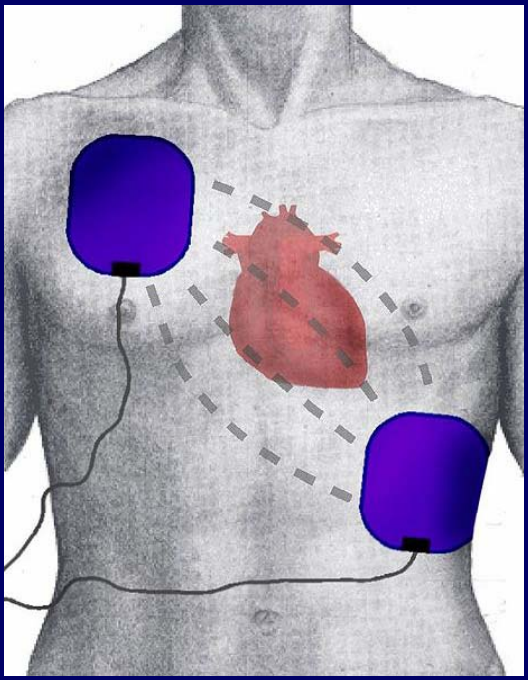
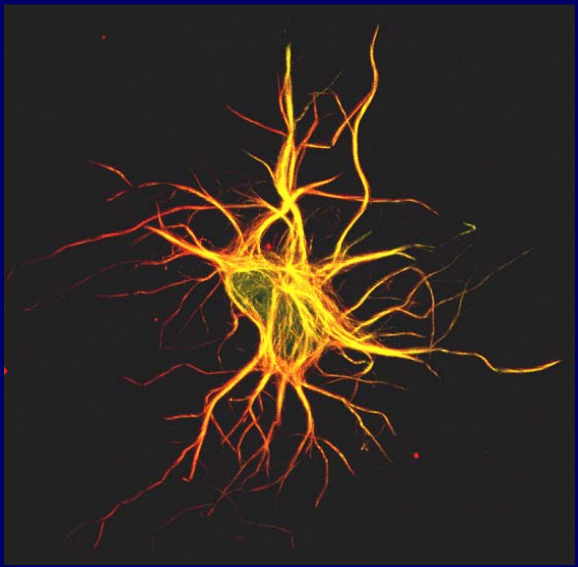
Physics 102: Lecture 7

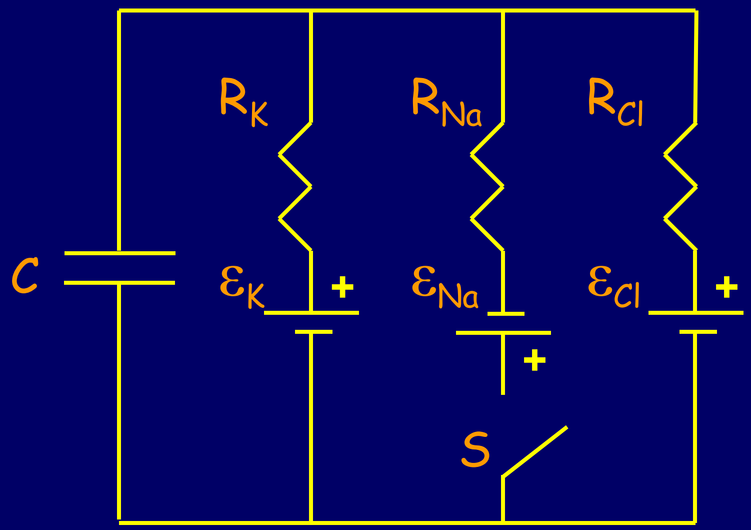
RC Circuits



RC Circuits

- Circuits that have both resistors *and* capacitors:





- With resistance in the circuits, capacitors do not charge and discharge instantaneously – it takes time (even if only fractions of a second).

RC Circuits

Used to controllably store and release energy

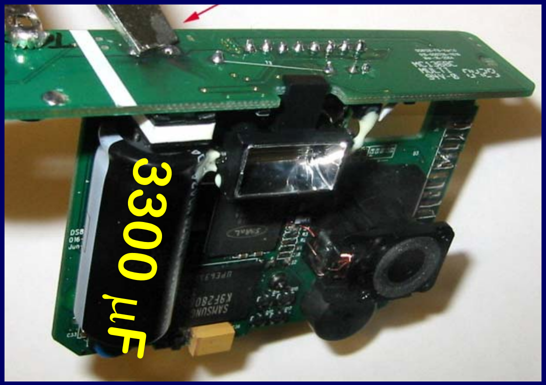
Today:

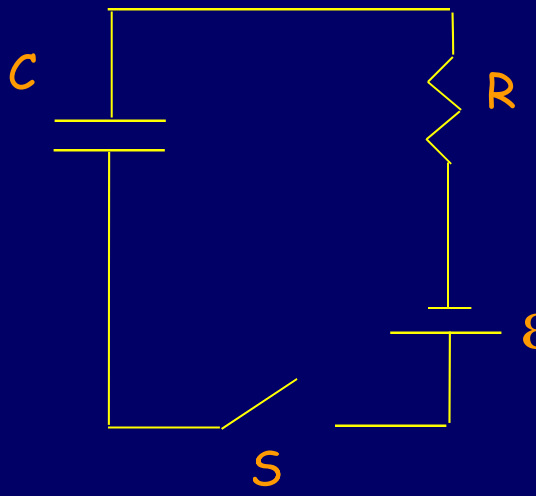
- RC Circuits
- Charging Capacitors
- Discharging Capacitors
- Intermediate Behavior

Charging Capacitors

Storing energy to use later

- Capacitor is initially uncharged and switch is open. Switch is then closed.
- What is current I_0 in circuit immediately thereafter?
- What is current I_∞ in circuit a long time later?

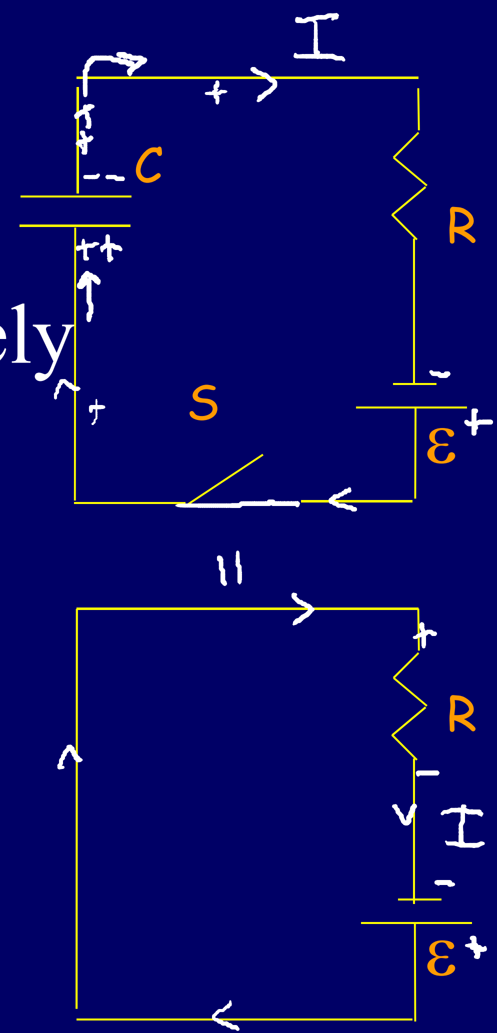




Charging Capacitors: t = 0

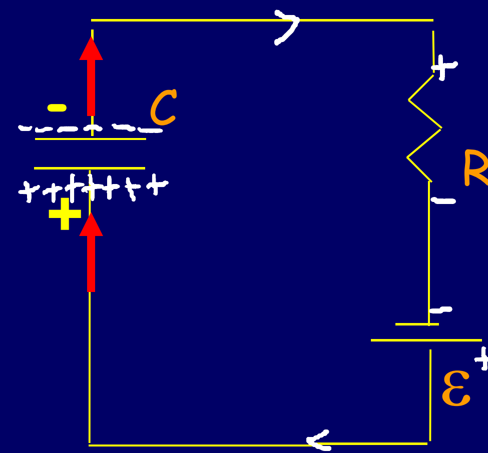
- Capacitor is initially uncharged and switch is open. Switch is then closed.
- What is current I_0 in circuit immediately thereafter?
  - Capacitor initially uncharged
  - Therefore $V_C = 0$ (since $V=Q/C$)
  - **Therefore C behaves as a wire (short circuit)** KLR: $+\mathcal{E} - IR = 0$
  - Ohm's law!

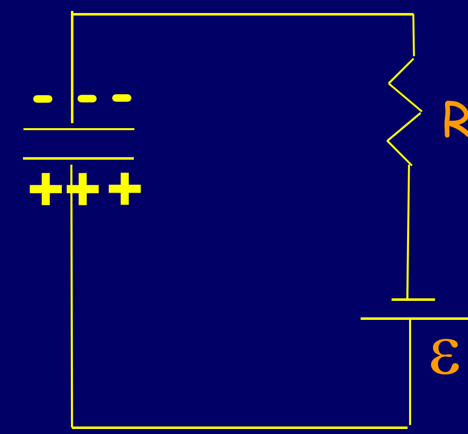
$I_0 = \varepsilon/R$



Charging Capacitors: $t > 0$

- $I_0 = \varepsilon/R$ $\quad Q_0 = 0$
- Positive charge flows
  - Onto bottom plate (+Q)
  - Away from top plate (-Q)
  - As charge builds up, V_C rises ($V_C = Q/C$)
  - Loop: $\varepsilon - V_C - I R = 0$
    - $I = (\varepsilon - V_C)/R < I_0$
    - Therefore I falls as Q rises
- When t is very large (∞)
  - **$I_\infty = 0$: no current flow into/out of capacitor at long times**
  - $V_C = \varepsilon$ $\quad Q_\infty = C\varepsilon$





ACT/CheckPoint 1

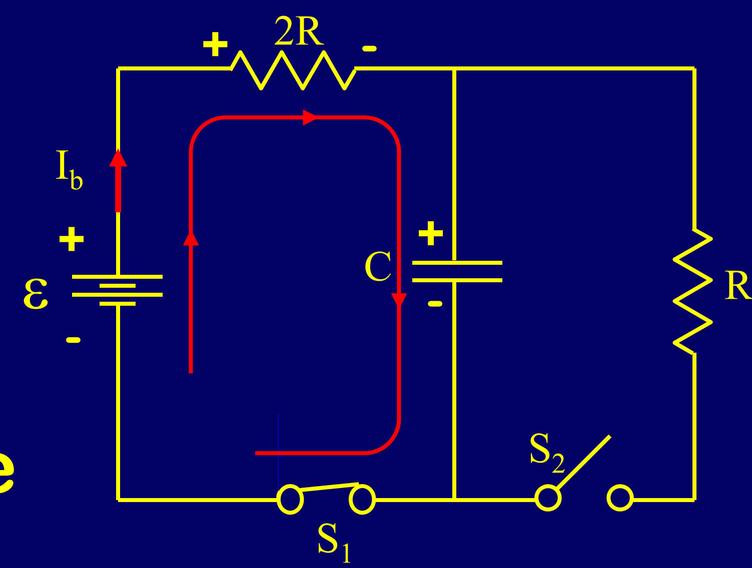
Both switches are initially open, and the capacitor is uncharged. What is the current through the battery just after switch S_1 is closed?

1) $I_b = 0$

2) $I_b = \mathcal{E}/(3R)$

3) $I_b = \mathcal{E}/(2R)$

4) $I_b = \mathcal{E}/R$

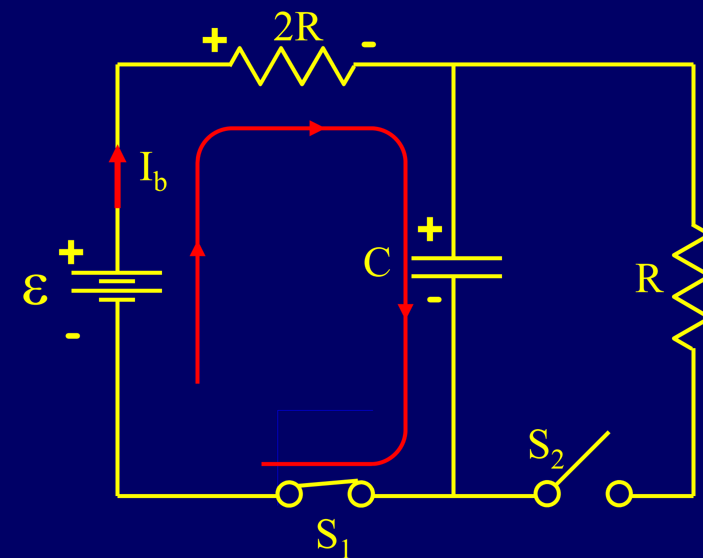


Capacitor acts like a wire the instant the switch is closed:

$\Rightarrow I = \varepsilon/(2R)$

ACT/CheckPoint 3

Both switches are initially open, and the capacitor is uncharged. What is the current through the battery after switch 1 has been closed a long time?



1) $I_b = 0$

2) $I_b = \mathcal{E}/(3R)$

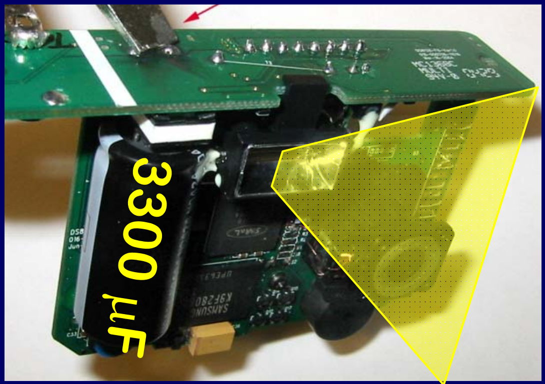
3) $I_b = \mathcal{E}/(2R)$

4) $I_b = \mathcal{E}/R$

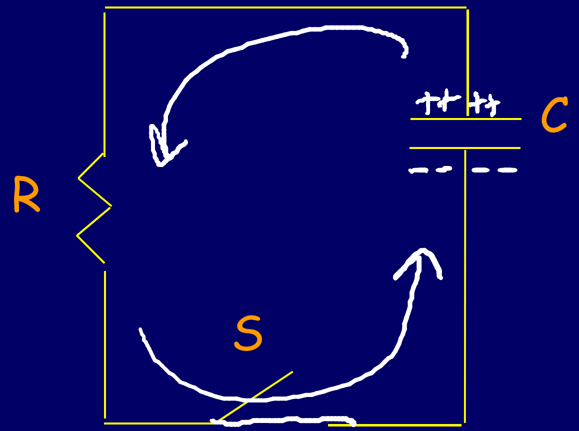
- Long time $\Rightarrow$ current through capacitor is zero
- I_b=0 because the battery and capacitor are in series.
- **KLR:** $\varepsilon - 0 - q_\infty/C = 0 \Rightarrow q_\infty = \varepsilon C$

Discharging Capacitors

Time to use that stored energy!

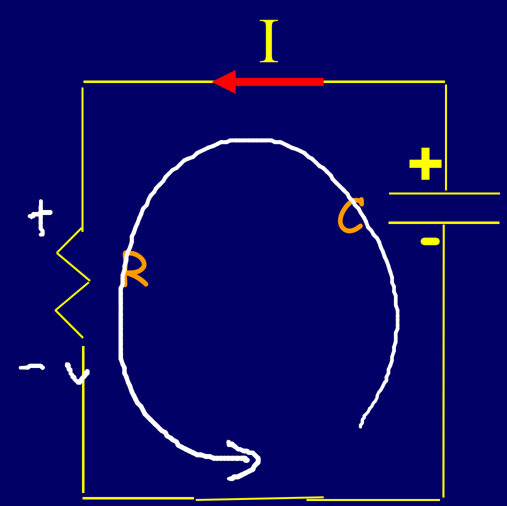


- Capacitor is initially charged (Q) and switch is open. Switch is then closed.
- What is current I_0 in circuit immediately thereafter?
- What is current I_∞ in circuit a long time later?



Discharging Capacitors

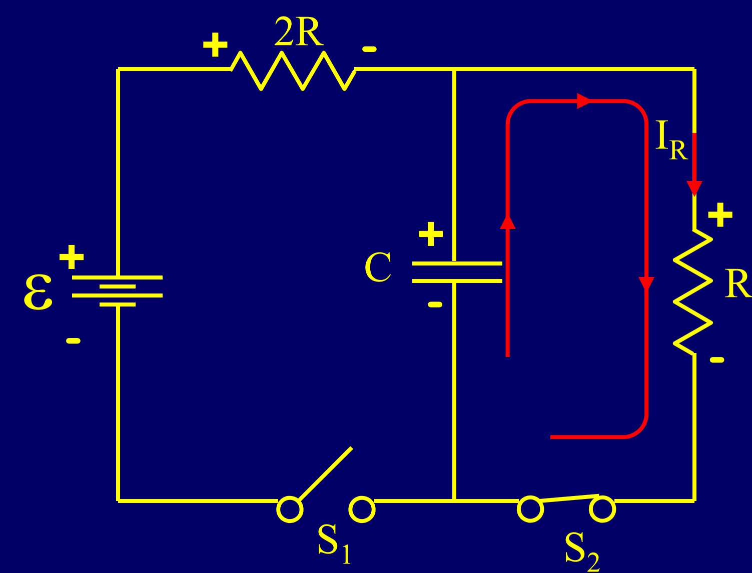
- Capacitor is initially charged (Q) and switch is open. Switch is then closed.
- What is current I_0 in circuit immediately thereafter?
  - KLR: $Q/C - I_0R = 0$
  - So, $I_0 = Q/RC$
- What is current I_∞ in circuit a long time later?
  - $I_\infty = 0$ $Q_\infty = 0$



ACT/CheckPoint 5

After switch 1 has been closed for a long time, it is opened and switch 2 is closed. What is the current through the right resistor just after switch 2 is closed?

$t = 0$



1) $I_R = 0$

2) $I_R = \varepsilon/(3R)$

3) $I_R = \varepsilon/(2R)$

4) $I_R = \varepsilon/R$

KLR: $q_0/C - IR = 0$

Recall q is charge on capacitor after charging:

$q_0 = \varepsilon C$ (since charged w/ switch 2 open!)

$\varepsilon - IR = 0 \Rightarrow I = \varepsilon/R$

Summary: charging & discharging

- Charge (and therefore voltage) on Capacitors cannot change instantly: remember $V_C = Q/C$

- Short term behavior of Capacitor:
  - If the capacitor starts with no charge, it has no potential difference across it and acts as a wire
  - If the capacitor starts with charge, it has a potential difference across it and acts as a battery.

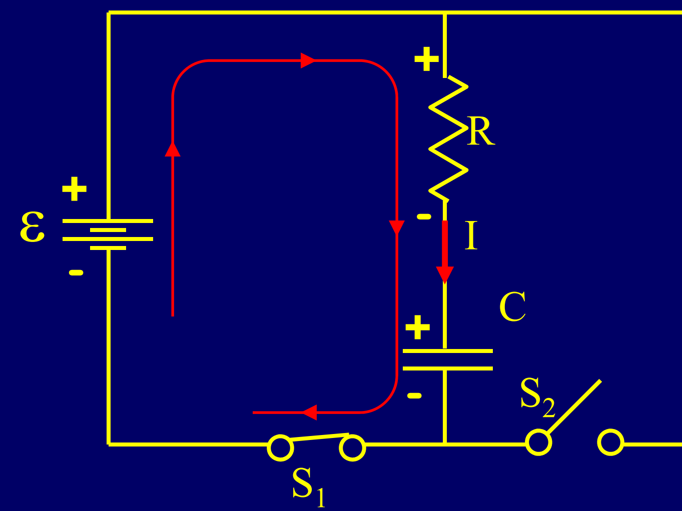
- Long term behavior of Capacitor: Current through a Capacitor is eventually zero.
  - If the capacitor is charging, when fully charged no current flows and capacitor acts as an open circuit
  - If capacitor is discharging, potential difference is zero and no current flows

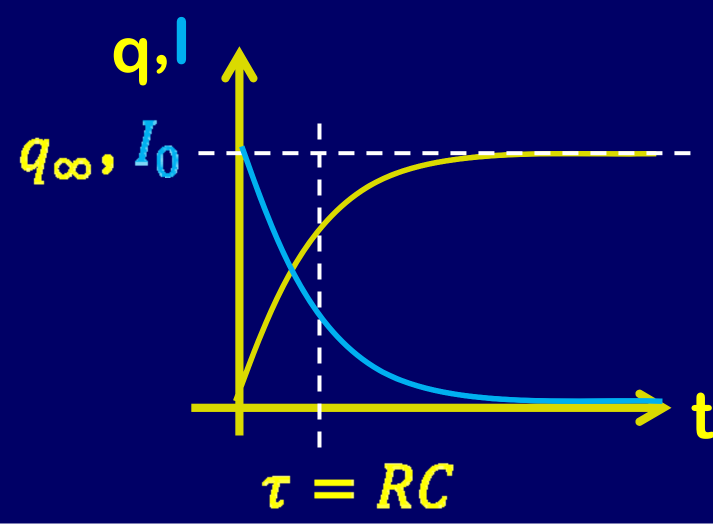
RC Circuits: Charging

The switches are originally open and the capacitor is uncharged. Then switch S_1 is closed.

- Loop: $\varepsilon - I(t)R - q(t) / C = 0$
- Just after…: $q = 0$ $\quad t = 0$
  - Capacitor is uncharged $\quad q_0 = 0$
  - $\varepsilon - I_0 R = 0 \Rightarrow I_0 = \varepsilon / R$
- Long time after: $I_\infty = 0$
  - Capacitor is fully charged
  - $\varepsilon - q_\infty / C = 0 \Rightarrow q_\infty = \varepsilon C$
- Intermediate (more complex)

$$q(t) = q_\infty(1 - e^{-t/RC})$$
$$I(t) = I_0 e^{-t/RC}$$



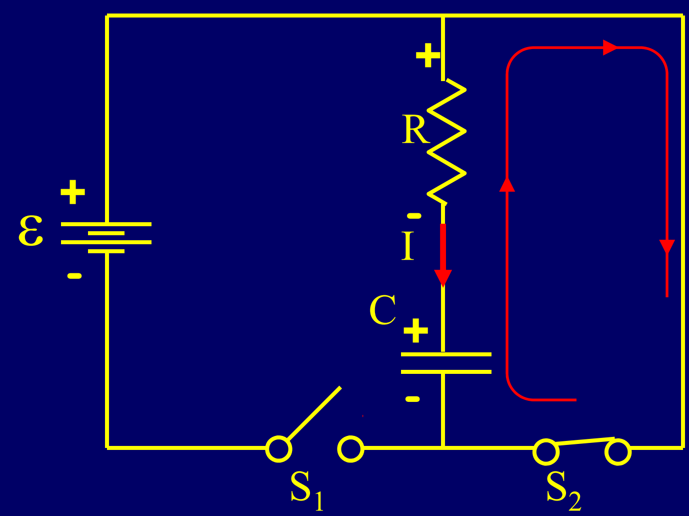


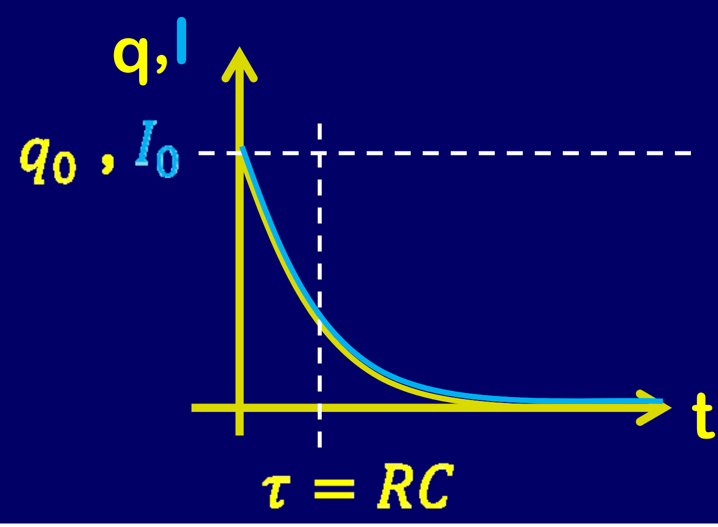
RC Circuits: Discharging

- Loop: $q(t) / C + I(t) R = 0$
- Just after…: $q=q_0$ $\quad t = 0$
  - Capacitor is still fully charged
  - $q_0 / C + I_0 R = 0 \Rightarrow I_0 = -q_0 / (RC)$
- Long time after: $I_\infty = 0$ $\quad t = \infty$
  - Capacitor is discharged
  - $q_\infty / C = 0 \Rightarrow q_\infty = 0$
- Intermediate (more complex)

  $q(t) = q_0 \, e^{-t/RC}$

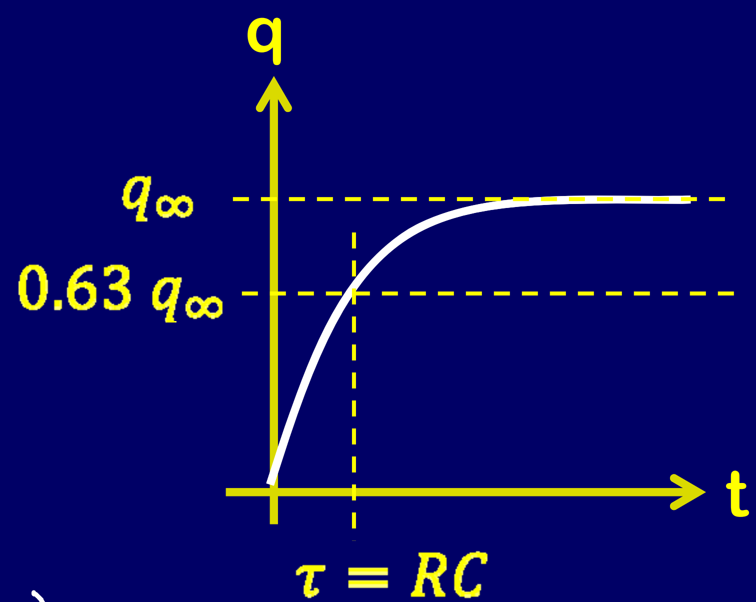
  $I(t) = I_0 \, e^{-t/RC}$

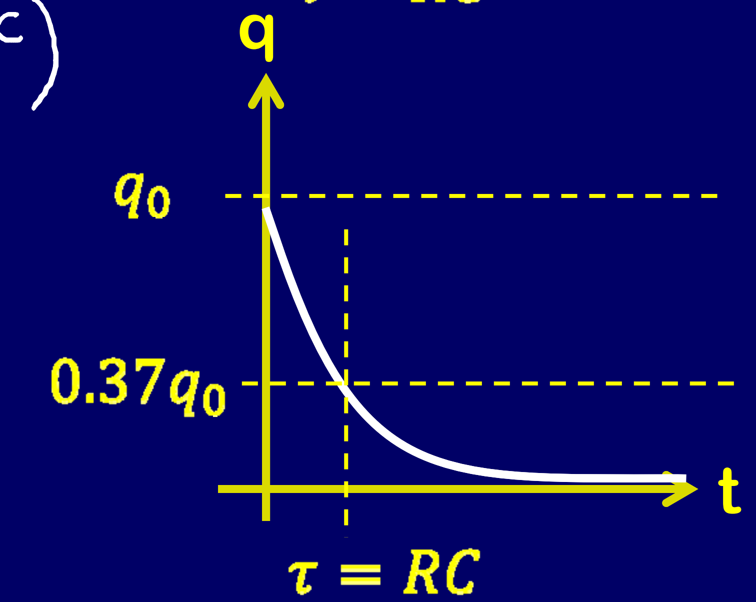




What is the time constant?

- The time constant $\tau = RC$.
- Given a capacitor starting with no charge, the time constant is the amount of time an RC circuit takes to charge a capacitor to about 63% of its final value. $q(t) = q_\infty (1 - e^{-t/RC})$
- The time constant is the amount of time an RC circuit takes to discharge a capacitor to about 37% of its original value. $q = q_0 e^{-t/RC}$





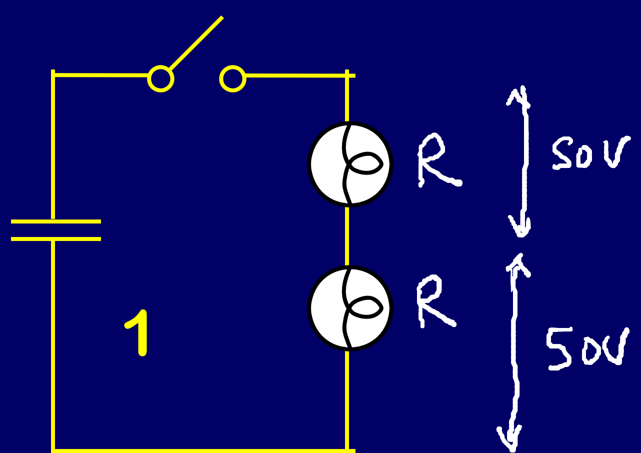
Example

Time Constant Demo

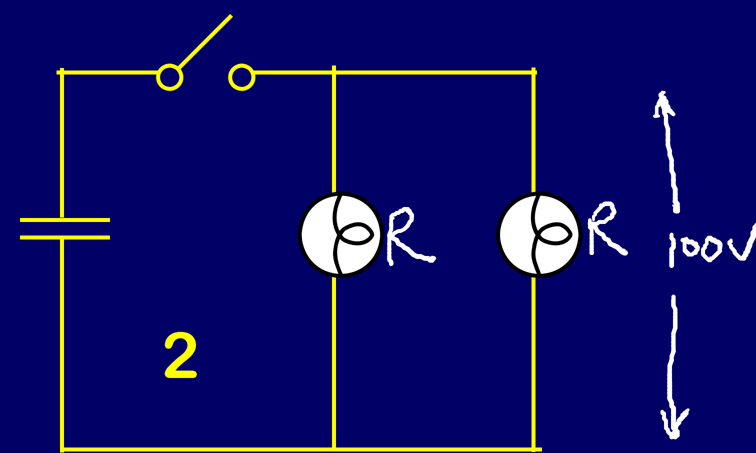
Each circuit has a 1 F capacitor charged to 100 Volts.
When the switch is closed:

- **Which system will be brightest?** — (2) $I=V/R$
- **Which lights will stay on longest?** — (1)
- **Which lights consumes more energy?** — Same

$U=1/2\ CV^2$



τ = 2RC



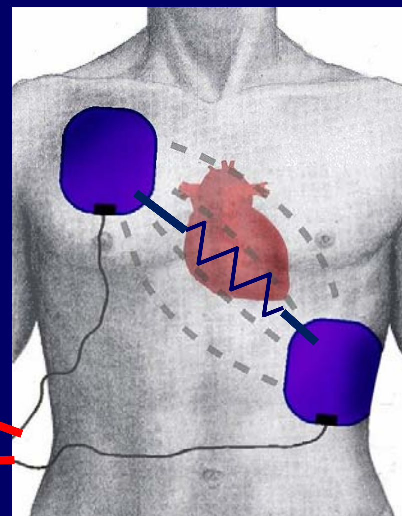
τ = RC/2

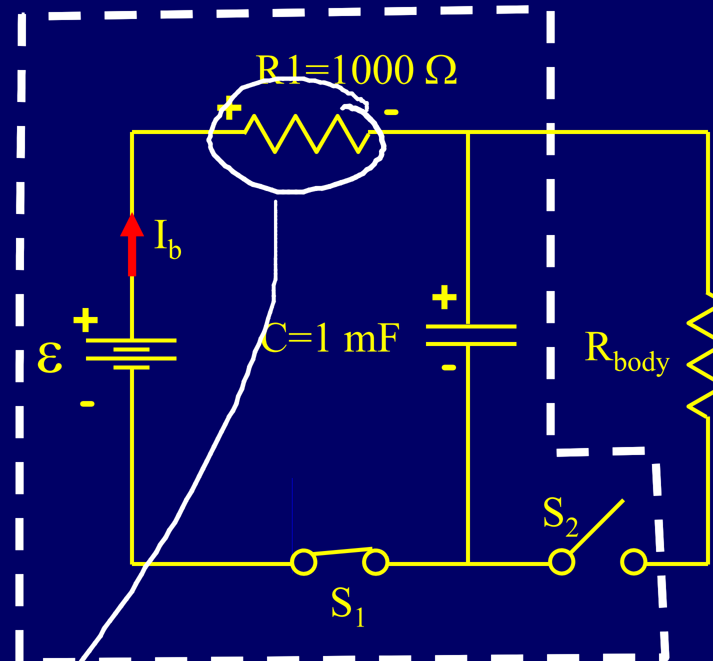
Summary of Concepts

- Charge (and therefore voltage) on Capacitors cannot change instantly: remember $V_C = Q/C$
- Short term behavior of Capacitor:
  - If the capacitor starts with no charge, it has no potential difference across it and acts as a wire
  - If the capacitor starts with charge, it has a potential difference across it and acts as a battery.
- Long term behavior of Capacitor: Current through a Capacitor is eventually zero.
  - If the capacitor is charging, when fully charged no current flows and capacitor acts as an open circuit.
  - If capacitor is discharging, potential difference is zero and no current flows.
- Intermediate behavior: Charge and current exponentially approach their long-term values $\tau = RC$

Example

Practice: defibrillator





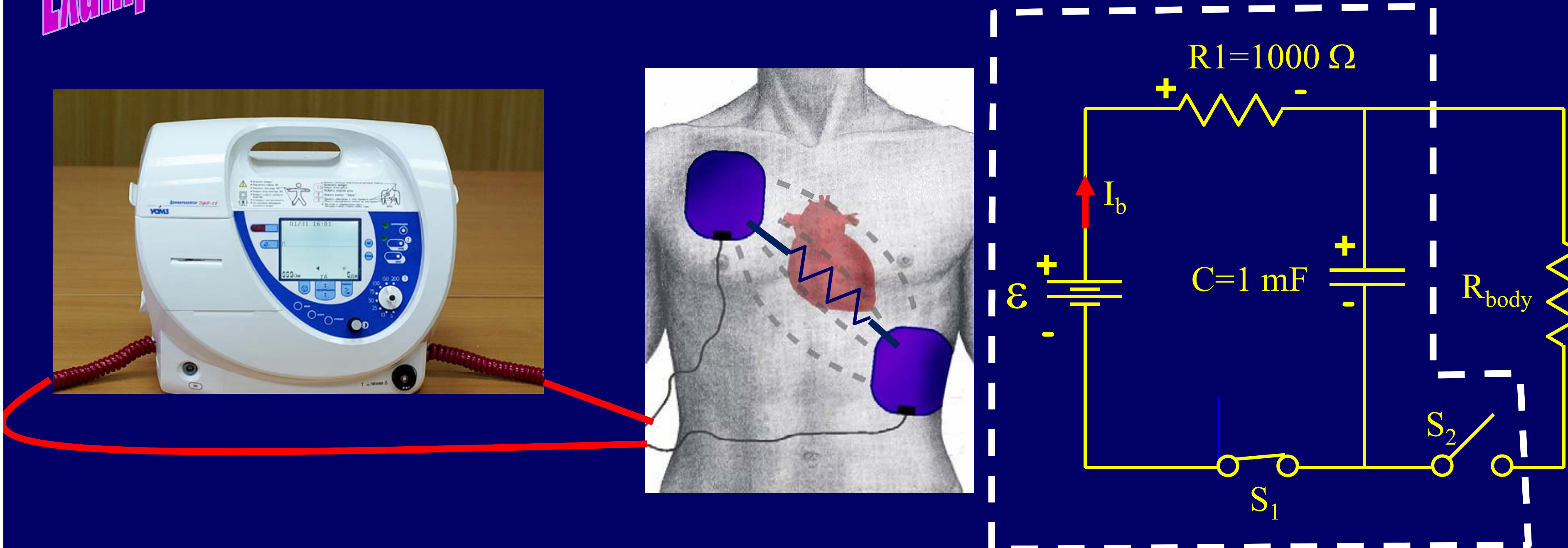
A 500 V battery is used to charge the 1 mF capacitor for 2 seconds. How much charge is stored on the capacitor?

$$q(t) = q_\infty(1 - e^{-t/RC}) \qquad q_\infty = C\epsilon = (0.001\ F)(500\ V) = 0.5\ C$$

$$q(2\ s) = 0.5\ C\left(1 - e^{-2s/1000\Omega\times 0.001F}\right) = 0.4\ \mathrm{C}$$

$$V_C = Q/C = 0.4C/0.001F = 400\ V$$

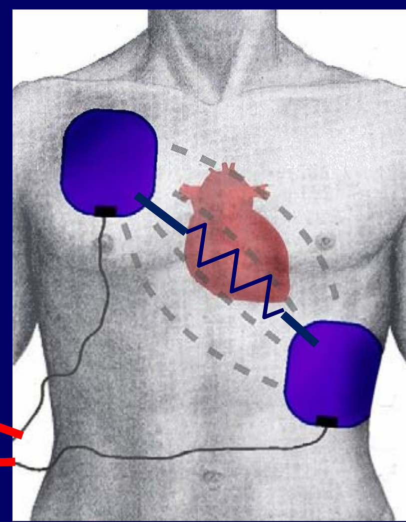
Practice: defibrillator

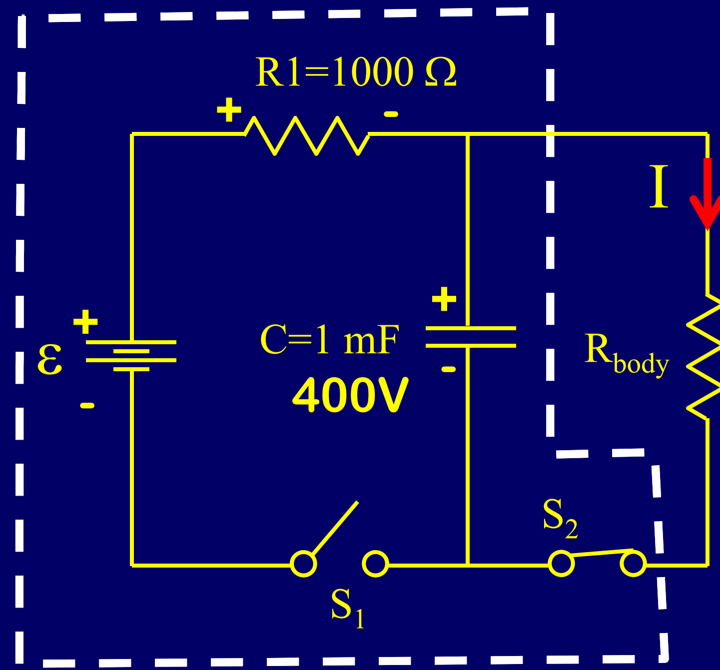


A 500 V battery is used to charge the 1 mF capacitor for 2 seconds. How much energy is stored in the capacitor?

$$U = Q^2/2C = (0.4C)^2/(2 \times 0.001\ F) = 80\ J$$

ACT: defibrillator





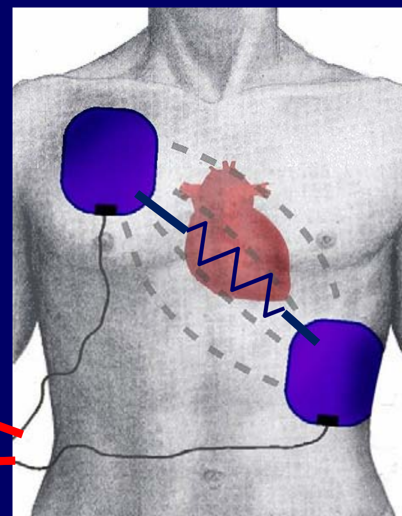
After charging for 2 seconds, S_1 is opened; What is the current through the patient right after S_2 is closed if R_{body}=100 Ω?

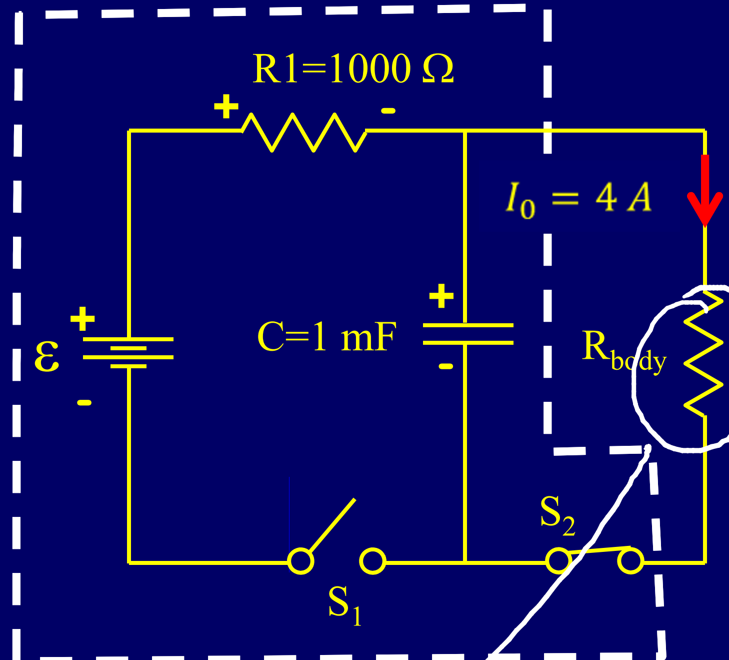
(A) 0 A **(B) 4 A** **(C) 0.25 A**

$$I = V/R = 400V/100\Omega = 4\ A$$

Example

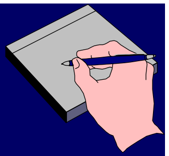
Practice: defibrillator





After charging for 2 s, S_1 is opened. What is the current through the patient 0.1 s after S_2 is closed if R_{body}=100 Ω?

$$I(0.1s) = I_0 e^{-t/RC} = 4\ \text{A}\ e^{-0.1\ \text{s}/100\Omega\times 0.001F} = 1.5\ \text{A}$$



RC Summary

Charging

$q(t) = q_{\infty}(1-e^{-t/RC})$

$V(t) = V_{\infty}(1-e^{-t/RC})$

$I(t) = I_0 e^{-t/RC}$

Discharging

$q(t) = q_0 e^{-t/RC}$

$V(t) = V_0 e^{-t/RC}$

$I(t) = I_0 e^{-t/RC}$

Time Constant $\tau = RC$

Large τ means long time to charge/discharge

Short term: Charge doesn't change (often zero or max)

Long term: Current through capacitor is zero.