Last Name: $\qquad$ First Name $\qquad$ Network-ID $\qquad$
Discussion Section: $\qquad$ Discussion TA Name:

This is an opportunity to improve your scaled score for hour exam 2. You must turn it in during lecture on Wednesday April $16^{\text {th }}$. All work must be your own, but you may consult others in preparing your solution.

1. You must both circle the correct answer and show your work. Full credit will be given for the correct answer being circled, and a clear solution/explanation for how you got that answer. No partial credit will be awarded. Each problem contains a short statement of what work should be included, and the first page is completed as an example. All work must be clearly legible on the exam paper given (no extra pages may be attached).
2. Your scaled score will be the average of the unscaled score you received on the actual exam, and the score you receive on this opportunity to redo the exam. Taking this redo cannot lower your scaled score.
3. Only a subset (approximately 10) of the questions you submit will be graded. Your score will be the average of your score on the graded questions. Each graded problem will count equally towards your score, since you must show your work for all questions. For example, if 10 questions are graded on your exam, and 8 have the correct answer and a clear solution, your score on this exam would be $80 \%$. If your raw score on the original exam was $60 \%$, then your scaled score for hour exam 2 would be $70 \%$.
4. Our goal is to have the results for this regraded exam available in the gradebook by reading day.
5. Good Luck!

The next three questions pertain to the situation described below.
A negatively-charged particle, moving at a speed $v=165 \mathrm{~m} / \mathrm{s}$, enters a region of width $d=0.87 \mathrm{~m}$ that contains a uniform magnetic field of magnitude $B=1.7 \mathrm{~T}$ pointing out of the page, as shown in the figure. The mass and the magnitude of the charge of the particle are unknown.


1) In which direction will the particle be deflected? Show how apply RHR (fingers point in $\qquad$ etc) $75 \%$

Fingers in direction of v (right/positive x )
$\frac{\text { a. Up }}{\text { b. Down }}$
Palm in direction of $B$ (out of page $+z$ )
Thumb = direction of force on positive charge (down/-y) Negative charge so force is opposite (up)
2) What is the minimum mass-to-charge ratio ( $\mathrm{m} / \mathrm{q}$ ) such that the particle can traverse the whole shaded region and exit through the right?
show work starting w/ $\mathrm{F}=\mathrm{ma}$, and solve

$$
\begin{aligned}
& \mathrm{F}=\mathrm{ma} \\
& \begin{aligned}
& \mathrm{qvB} \sin (90)=m v^{\wedge} 2 / \mathrm{r} \\
& \mathrm{~m} / \mathrm{q}=\mathrm{Br} / \mathrm{v} \\
&=(1.7)(0.87) /(165) \\
&=8.96 \mathrm{e}-3
\end{aligned}
\end{aligned}
$$

a. $m / q=0.0064 \mathrm{~kg} / \mathrm{C}$
b. $m / q=0.0112 \mathrm{~kg} / \mathrm{C}$
c. $m / q=0.00427 \mathrm{~kg} / \mathrm{C}$
d. $m / q=0.00345 \mathrm{~kg} / \mathrm{C}$
e. $m / q=0.00896 \mathrm{~kg} / \mathrm{C}$
3) Now an electric field of magnitude $E=78 \mathrm{~N} / \mathrm{C}$ is added to the shaded region. What should the speed of the particle be such that it travels in a straight line across the shaded region?

Show work starting w/F=ma
a. $v=39.9 \mathrm{~m} / \mathrm{s}$

$$
\text { b. } v=1.22 \mathrm{~m} / \mathrm{s}
$$

$$
\text { c. } v=45.9 \mathrm{~m} / \mathrm{s}
$$

$$
\text { d. } v=76.6 \mathrm{~m} / \mathrm{s}
$$

$$
\text { e. } v=4.13 \mathrm{~m} / \mathrm{s}
$$

$$
\begin{aligned}
& F=m a \\
& q v B \sin (90)-q E=0 \\
& v=E / B \\
& \\
& =78 / 1.7 \\
& \\
& =45.9
\end{aligned}
$$



Front view


Top view

A rectangular loop of area $A=0.0245 \mathrm{~m}^{2}$ and carrying a current $I=3.9 \mathrm{~A}$ is exposed to a uniform magnetic field of magnitude $B=4.6 \mathrm{~T}$, as shown in the figure.
4) What is the magnitude of the torque exerted on the loop? Show work starting w/ expression for torque
a. 0.136 Nm
b. 0.153 Nm
c. 0.418 Nm
5) As seen from the front, in which direction will the loop rotate? Either show forces on diagram (front view) or explain using dipole moment.
a. Clockwise
b. Counterclockwise

Three long, straight wires, each carrying a current $I=4.8 \mathrm{~A}$, are arranged as shown in the figure, with $d=2$ m and $\theta=30$ degrees.

6) What is the magnitude of the total magnetic field at the origin, $B_{\text {total }}$, due to the three wires?

Draw $B$ from each wire, then show calc of $x$ and $y$ components be sure to show angle, and combine to get Btotal
a. $B_{\text {total }}=1.07 \times 10^{-6} \mathrm{~T}$
b. $B_{\text {total }}=2.07 \times 10^{-6} \mathrm{~T}$
c. $B_{\text {total }}=1.92 \times 10^{-6} \mathrm{~T}$
d. $B_{\text {total }}=1.36 \times 10^{-6} \mathrm{~T}$
e. $B_{\text {total }}=1.47 \times 10^{-6} \mathrm{~T}$
7) What is the $x$ component of the net force on one meter of the top wire due to the other two wires?

Draw forces from lower currents, then calculate magnitude using expression for force between two wires, then calculate x component. Be sure to label angles
a. $F_{x}=-2.3 \times 10^{-6} \mathrm{~N}$
b. $F_{x}=3.99 \times 10^{-6} \mathrm{~N}$
c. $F_{x}=-3.99 \times 10^{-6} \mathrm{~N}$
d. $F_{x}=2.3 \times 10^{-6} \mathrm{~N}$
e. $F_{x}=0 \mathrm{~N}$


A coil of wire turns between the poles of a permanent magnet as shown in the diagram. The coil has $N=34$ turns of wire. The magnet produces a constant field of magnitude $B=0.119 \mathrm{~T}$. The coil has a cross-sectional area $A=0.0411 \mathrm{~m}^{2}$.
8) The coil is driven at an angular frequency $\omega=4.08 \mathrm{rad} / \mathrm{s}$. What is the peak emf, $\varepsilon$, this generator can produce? Just write down correct formula, then show inserted numbers and final result.
a. $\varepsilon=0.02 \mathrm{~V}$
b. $\varepsilon=5.7 \mathrm{~V}$
c. $\varepsilon=16.5 \mathrm{~V}$
d. $\varepsilon=0.678 \mathrm{~V}$
e. $\varepsilon=0.166 \mathrm{~V}$
9) If the coil is driven in the counter-clockwise direction, in what direction is the induced field at the instant shown?

Explain using flux and Lens' law.
a. The induced field is directed toward the left.
b. The induced field is directed toward the right.
c. There is no induced field.

## The next three questions pertain to the situation described below.

The series LRC circuit is driven by a voltage generator with $\mathrm{V}(\mathrm{t})=12 \sin (260 \mathrm{t})$ Volts. The remaining circuit elements have the following values; $\mathrm{R}=12.5 \Omega, \mathrm{C}$ $=8.5 \times 10^{-5} \mathrm{~F}$ and $\mathrm{L}=0.25 \mathrm{H}$.

10) The voltage across the generator $\qquad$ the current through the generator.

Explain using reactance for L and C
a. lags
b. leads
c. is in phase with
11) Which of the following circuit elements has the largest peak voltage across it?

Explain using reactance and $R$
a. Resistor
b. Generator
c. Capacitor
12) What is the average power delivered by the generator?

Start with formula, show all values you use.
a. $\mathrm{P}_{\text {average }}=3.3 \mathrm{~W}$
b. $\mathrm{P}_{\text {average }}=1.65 \mathrm{~W}$
c. $\mathrm{P}_{\text {average }}=2.33 \mathrm{~W}$

The next three questions pertain to the situation described below.
The series LRC circuit is driven by a voltage from the antenna recieving a signal at 915 MHz with peak voltage of 0.0085 Volts. The remaining circuit elements have the following values; $\mathrm{R}=3.8 \Omega, \mathrm{~L}=2$ $\times 10^{-9} \mathrm{H}$ and the capacitance is adjustable.

13) What value of capacitance will provide the largest peak current in the circuit?

Explain choice of equation, then show values used.
a. $\mathrm{C}=1.74 \times 10^{-10} \mathrm{~F}$
b. $\mathrm{C}=1.51 \times 10^{-11} \mathrm{~F}$
c. $\mathrm{C}=2 \times 10^{-9} \mathrm{~F}$
14) Assuming the above capacitance, what is the peak value of the voltage across the inductor?

Show choice of equation and values inserted.
a. $\mathrm{V}_{\mathrm{L} \max }=0.0257 \mathrm{~V}$
b. $\mathrm{V}_{\mathrm{L} \max }=0.0085 \mathrm{~V}$
c. $\mathrm{V}_{\mathrm{L} \text { max }}=0.00601 \mathrm{~V}$
15) If you are traveling toward the radio station at $30 \mathrm{~m} / \mathrm{s}$ the radio waves will be shifted to a slightly
a. lower frequency.

Explain based on general principle for when gets higher/
b. lower wavelength.

An ideal transformer has 50 turns in the primary coil and 10 turns in the secondary coil. A 60 Hz AC voltage source with RMS voltage of 80 V is connected to the primary coil. A $10 \Omega$ resistor is connected to the secondary coil as shown in the figure.

16) What is the average power dissipated in the resistor?

Show equations started from, and values inserted.
a. 25.6 W
b. $1.6 \times 10^{4} \mathrm{~W}$
c. 51.2 W
d. 640 W
e. $3.2 \times 10^{4} \mathrm{~W}$

## The next three questions pertain to the situation described below.

The electric field for a plane electromagnetic wave in vacuum s given by
$\mathbf{E}=2100(\mathrm{~N} / \mathrm{C}) * \sin \left(-0.6 \mathrm{~m}^{-1} \mathrm{z}+\omega \mathrm{t}\right) \hat{x}$.
17) What is the frequency of the wave? Show equations used and values inserted.

$$
\mathrm{k}=2 \mathrm{pi} / \text { wavelength (from lecture) }
$$

a. $\mathrm{f}=6 \times 10^{5} \mathrm{~Hz}$

$$
\begin{aligned}
\text { wavelength } & =2 \mathrm{pi} / \mathrm{k} \\
& =2 \mathrm{pi} / 0.6
\end{aligned}
$$

b. $\mathrm{f}=1.8 \times 10^{8} \mathrm{~Hz}$

$$
\begin{aligned}
\mathrm{f} & =\mathrm{c} / \text { wavelength } \\
& =\mathrm{c} * 0.6 /(2 \mathrm{pi}) \\
& =2.86 \mathrm{e} 7
\end{aligned}
$$

$$
\begin{array}{ll}
\text { c. } \mathrm{f}=2.86 \times 10^{7} \mathrm{~Hz} & \mathrm{f}=\mathrm{c} / \text { wavelength } \\
& =\mathrm{c} * 0.6 /(2 \mathrm{pi})
\end{array}
$$

18) What is the magnitude of the magnetic field oscillation?

Show equation used and values inserted
a. $B=1.4 \times 10^{5} \mathrm{~T}$

$$
E=c B
$$

b. $B=7 \times 10^{-6} \mathrm{~T}$

$$
\begin{aligned}
B & =E / c \\
& =2100 / 3 e 8 \\
& =7 e-6
\end{aligned}
$$

19) In what direction does the wave propogate?

Describe what information in equation tells the direction.
a. +x
b. -z
c. +Z

The direction can be determined by looking at the argument of the sine function. Since it depends on the value of $z$, it must be moving in the $+-z$ direction. Since there is a relative minus sign between the two terms, it moves in the + direction.

## The next two questions pertain to the situation described below.

A beam of unpolarized light of intensity $\mathrm{I}_{0}$ travels in the positive z-direction and is incident from the left on a series of two linear polarizers as shown. The transmission axis of the two polarizers make angles of $\theta_{1}=54$ degrees and $\theta_{2}=122$ degrees, respectively, with respect to the positive x -axis. The intensity of the beam immediately after the first polarizer is $\mathrm{I}_{1}=214 \mathrm{~W} / \mathrm{m}^{2}$.

end view


20) What is the intensity of the incident beam?
a. $\mathrm{I}_{0}=619 \mathrm{~W} / \mathrm{m}^{2}$
b. $\mathrm{I}_{0}=428 \mathrm{~W} / \mathrm{m}^{2}$
c. $\mathrm{I}_{0}=74 \mathrm{~W} / \mathrm{m}^{2}$
21) What is the intensity of the beam immediately after the second polarizer?

Show equation and explain reason
a. $\mathrm{I}_{2}=30.1 \mathrm{~W} / \mathrm{m}^{2}$
b. $\mathrm{I}_{2}=59.9 \mathrm{~W} / \mathrm{m}^{2}$
c. $\mathrm{I}_{2}=74 \mathrm{~W} / \mathrm{m}^{2}$

22) In figure (a) above, a coil is produced by wrapping a copper wire around a cylinder of iron. The iron cylinder is fixed within the wire. Which of the following statements is true:

Briefly explain reason (2 sentences)
a. Neither of these.
b. The iron will behave like a magnet if current flows through the wire.
c. The wire will have an induced current if the iron is magnetized.
23) In figure (b) above, a magnetic iron cylinder moves through the coil in the direction shown. Which of the following statements is true:

Explain using flux and Lens's law
a. The induced current flows right to left across the front of the coil.
b. The induced current flows left to right across the front of the coil.
c. There is no induced current.
24) In figure (b) the cylinder moves through the coil for $t=3.95 \mathrm{~s}$ and produces $|\varepsilon|=0.119 \mathrm{~V}$. What is the magnitude of the change flux?

Show equation, and values used for variables.
a. $\Delta \Phi=0.94 \mathrm{Tm}^{2}$
b. $\Delta \Phi=0.235 \mathrm{Tm}^{2}$
c. $\Delta \Phi=0.0301 \mathrm{Tm}^{2}$
d. $\Delta \Phi=0.47 \mathrm{Tm}^{2}$
e. $\Delta \Phi=0.157 \mathrm{Tm}^{2}$
25) In figure (b) the coil has a diameter $d=0.0411 \mathrm{~m}$ and 100 turns of wire. The resistance per unit length is $34.3 \Omega / m$. The emf is $|\varepsilon|=0.119 \mathrm{~V}$. What is the magnitude current in the coil?

Show equations used, and values input to get answer.
a. $I=269 \mu A$
b. $I=844 \mu \mathrm{~A}$
c. $I=53.7 \mu \mathrm{~A}$
d. $I=3470 \mu \mathrm{~A}$
e. $I=537 \mu \mathrm{~A}$

## Mechanics:

$x=x_{0}+v_{0} t+\frac{1}{2} a t^{2}$
$v=v_{0}+a t$
$F=m a$
$a_{c}=\frac{v^{2}}{r}$
$E_{t o t}=K . E .+P . E$.
K.E. $=\frac{1}{2} m v^{2}=\frac{p^{2}}{2 m}$
$p=m v$
$W_{F}=F d \cos \theta$

## Electrostatics:

$F_{12}=\frac{k q_{1} q_{2}}{r^{2}}$
$E \equiv \frac{F}{q_{0}}$
$V \equiv \frac{U}{q_{0}}$
Point charge: $E=\frac{k q}{r^{2}}, \quad V=\frac{k q}{r}$
$U_{12}=\frac{k q_{1} q_{2}}{r}$
$W_{E}=-\Delta U=-W_{\text {you }}$

## Capacitance:

$C \equiv \frac{Q}{V}$
Parallel plate capacitor: $C=\frac{\kappa \varepsilon_{0} A}{d}, V=E d$
$U_{C}=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{1}{2} \frac{Q^{2}}{C}$
$C_{P}=C_{1}+C_{2}+\cdots$
$\frac{1}{C_{S}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}+\cdots$

## Resistance:

$R \equiv \frac{V}{I}$
$I=\frac{\Delta q}{\Delta t}$
Physical resistance: $R=\rho \frac{L}{A}$
$P=I V=I^{2} R=\frac{V^{2}}{R}$

$$
R_{S}=R_{1}+R_{2}+\cdots
$$

$$
\frac{1}{R_{P}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\cdots
$$

## Circuits:

$\begin{array}{ll}\sum \Delta V=0 & \sum I_{\text {in }}=\sum I_{\text {out }} \\ q(t)=q_{\infty}\left(1-e^{-t / \tau}\right) & q(t)=q_{0} e^{-t / \tau}\end{array}$

$$
I(t)=I_{0} e^{-t / \tau} \quad \tau=R C
$$

## Magnetism:

$F=q v B \sin \theta$
$r=\frac{m v}{q B}$
$F=I L B \sin \theta$
$\tau=N I A B \sin \varphi$
$B_{\text {wire }}=\frac{\mu_{0} I}{2 \pi r}$
$B_{\text {sol }}=\mu_{0} n I$

Induction and inductance:
$\varepsilon=-N \frac{\Delta \Phi}{\Delta t}$
$\varepsilon_{b a r}=B L v$
$L \equiv \frac{N \Phi}{I}$
$\varepsilon=-L \frac{\Delta I}{\Delta t}$
$\varepsilon_{\text {gen }}=\varepsilon_{\max } \sin \omega t=\omega N A B \sin \omega t$
$\omega=2 \pi f$
Solenoid inductor: $L=\mu_{0} n^{2} A \ell$
$U_{L}=\frac{1}{2} L I^{2}$
$V_{r m s}=\frac{V_{\max }}{\sqrt{2}} \quad I_{r m s}=\frac{I_{\max }}{\sqrt{2}}$
$\frac{V_{p}}{V_{s}}=\frac{I_{s}}{I_{p}}=\frac{N_{p}}{N_{s}}$
$V_{R}(t)=V_{R, \max } \sin (\omega t)=I_{\max } R \sin (\omega t)$

$$
\omega=2 \pi f
$$

$V_{C}(t)=V_{C, \max } \sin (\omega t-\pi / 2)=I_{\max } X_{C} \sin (\omega t-\pi / 2)$
$X_{C} \equiv \frac{1}{\omega C}$
$V_{L}(t)=V_{L, \max } \sin (\omega t+\pi / 2)=I_{\max } X_{L} \sin (\omega t+\pi / 2)$
$X_{L} \equiv \omega L$
$V_{\text {gen }}(t)=V_{\text {gen }, \max } \sin (\omega t+\varphi)=I_{\max } Z \sin (\omega t+\varphi)$
$Z \equiv \sqrt{R^{2}+\left(X_{L}-X_{C}\right)^{2}}$
$\tan \varphi=\frac{X_{L}-X_{C}}{R}$
$\bar{P}=I_{r m s} V_{R, r m s}=I_{r m s} V_{\text {gen }, r m s} \cos \varphi$

## Electromagnetic waves:

$$
\begin{array}{ll}
\lambda=\frac{c}{f} & E=c B \\
u_{E}=\frac{1}{2} \varepsilon_{0} E^{2} & u_{B}=\frac{1}{2 \mu_{0}} B^{2} \\
f^{\prime}=f\left(1 \pm \frac{u}{c}\right) & \\
& \\
&
\end{array}
$$

## Reflection and refraction:

$\theta_{r}=\theta_{i}$
$\frac{1}{d_{o}}+\frac{1}{d_{i}}=\frac{1}{f}$
$f= \pm \frac{R}{2}$
$m=\frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}}$
$n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}$
$v=\frac{c}{n}$
$\sin \theta_{c}=\frac{n_{2}}{n_{1}}$
$M=\frac{\theta^{\prime}}{\theta} \approx \frac{d_{\text {near }}}{f}$

## Interference and diffraction:

Double slit interference

$$
d \sin \theta=m \lambda
$$

$$
d \sin \theta=\left(m+\frac{1}{2}\right) \lambda
$$

$$
m=0, \pm 1, \pm 2 \ldots
$$

Single-slit diffraction:

$$
w \sin \theta=m \lambda \quad m= \pm 1, \pm 2 \ldots
$$

Circular aperture:
$D \sin \theta \approx 1.22 \lambda$
Thin film: $\quad \delta_{1}=\left(0\right.$ or $\left.\frac{1}{2}\right) \quad \delta_{2}=\left(0\right.$ or $\left.\frac{1}{2}\right)+2 t \frac{n_{\text {film }}}{\lambda_{0}} \quad\left|\delta_{2}-\delta_{1}\right|=\left(m\right.$ or $\left.m+\frac{1}{2}\right) \quad m=0,1,2 \ldots$

## Quantum mechanics:

$$
E=h f=\frac{h c}{\lambda} \quad \lambda=\frac{h}{p}
$$

Blackbody radiation: $\lambda_{\max } T=2.898 \times 10^{-3} \mathrm{~m} \cdot \mathrm{~K}$
$\Delta p_{x} \Delta x \geq \frac{\hbar}{2}$
Photoelectric effect: K.E. $=h f-W_{0}$

$$
\hbar \equiv \frac{h}{2 \pi}
$$

Bohr atom: $\quad 2 \pi r_{n}=n \lambda \quad n=1,2,3 \ldots$

$$
L_{n}=m v_{n} r_{n}=n \hbar
$$

$r_{n}=\left(\frac{\hbar^{2}}{m k e^{2}}\right) \frac{n^{2}}{Z} \approx\left(5.29 \times 10^{-11} m\right) \frac{n^{2}}{Z}$
$E_{n}=-\left(\frac{m k^{2} e^{4}}{2 \hbar^{2}}\right) \frac{Z^{2}}{n^{2}} \approx-(13.6 e V) \frac{Z^{2}}{n^{2}}$
$\frac{1}{\lambda} \approx\left(1.097 \times 10^{7} \mathrm{~m}^{-1}\right) Z^{2}\left(\frac{1}{n_{f}^{2}}-\frac{1}{n_{i}^{2}}\right)$
Quantum atom: $\quad L=\sqrt{\ell(\ell+1)} \hbar$

$$
L_{z}=m_{\ell} \hbar
$$

## Nuclear physics and radioactive decay:

$A=Z+N$
$r \approx\left(1.2 \times 10^{-15} m\right) A^{1 / 3}$
$E_{0}=m c^{2}$
$N(t)=N_{0} e^{-\lambda t}=N_{0} 2^{-t / T_{1 / 2}}$
$T_{1 / 2} \equiv \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$

## Special relativity:

$\Delta t=\gamma \Delta t_{0}$

$$
L=\frac{L_{0}}{\gamma}
$$

$$
\gamma \equiv \frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}}
$$

## Constants and unit conversions:

$g=9.8 \mathrm{~m} / \mathrm{s}^{2}$
$\varepsilon_{0}=8.85 \times 10^{-12} C^{2} /{N m^{2}}^{2}$
$c=\frac{1}{\sqrt{\varepsilon_{0} \mu_{0}}}=3 \times 10^{8} \mathrm{~m} / \mathrm{s}$ $e=1.60 \times 10^{-19} C$
$k \equiv \frac{1}{4 \pi \varepsilon_{0}}=8.99 \times 10^{9} \mathrm{Nm}^{2} / \mathrm{C}^{2}$ $\mu_{0}=4 \pi \times 10^{-7} T \cdot m / A$ $h=6.626 \times 10^{-34} \mathrm{~J} \cdot \mathrm{~s}$

$$
m_{\text {proton }}=1.67 \times 10^{-27} \mathrm{~kg}=938 \mathrm{MeV} \quad m_{\text {electron }}=9.11 \times 10^{-31} \mathrm{~kg}=511 \mathrm{keV}
$$

| SI Prefixes |  |  |
| :---: | :---: | :---: |
| Power | Prefix | Symbol |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | kilo | k |
| $10^{0}$ | - | - |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |

Physics 102 Exam 2 -Spring 2014

1. $a$
2. e
3. c
4. c
5. a
6. e
7. c
8. d
9. a
10. b
11. c
12. b
13. b
14. a
15. b
16. a
17. c
18. b
19. c
20. b
21. a
22. b
23. a
24. d
25. a
