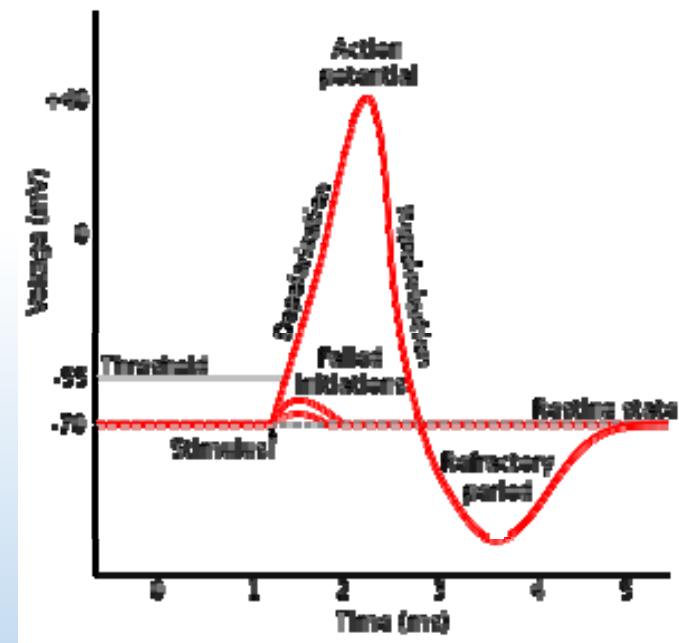
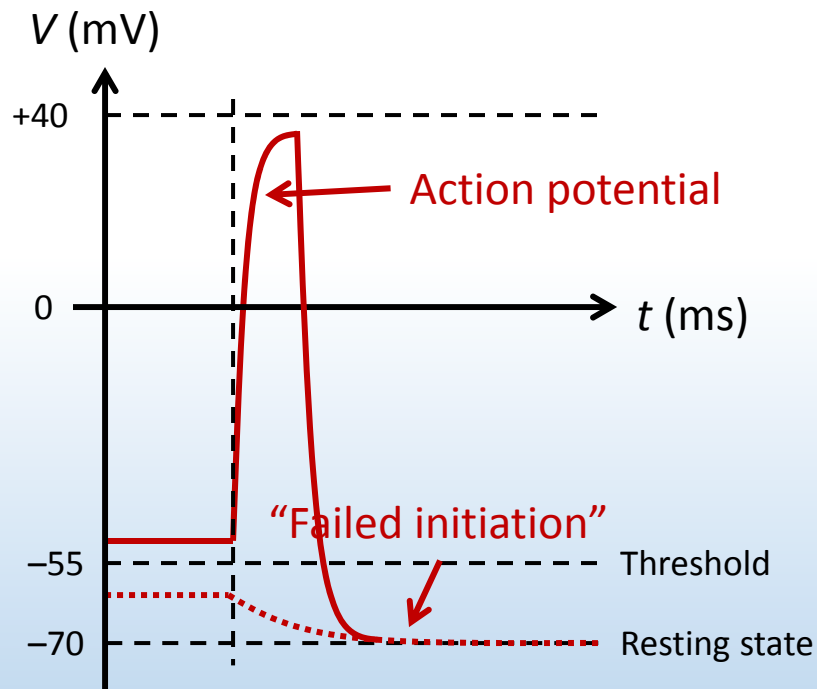
- A. All currents are 0
- B. The currents $I_K = I_{Na} \neq 0$
- C. Voltage across C_m is 0



$$\begin{aligned} \epsilon_K &= 70 \text{ mV}, \quad \epsilon_{Na} = 60 \text{ mV}, \\ R_K &= 2 \text{ M}\Omega, \quad R_{Na} = 0.4 \text{ M}\Omega, \quad C_m = 300 \text{ pF} \end{aligned}$$

Action potential summary

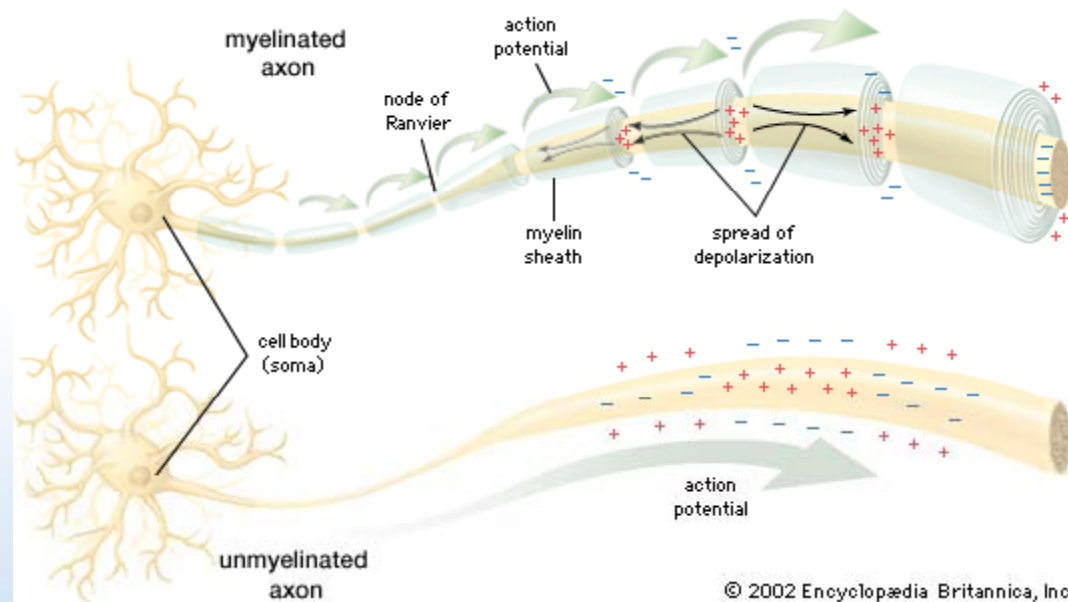
If the stimulus exceeds -55 mV, the Na^+ channels open, *depolarize* the cell & trigger an *action potential*.



Once a $+40$ mV potential is reached, the Na^+ channels close again & the cell *repolarizes* to its resting potential.

Myelinated nerve cells

Action potentials propagate down nerve cell at rate determined by the cell's RC time constant.



With very few exceptions (ex: C fibres) human neuron fibres are *myelinated*. Myelin reduces C , decreasing time constant & increasing propagation speed.

Many neurodegenerative diseases (ex: MS) cause progressive de-myelination.

Summary of today's lecture

- RC circuits depend on time

Charge on capacitors cannot change instantly

- Short/long times & charging/discharging

$t = 0$: I flows Q onto/off of C , Q increases/decreases
(charging/discharging)

$t = \infty$: I through C decays to 0, Q reaches maximum/minimum
(charging/discharging)

$\tau = RC$: provides time to charge/discharge

Next week magnetism!