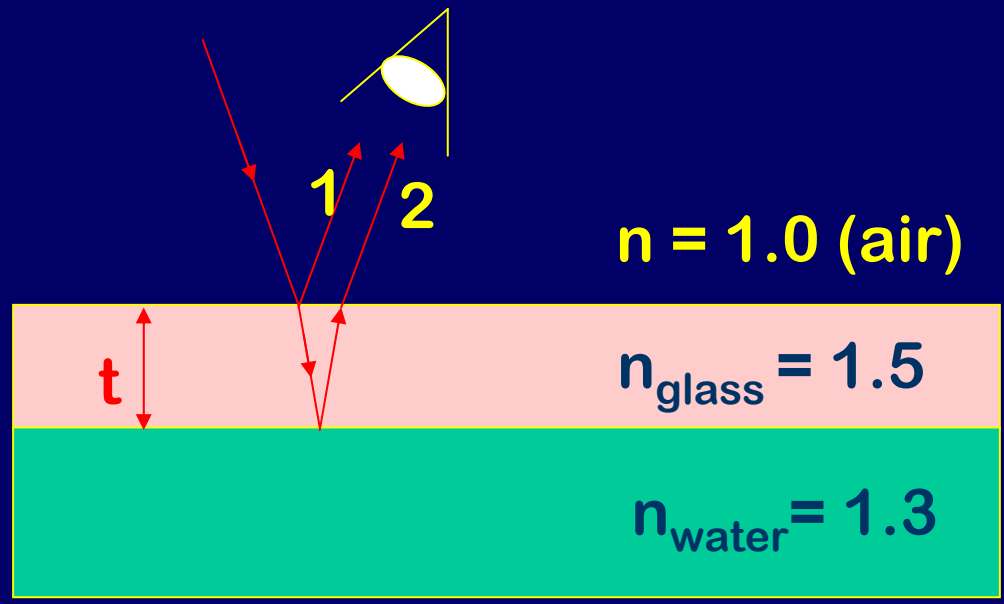
Physics 102: Lecture 21

Diffraction, Gratings, Resolving Power

Recall

- Interference (at least 2 coherent waves)
  - Constructive (full wavelength difference)
  - Destructive (half wavelength difference)
- Light (1 source, but different paths)
  - last lecture
    - Thin Films
    - Double slit
  - today
    - Multiple slit
    - x-ray diffraction from crystal
    - Diffraction/single slit

But first: Thin Film Practice



Blue light (λ_o = 500 nm) incident on a glass (n_{glass} = 1.5) cover slip (t = 167 nm) floating on top of water (n_{water} = 1.3).

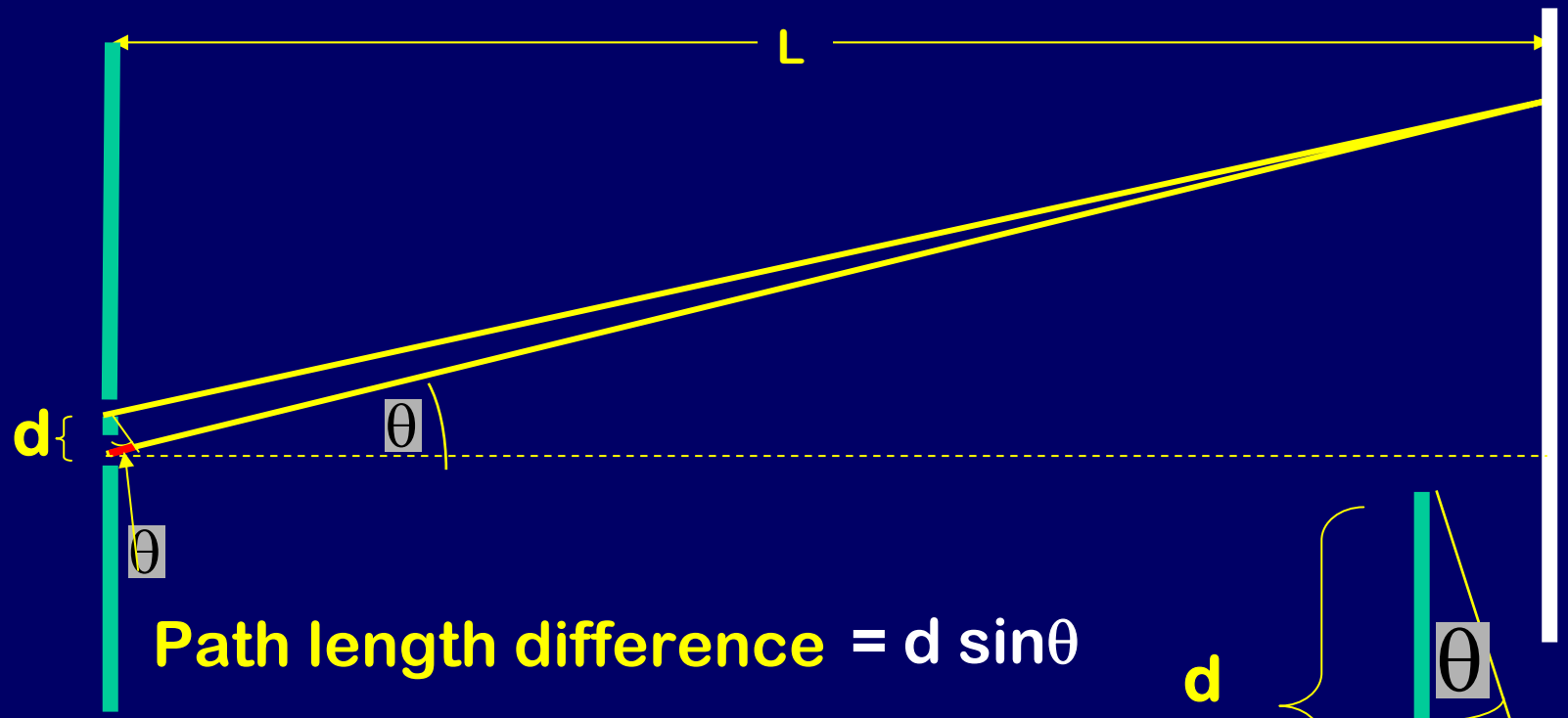
Is the interference constructive or destructive or neither?

$\delta_1 = ½$ Reflection at air-film interface only

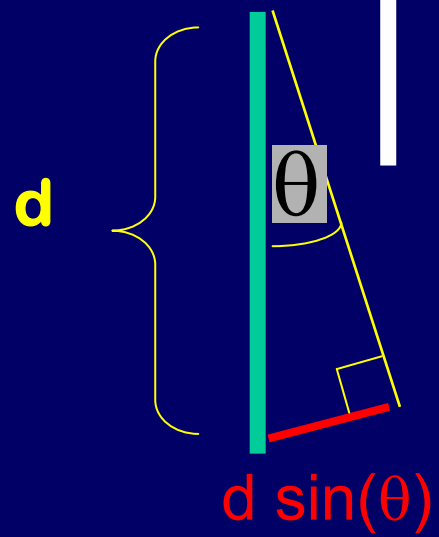
$\delta_2 = 0 + 2t / \lambda_{glass} = 2t\, n_{glass} / \lambda_0 = (2)(167)(1.5)/500) = 1$

Phase shift = $\delta_2 - \delta_1$ = ½ wavelength

Double Slit Review/ACT



Path length difference = d sinθ



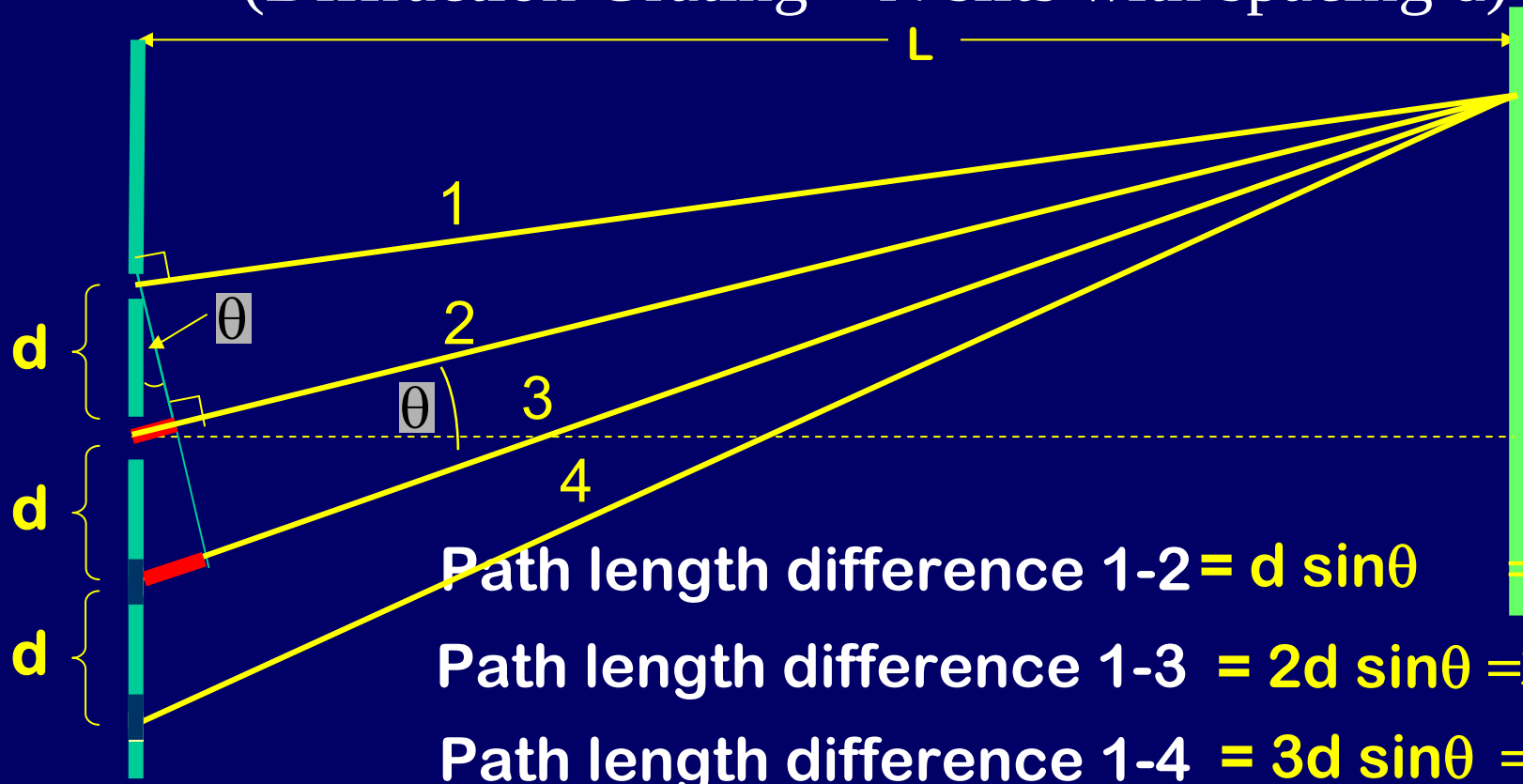
Which condition gives <u>destructive</u> interference?

1) $d \sin\theta = m\lambda$

2) $d \sin\theta = (m + \frac{1}{2})\lambda$

Multiple Slits:

(Diffraction Grating – N slits with spacing d)



Path length difference 1-2 = d sinθ =λ

Path length difference 1-3 = 2d sinθ =2λ

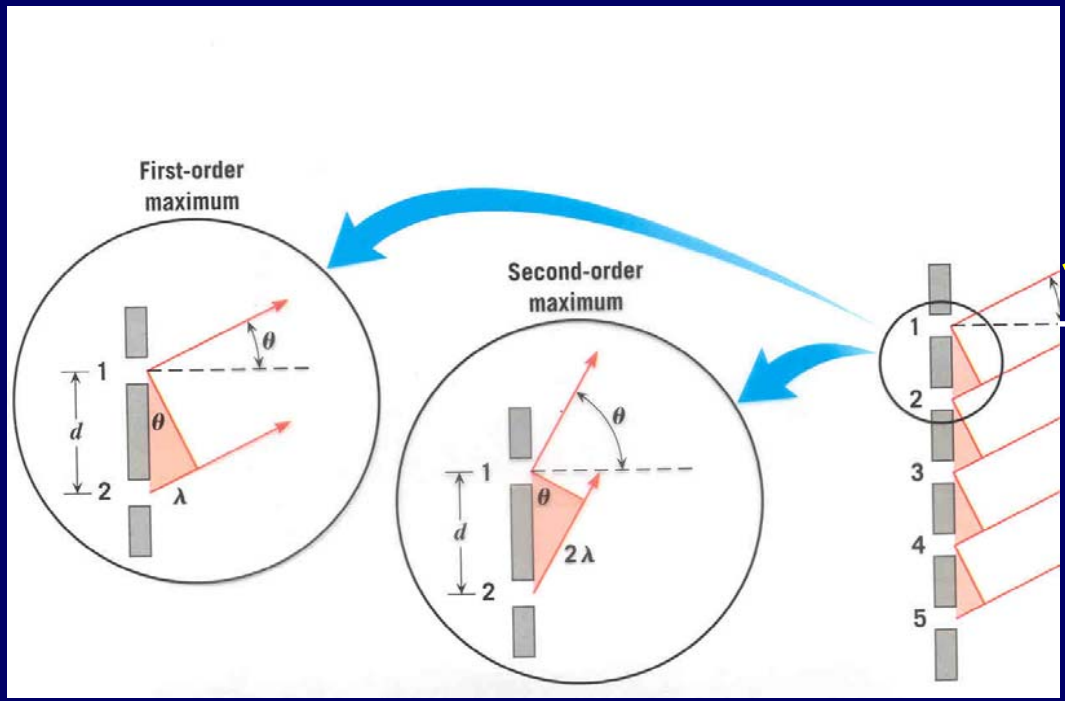
Path length difference 1-4 = 3d sinθ =3λ

Constructive interference for all paths when

$$d \sin\theta = m\lambda$$

Diffraction Grating

N slits with spacing d



θ

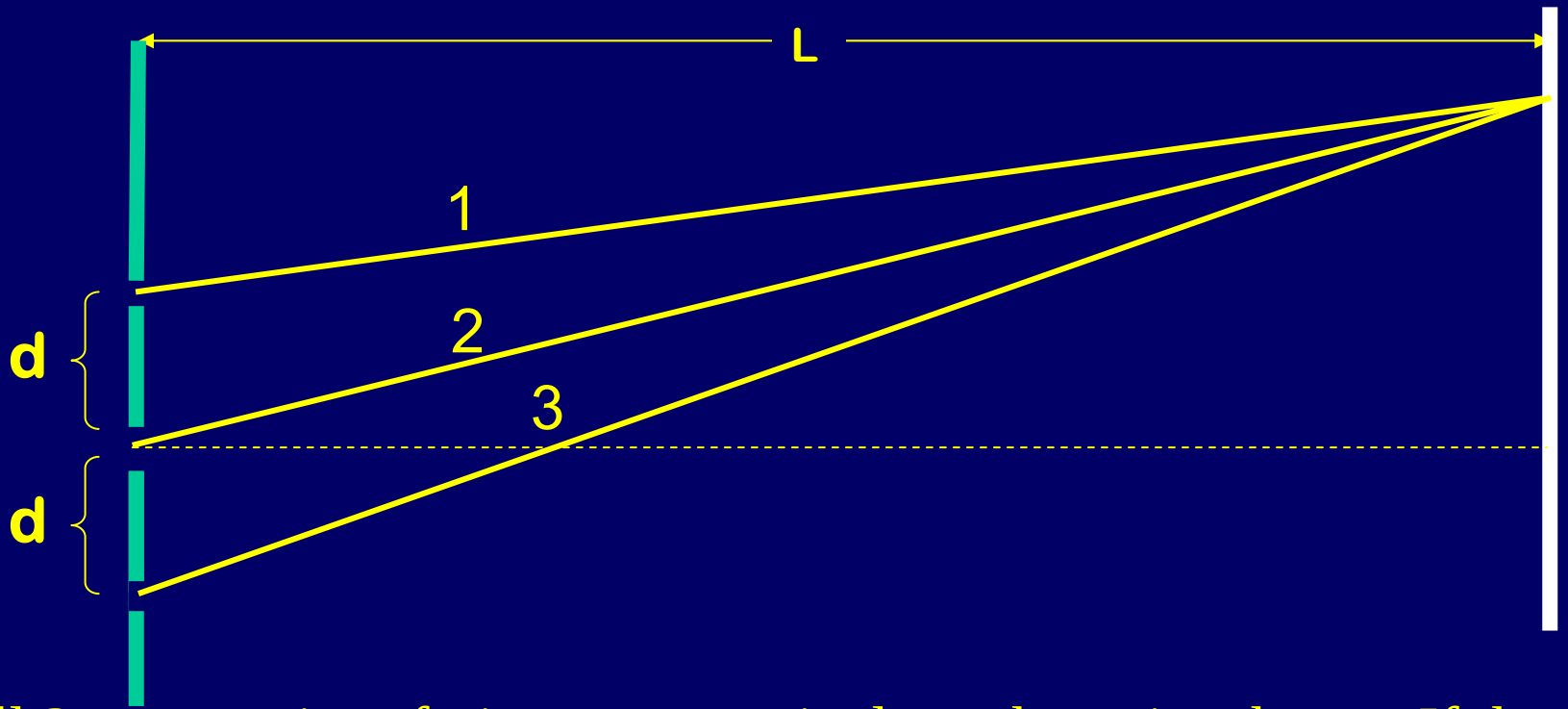
*** screen VERY far away**

<u>Constructive Interference</u> Maxima are at:

$$\sin\theta = m\frac{\lambda}{d}$$

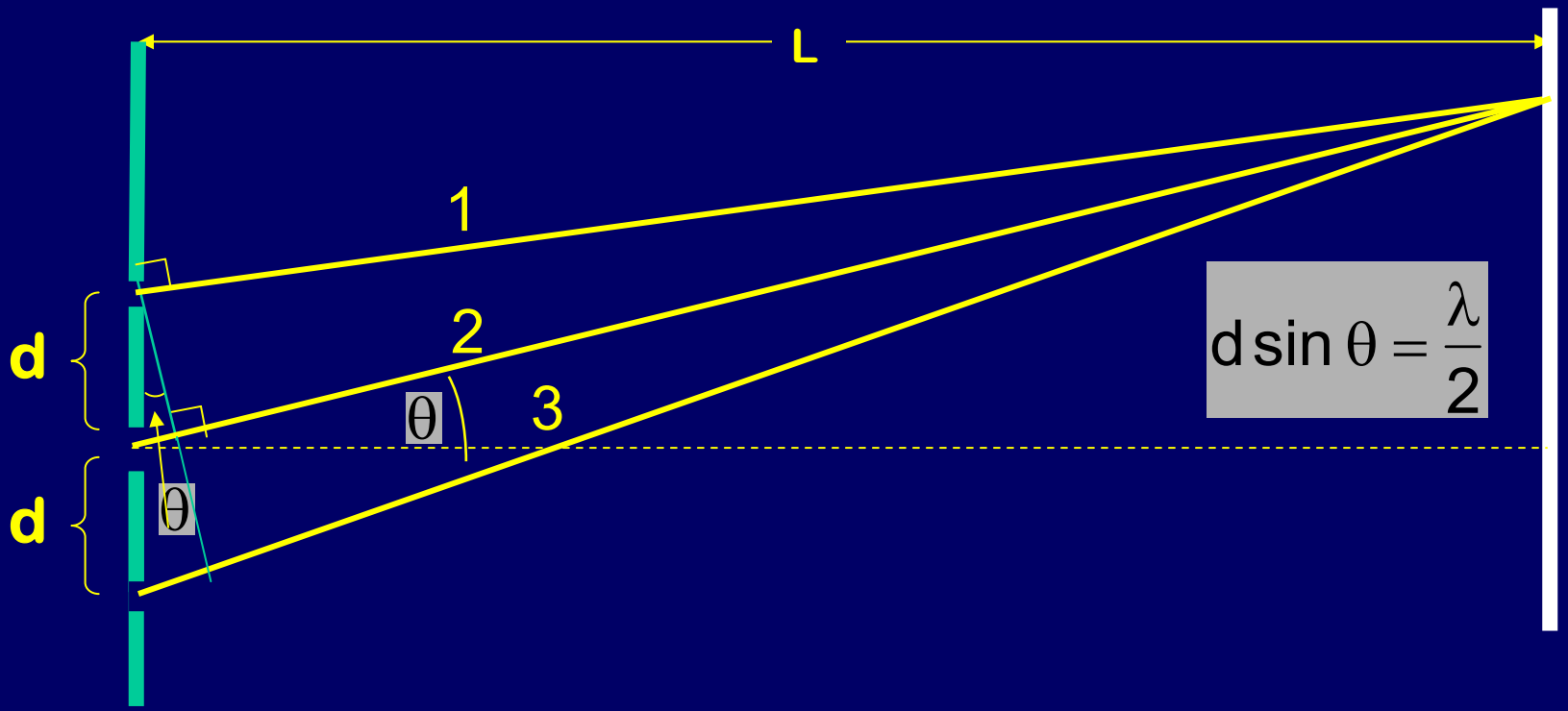
Same as for Young's Double Slit !

Preflight 21.1



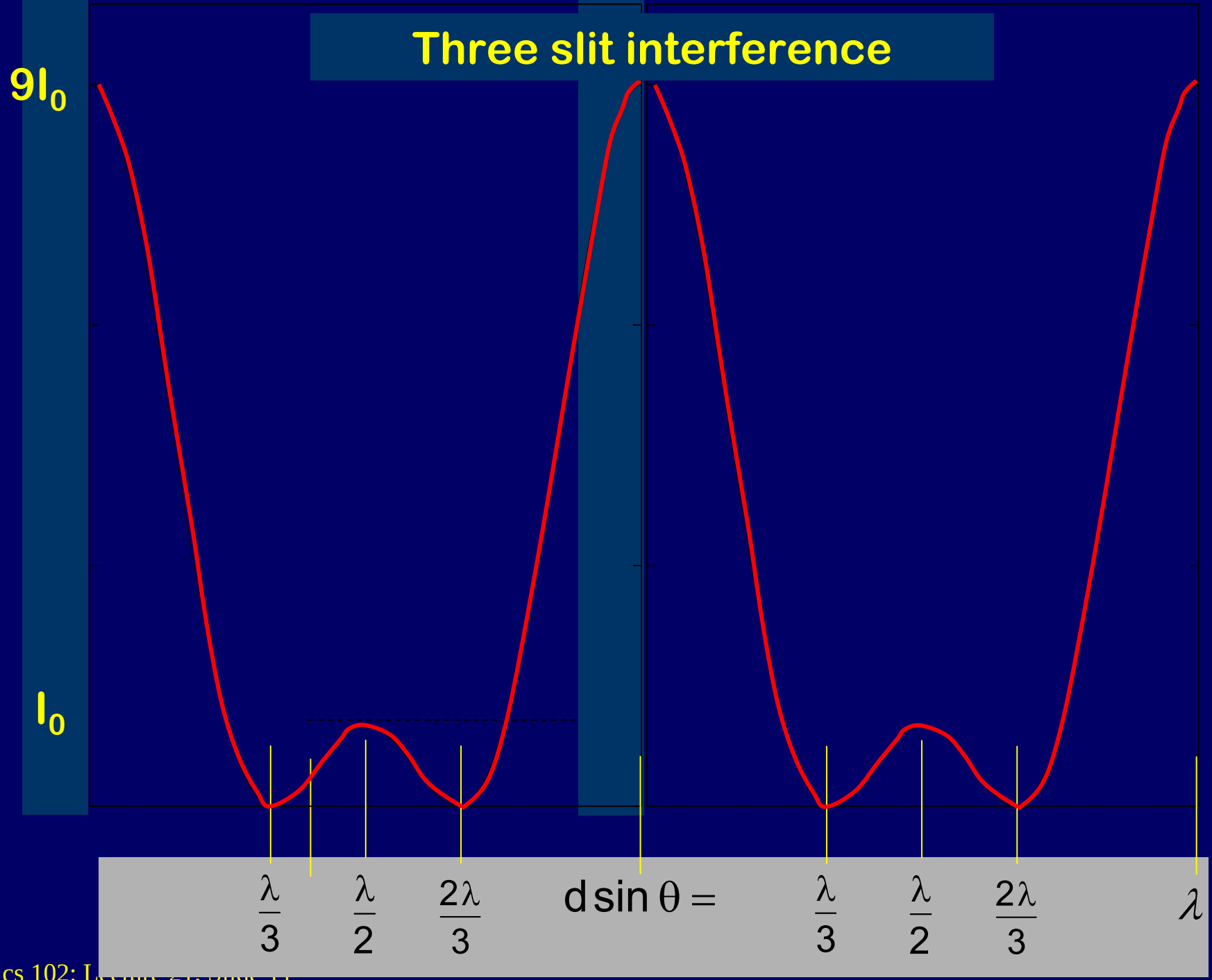
All 3 rays are interfering constructively at the point shown. If the intensity from ray 1 is I_0 , what is the combined intensity of all 3 rays? 1) I_0 2) 3 I_0 3) 9 I_0

ACT/Preflight 21.2



When rays 1 and 2 are interfering destructively, is the intensity from the **three** rays a minimum? 1) Yes 2) No

Three slit interference



Multiple Slit Interference

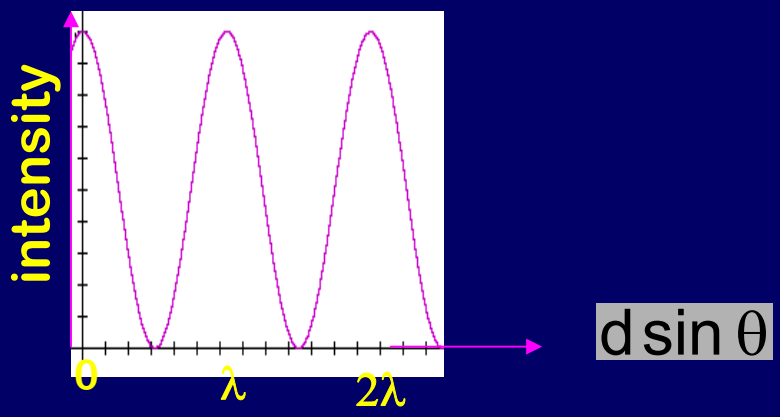
(Diffraction Grating)

For many slits, maxima are still at $\sin\theta = m\frac{\lambda}{d}$

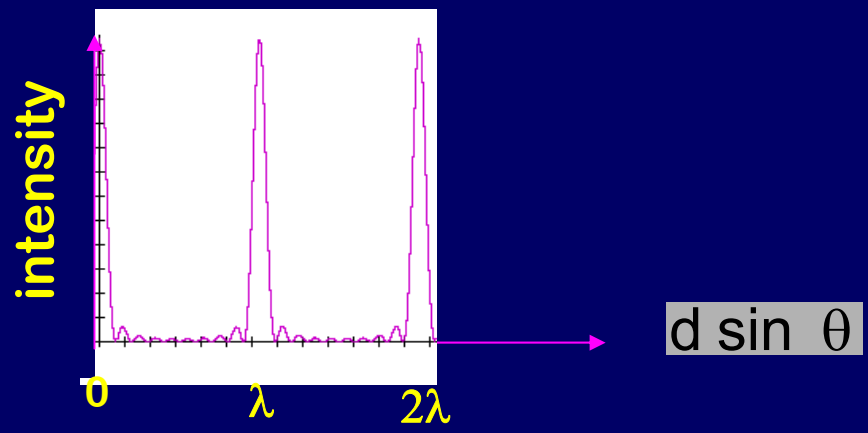
Peak location depends on wavelength!

Region between maxima gets suppressed more and more as no. of slits increases – bright fringes become narrower and brighter.

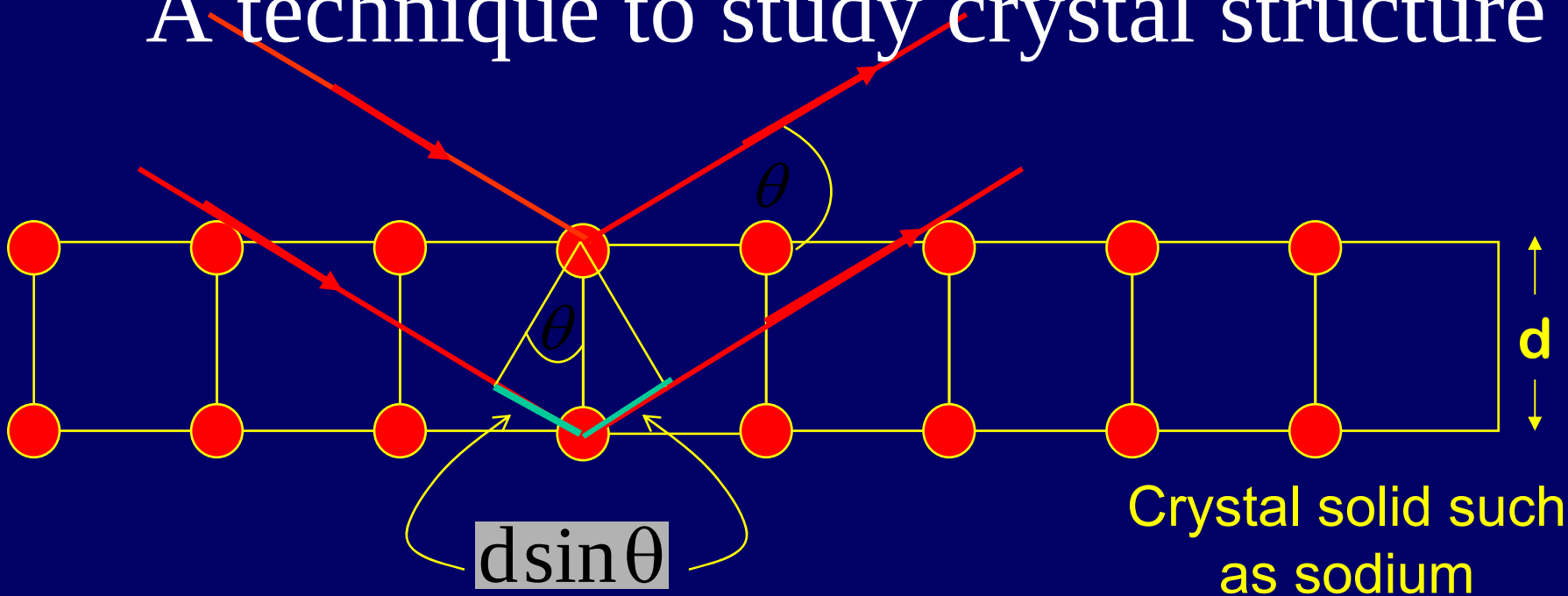
2 slits (N=2)



10 slits (N=10)



X-Ray Diffraction:
A technique to study crystal structure



Crystal solid such as sodium

Constructive interference: $2d\sin\theta = m\lambda$

$d \approx 0.5\,\text{nm}$ **in NaCl**

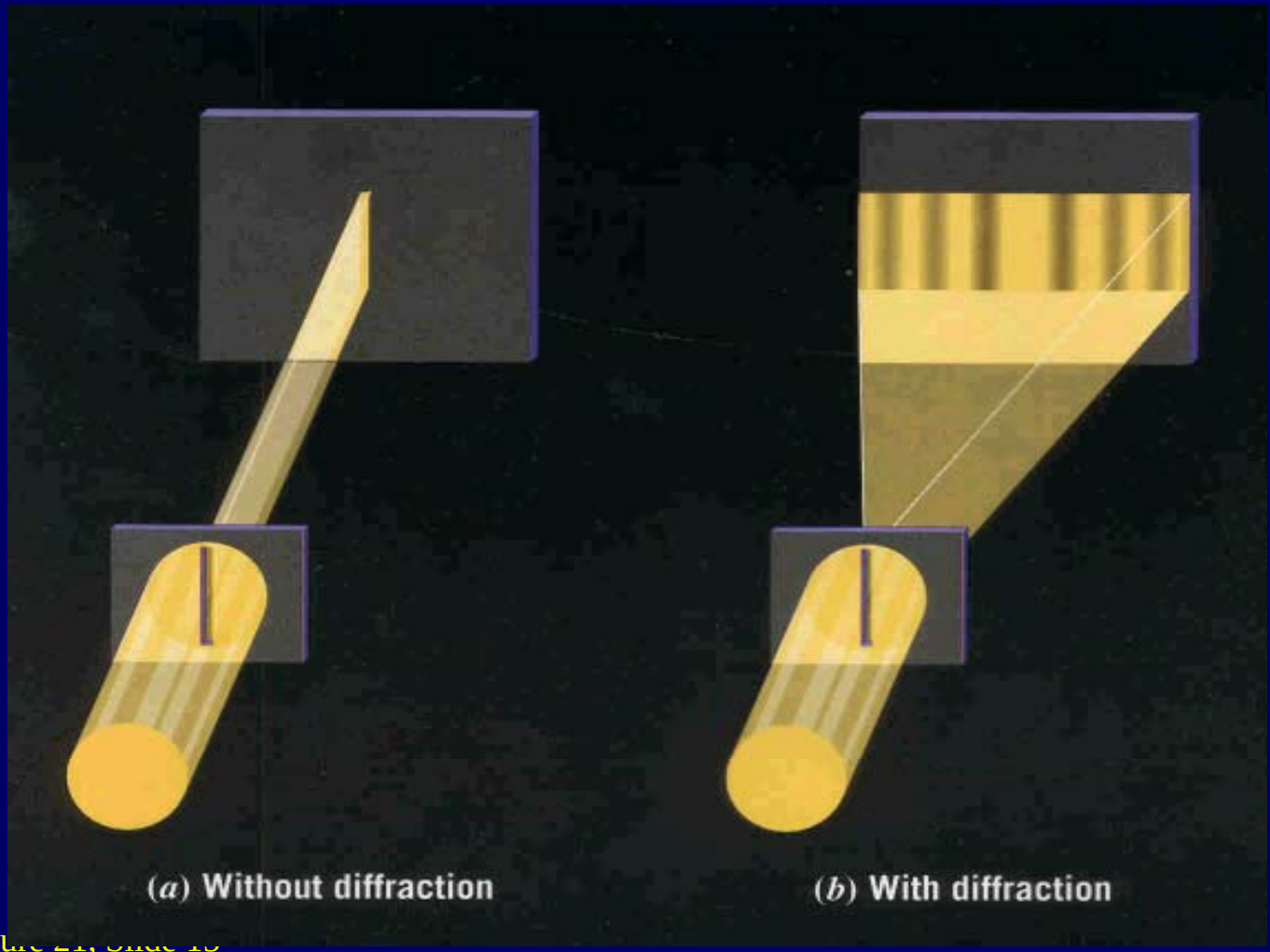
For λ=.017nm

X-ray

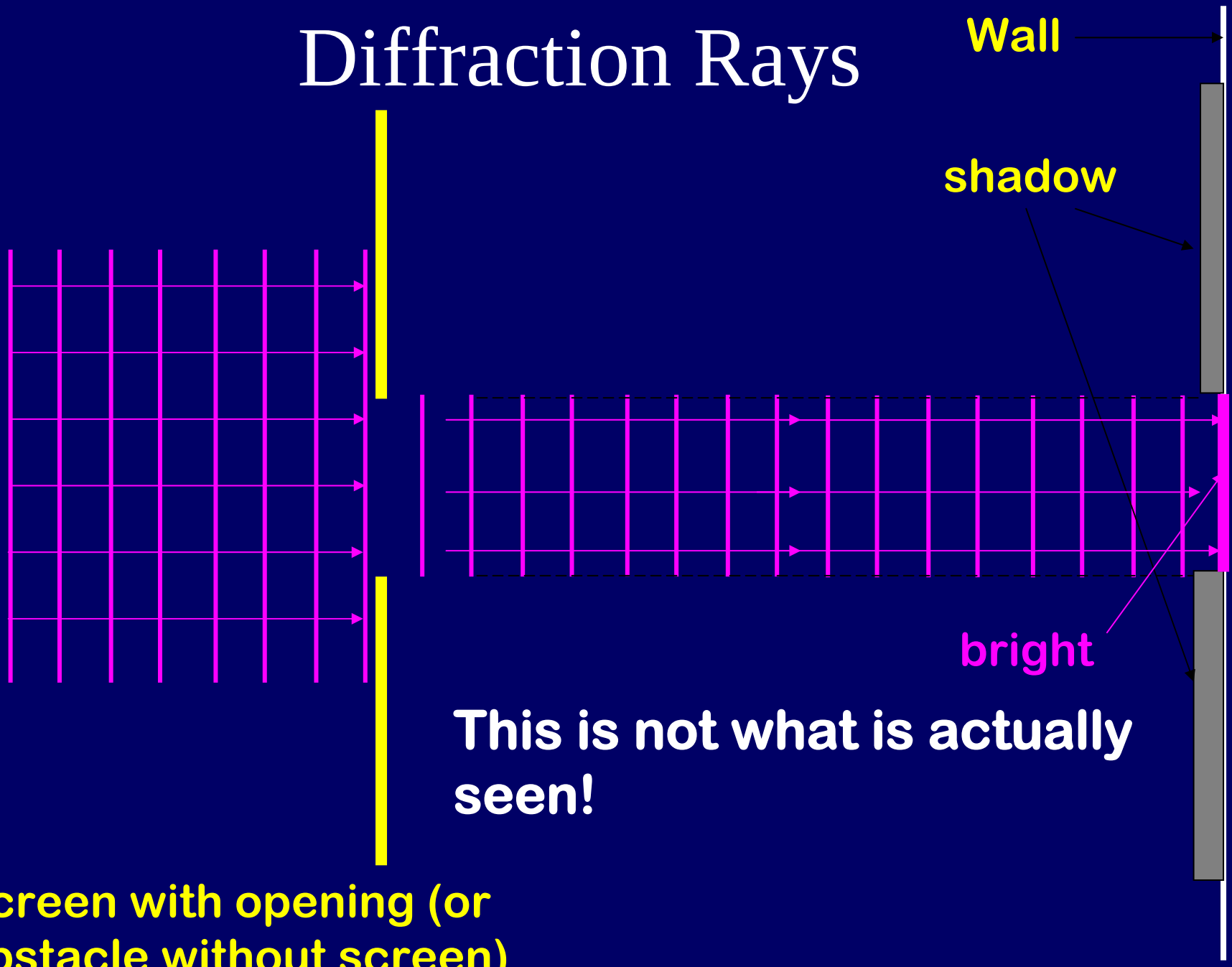
1st maximum will be at 10^0

Measure θ, determine d

Single Slit Interference?!

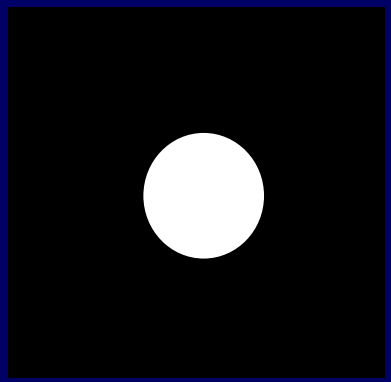


Diffraction Rays

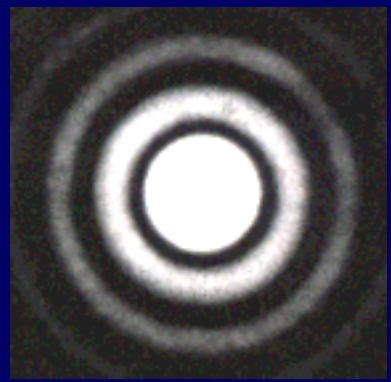


A laser is shined onto a screen through a very small hole. If you make the hole even smaller, the spot on the screen will get:

Which drawing correctly depicts the pattern of light on the screen?



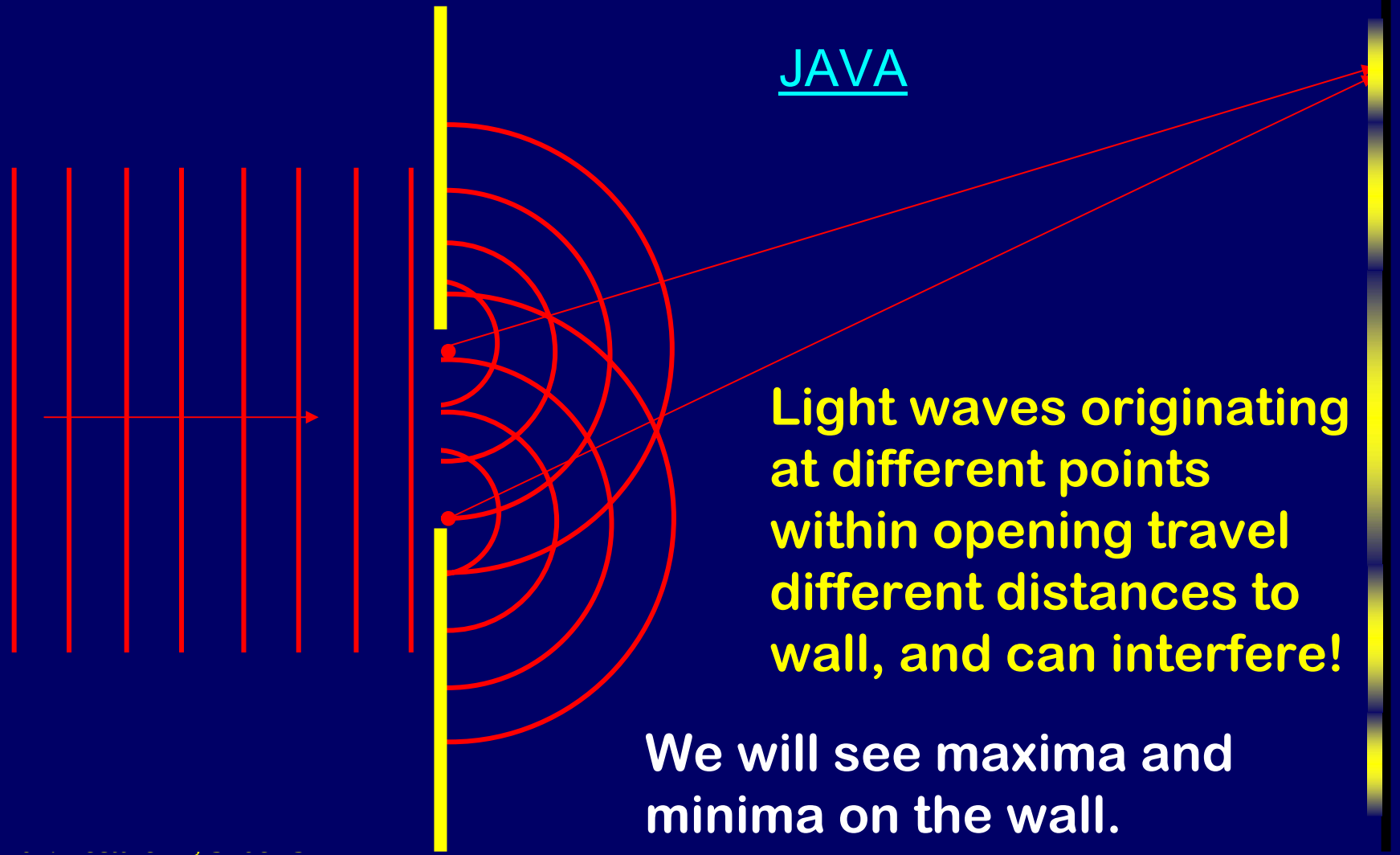
(1)



(2)

Diffraction/ Huygens

Every point on a wave front acts as a source of tiny wavelets that move forward.

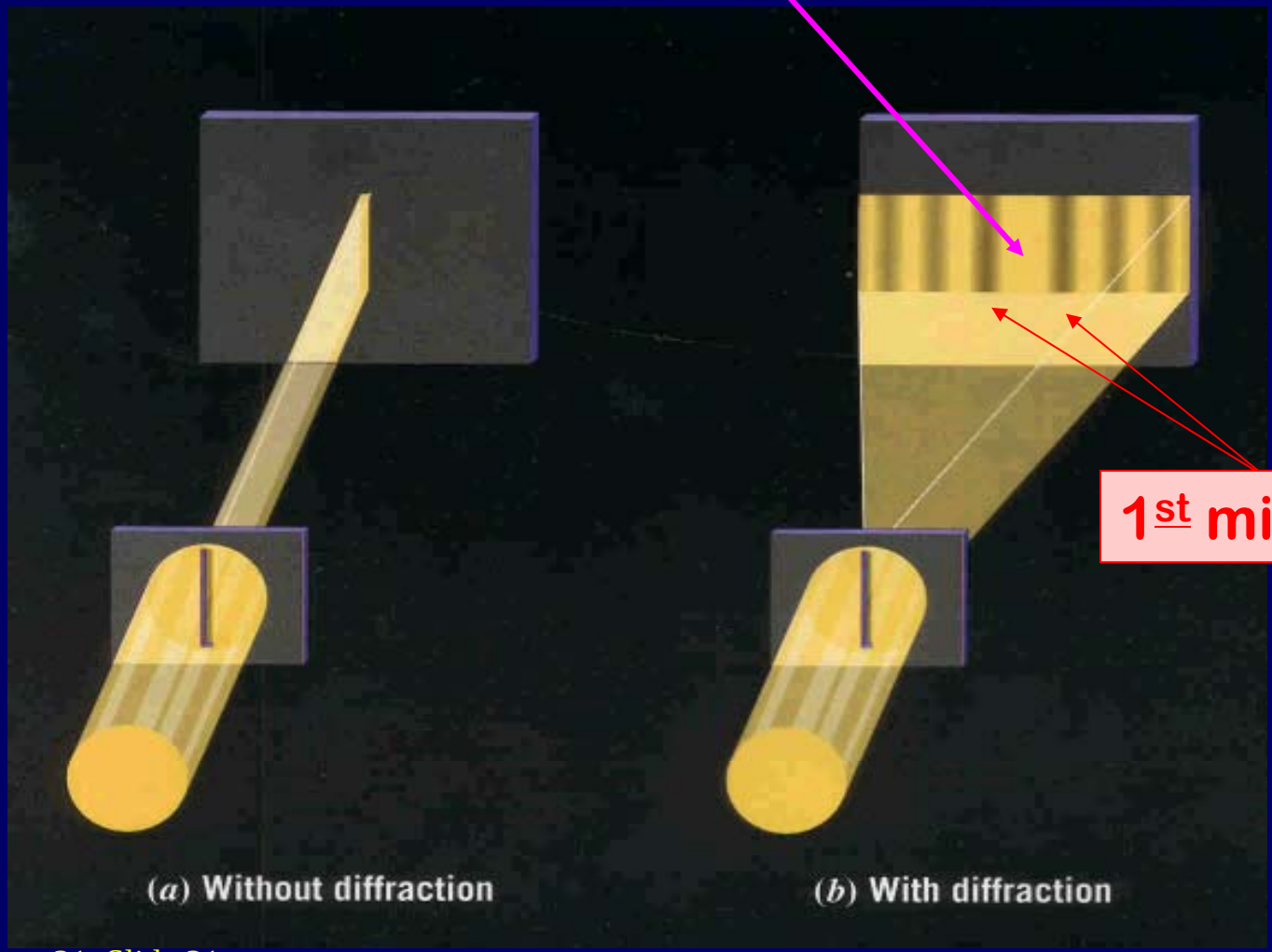


JAVA

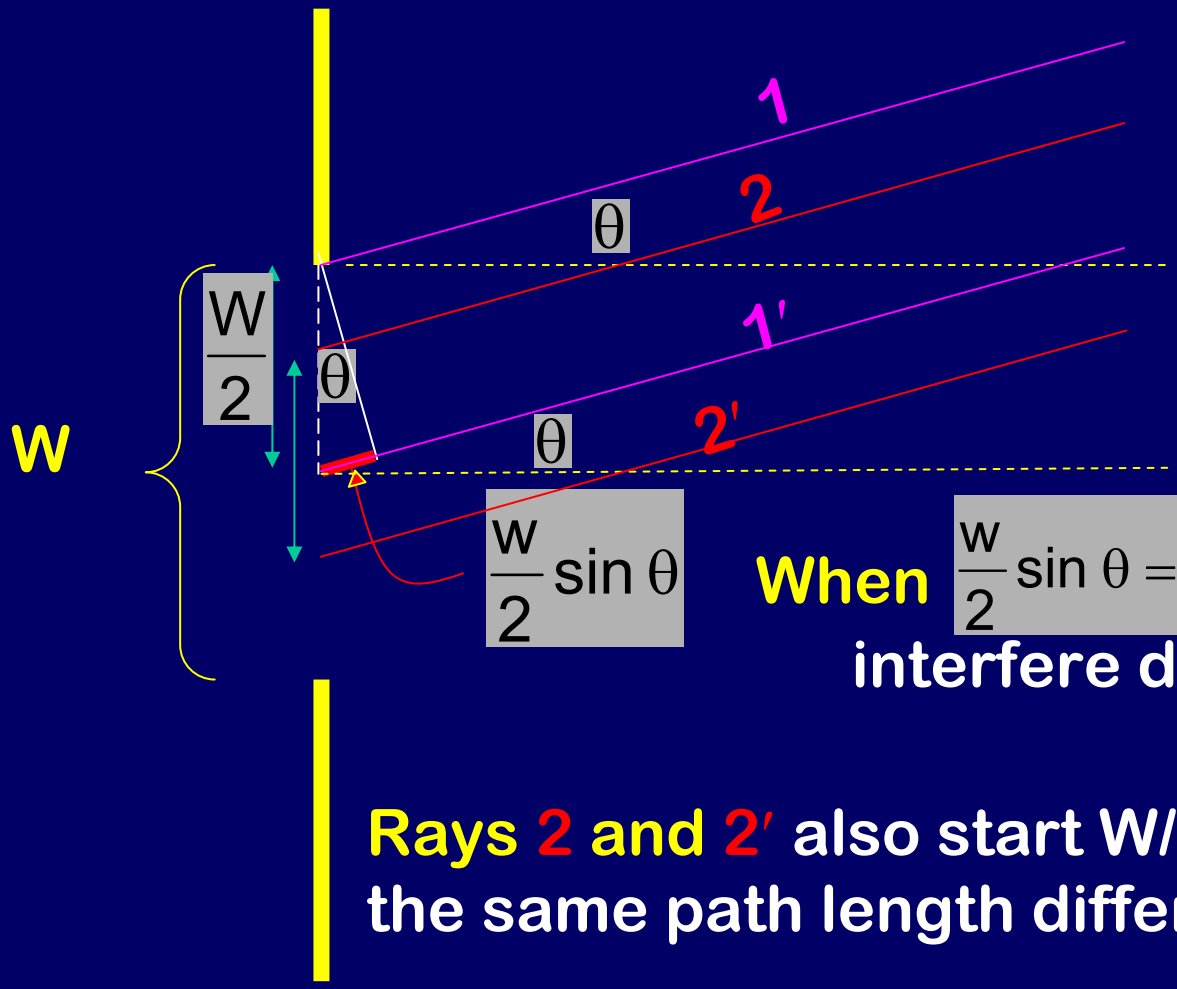
Light waves originating at different points within opening travel different distances to wall, and can interfere!

We will see maxima and minima on the wall.

Central maximum



Single Slit Diffraction

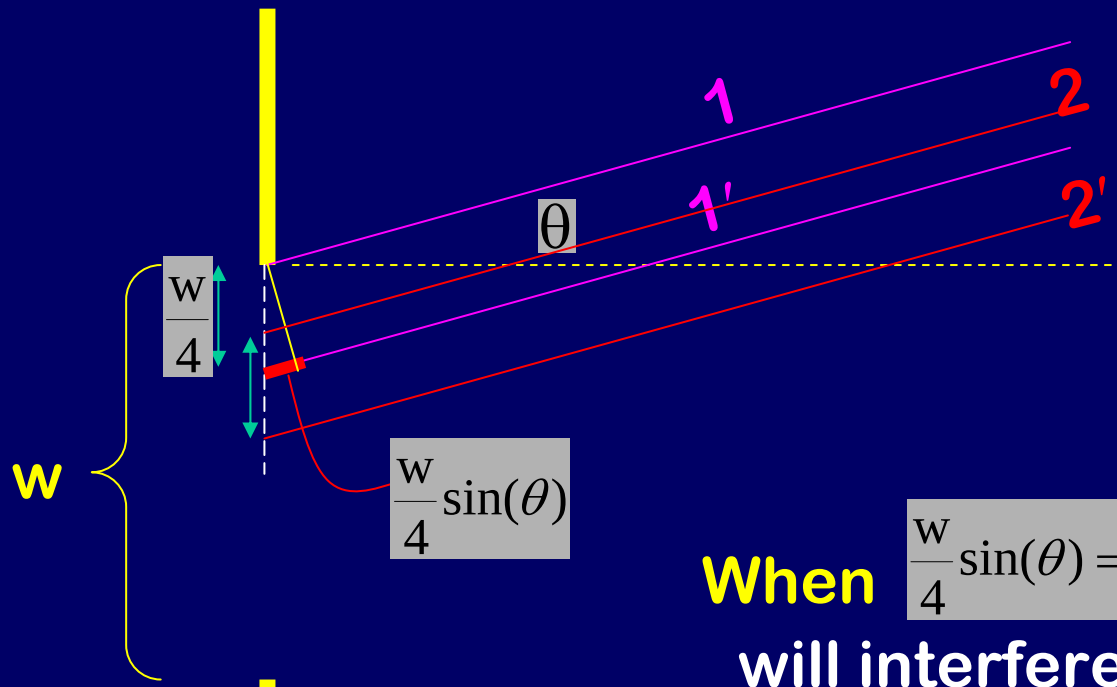


When $\frac{w}{2}\sin\theta = \frac{\lambda}{2}$ rays 1 and 1′ interfere destructively.

Rays 2 and 2′ also start W/2 apart and have the same path length difference.

Under this condition, every ray originating in top half of slit interferes destructively with the corresponding ray originating in bottom half. 1st minimum at $\sin\theta = \lambda/w$

Single Slit Diffraction



When $\frac{w}{4}\sin(\theta) = \frac{\lambda}{2}$ rays 1 and 1′ will interfere destructively.

Rays 2 and 2′ also start w/4 apart and have the same path length difference.

Under this condition, every ray originating in top quarter of slit interferes destructively with the corresponding ray originating in second quarter.

2nd minimum at $\sin\theta = 2\lambda/w$

Single Slit Diffraction Summary

Condition for halves of slit to destructively interfere

$$\sin(\theta) = \frac{\lambda}{w}$$

Condition for quarters of slit to destructively interfere

$$\sin(\theta) = 2\frac{\lambda}{w}$$

Condition for sixths of slit to destructively interfere

$$\sin(\theta) = 3\frac{\lambda}{w}$$

All together… $\sin\theta = m\frac{\lambda}{w}$ **(m=1, 2, 3, …)**

THIS FORMULA LOCATES MINIMA!!

Narrower slit => broader pattern

Preflight 21.3

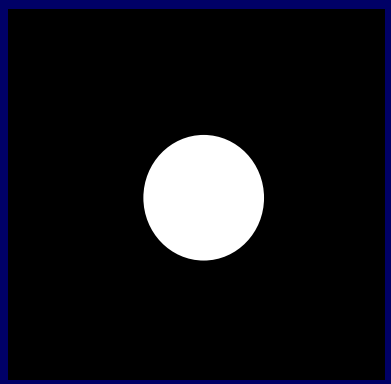
Note: interference only occurs when w > λ

ACTS/Preflights 21.4, 21.5

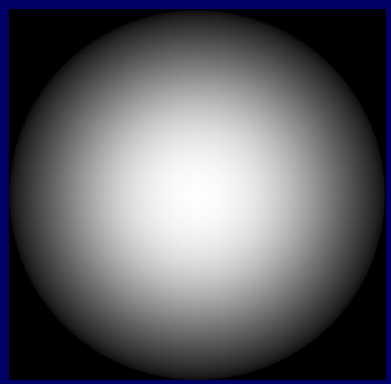
A laser is shined onto a screen through a very small hole. If you make the hole even smaller, the spot on the screen will get:

(1) Larger **(2) Smaller**

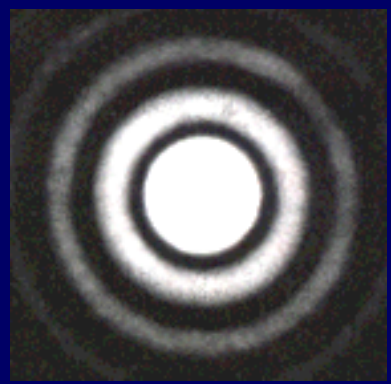
Which drawing correctly depicts the pattern of light on the screen?



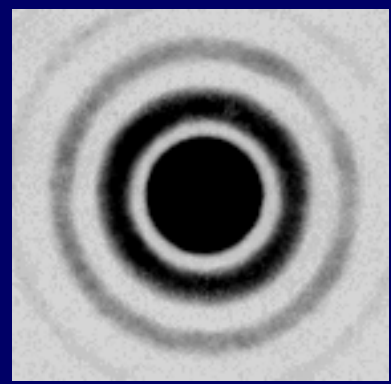
(1)



(2)

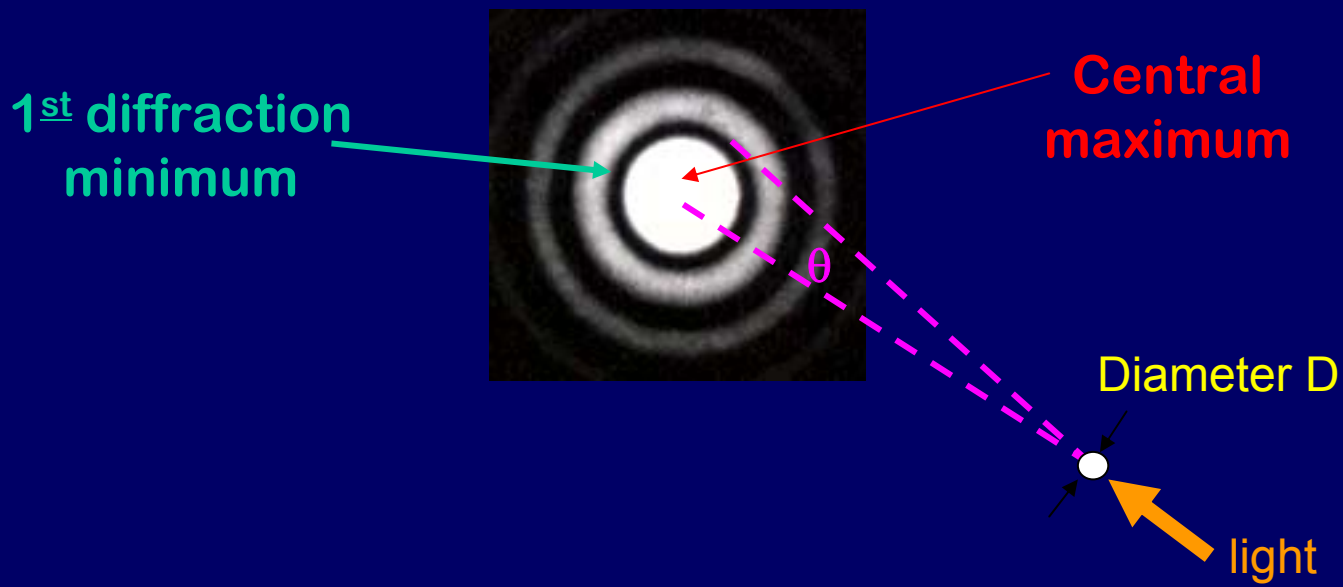


(3)



(4)

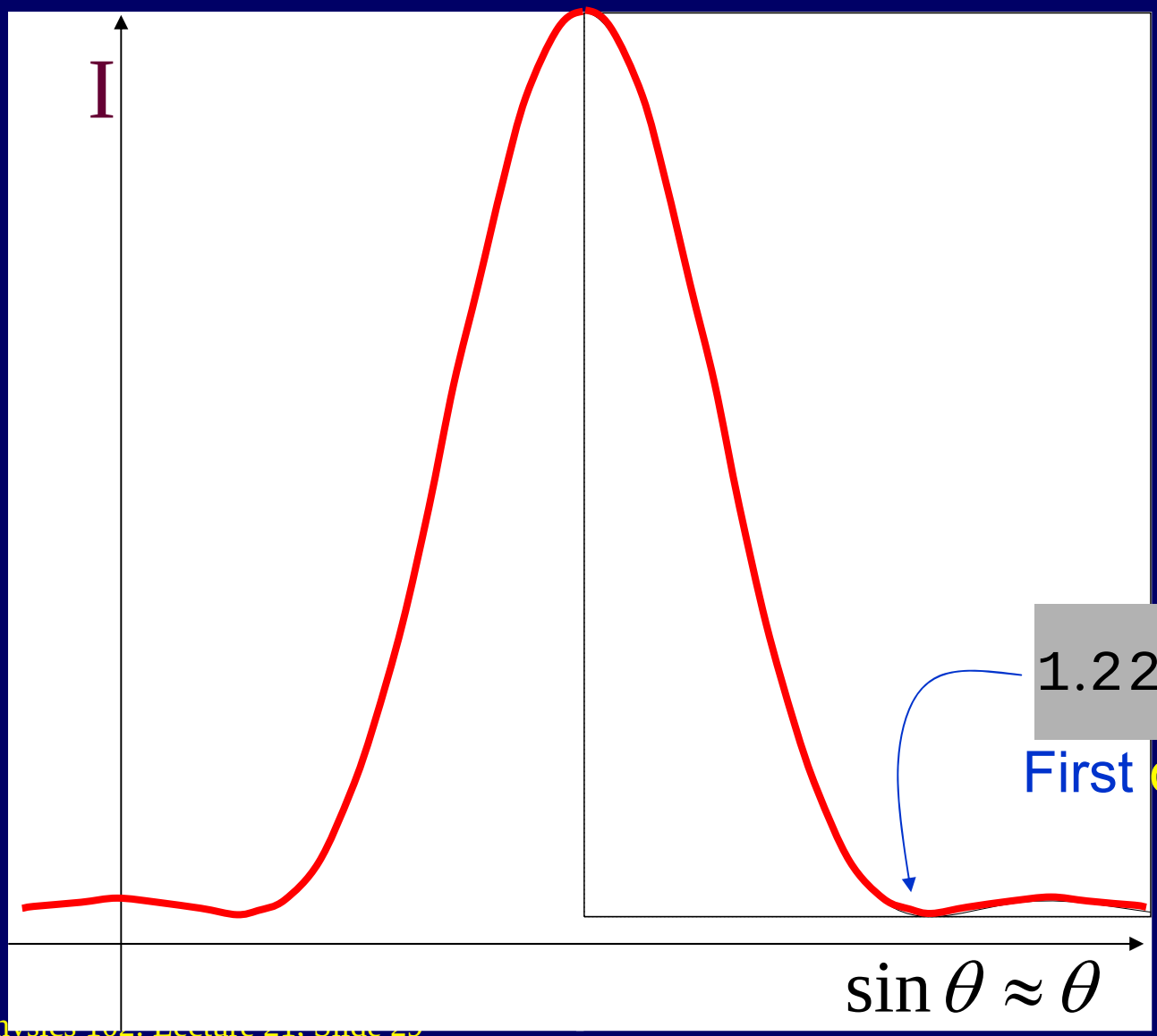
Diffraction from Circular Aperture

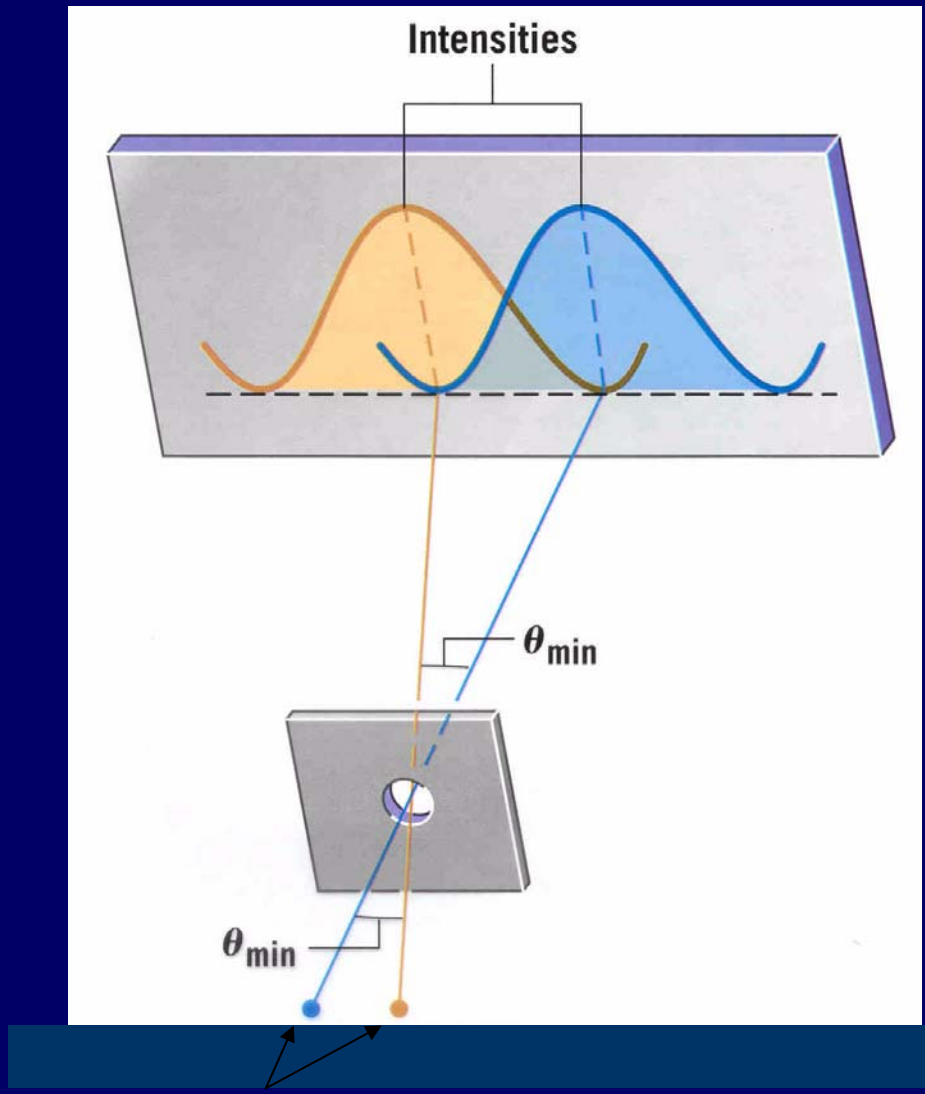


Maxima and minima will be a series of **bright** and **dark** rings on screen

First diffraction minimum is at $\sin\theta = 1.22\frac{\lambda}{D}$

Intensity from Circular Aperture





These objects are *just* resolved

Two objects are just resolved when the maximum of one is at the minimum of the other.

Resolving Power

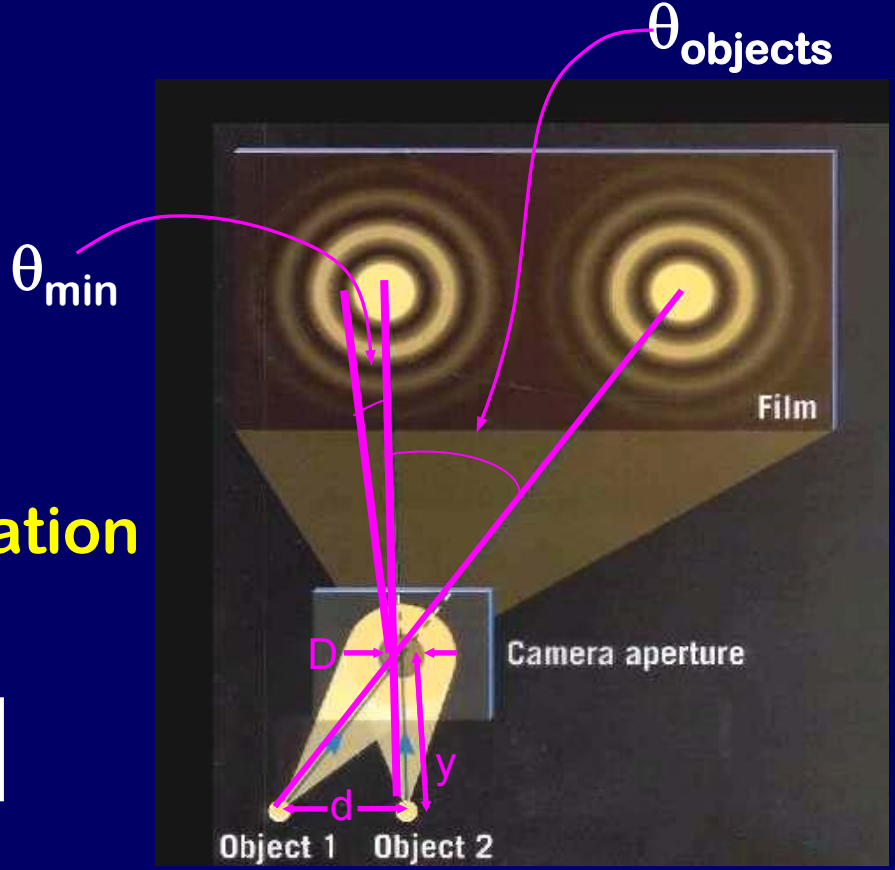
To see two objects distinctly, need $\theta_{objects} > \theta_{min}$

$\theta_{objects}$ is angle between objects and aperture:

$$\theta_{objects} \approx \tan (d/y)$$

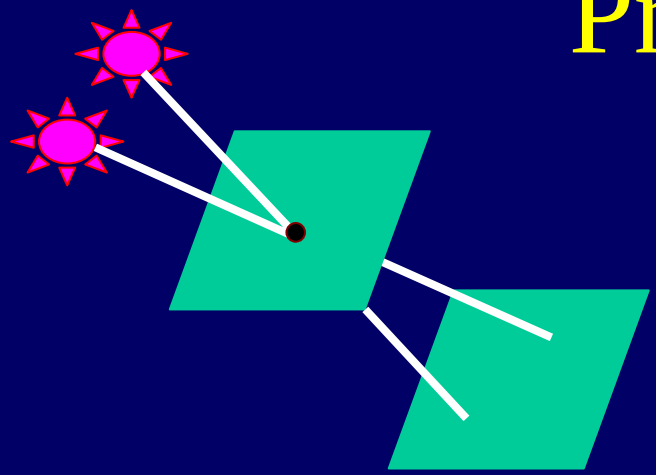
θ_{min} is minimum angular separation that aperture can resolve:

$$\sin \theta_{min} \approx \theta_{min} = 1.22\ \lambda/D$$



Improve resolution by increasing $\theta_{objects}$ or decreasing θ_{min}

Preflight 21.6



Want $\theta_{objects} > \theta_{min}$

Decrease $\theta_{min} = 1.22\lambda / D$

Increase D !

Astronaut Joe is standing on a distant planet with binary suns. He wants to see them but knows it's dangerous to look straight at them. So he decides to build a pinhole camera by poking a hole in a card. Light from both suns shines through the hole onto a second card.

But when the camera is built, Astronaut Joe can only see one spot on the second card! To see the two suns clearly, should he make the pinhole larger or smaller?

ACT: Resolving Power

$$\sin\theta_{min} \approx \theta_{min} = 1.22\frac{\lambda}{D}$$

How does the maximum resolving power of your eye change when the brightness of the room is decreased.

1) Increases 2) Constant 3) Decreases

Recap.

- Interference: Coherent waves
  - Full wavelength difference = Constructive
  - ½ wavelength difference = Destructive
- Multiple Slits
  - Constructive $d \sin(\theta) = m \lambda$ $(m=1,2,3\ldots)$
  - Destructive $d \sin(\theta) = (m + 1/2) \lambda$ 2 slit only
  - More slits = brighter max, darker mins
- Huygens' Principle: Each point on wave front acts as coherent source and can interfere.
- Single Slit:
  - Destructive: $w \sin(\theta) = m \lambda$ $(m=1,2,3\ldots)$
  - Resolution: Max from 1 at Min from 2

opposite!