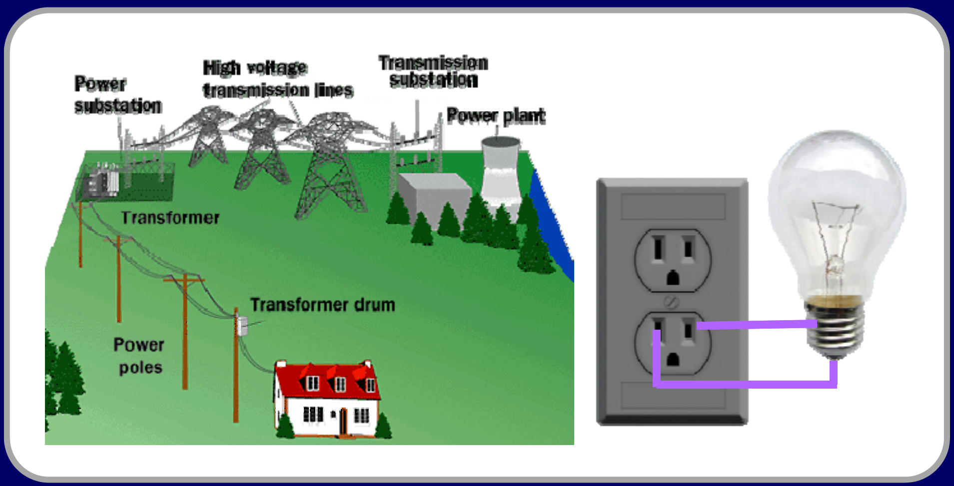
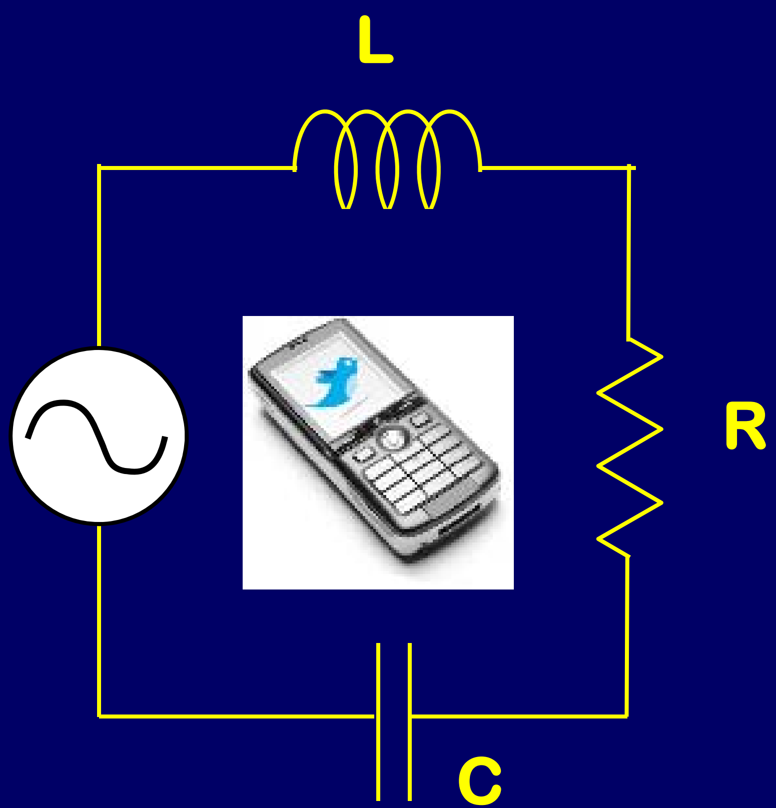
Physics 102: Lecture 12

AC & RLC Circuits

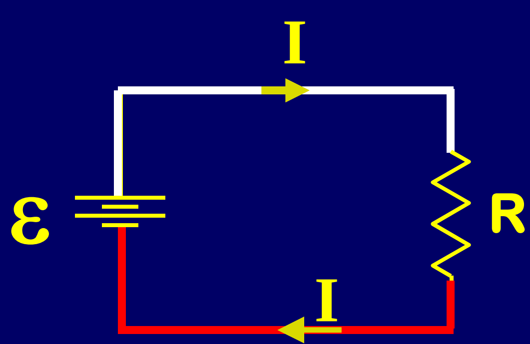




DC vs. AC circuits

DC

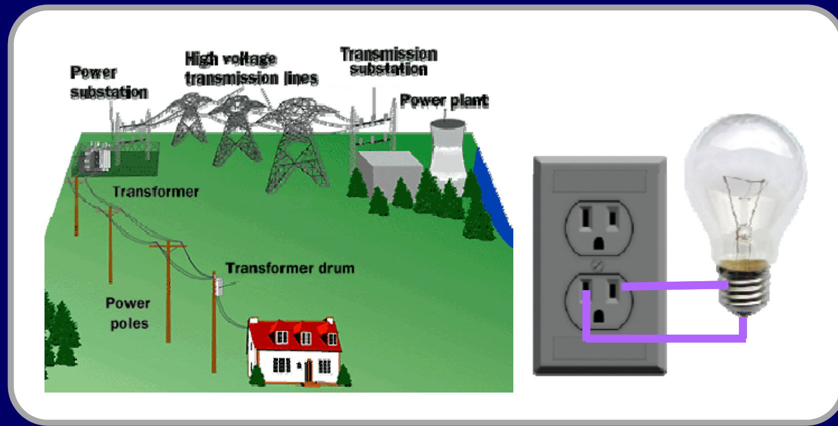
Direct Current

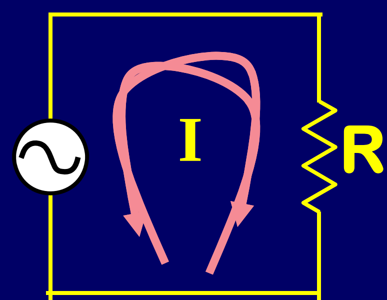


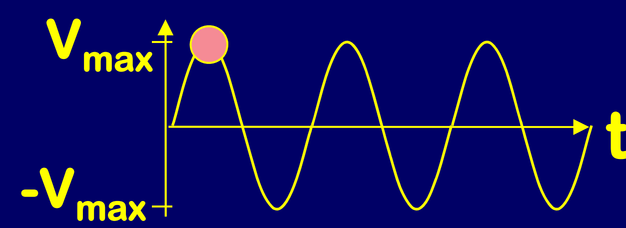
Direction of current is fixed

AC

Alternating Current







Direction of current alternates!

Review: Generators and EMF

Voltage across generator:

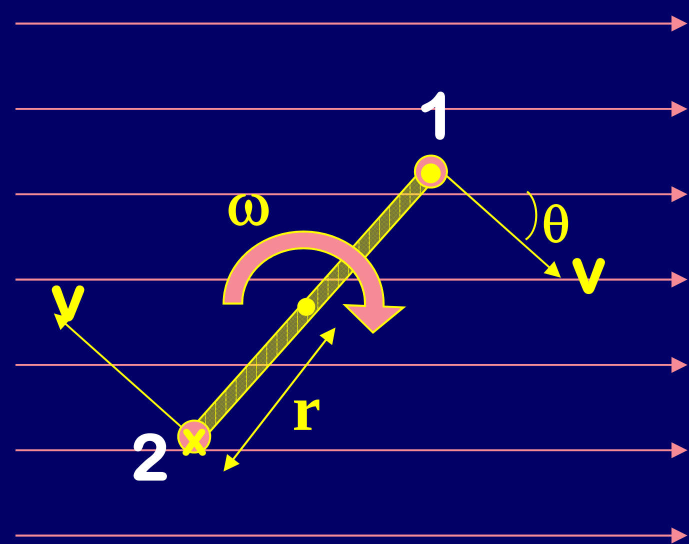
$$\varepsilon = \omega A B \sin(\theta)$$

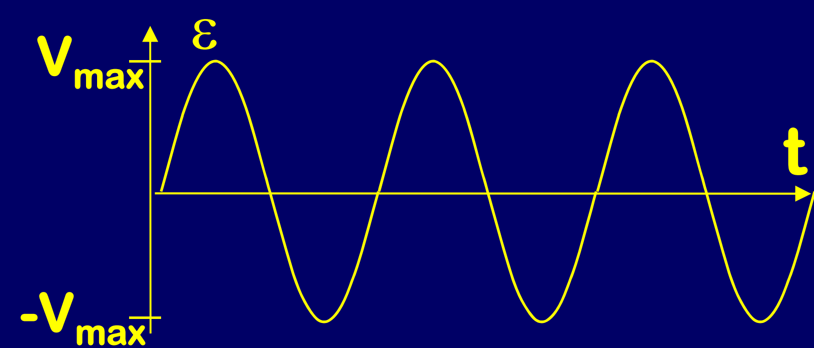
$$\varepsilon = \omega A B \sin(\omega t)$$

$$\varepsilon = V_{max} \sin(\omega t)$$

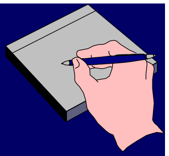
V_{max} = Amplitude

= Maximum voltage

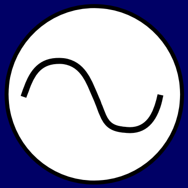




Example



AC Source



$V(t) = V_{max} \sin(\omega t) = V_{max} \sin(2\pi f\, t)$

V_{max} = maximum voltage

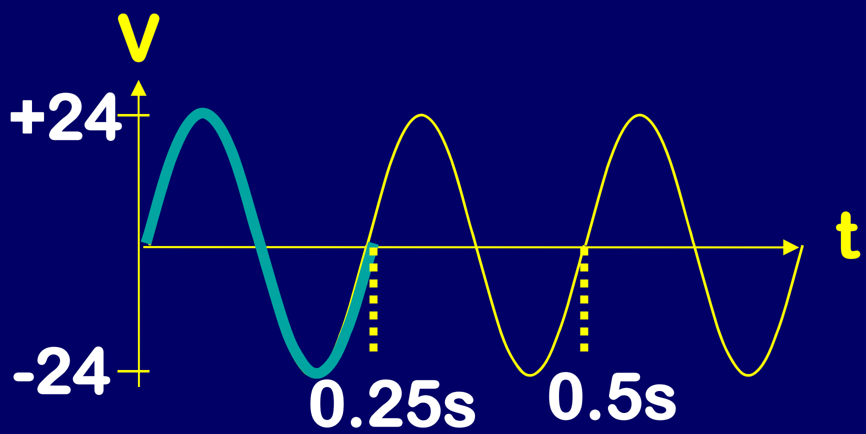
f = frequency (cycles/second)

$V(t) = 24 \sin(8\pi\, t)$

$2\pi f\, t = 8\pi t$

$f = 4$ Hz

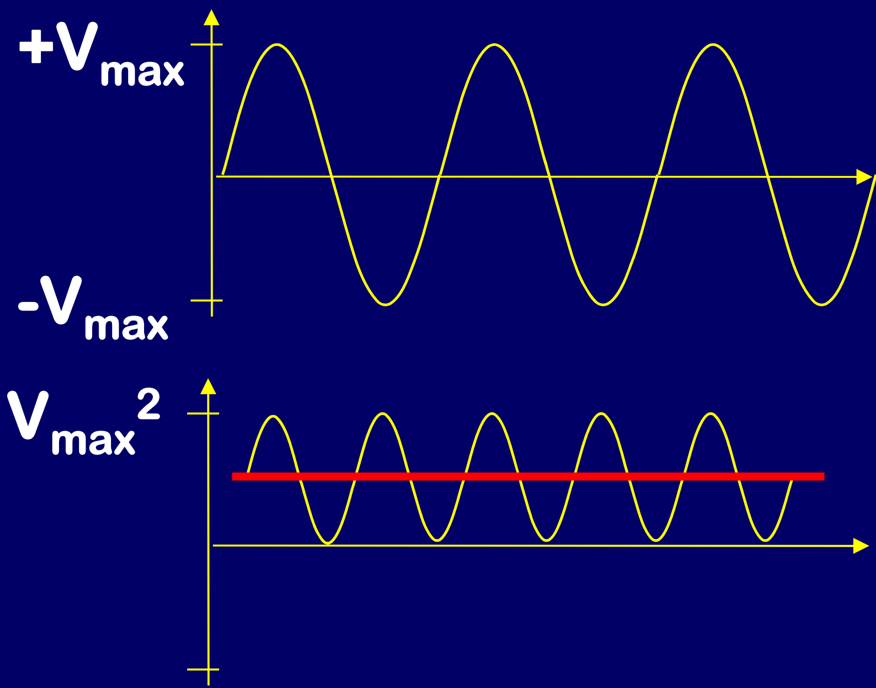
$T = (1/4)$ seconds



RMS: Root Mean Square $V_{rms} = V_{max}/\sqrt{2}$

RMS?

$V(t) = V_{max} \sin(2\pi f\, t)$



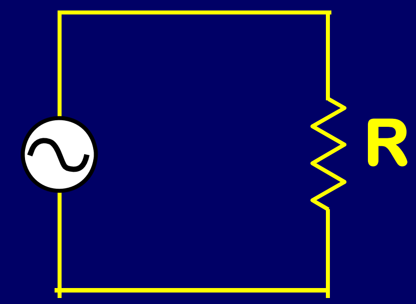
Square:

Mean: $V_{max}^2 / 2$

square Root: $V_{max} / \sqrt{2}$

RMS: Root Mean Square $V_{rms} = V_{max} / \sqrt{2}$

CheckPoint 2.1, 2.2



$$I(t) = \frac{V_{max}}{R} \sin(377t) = 10 \sin(377t)$$

Find I_{max} — **Well… We know that the maximum value of sin is 1. So the maximum current is 10!**

$I_{max} = 10$ A

Find I_{rms} — **Just like $V_{rms} = V_{max}/\sqrt{2}$ …**

$I_{rms} = I_{max}/\sqrt{2} = 10/\sqrt{2}$ A $= 7.07$ A

Average power dissipated: $\bar{P} = I_{rms} V_{rms} = \frac{1}{2} I_{max} V_{max}$

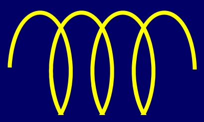
(For an AC circuit with only a resistor)

ACT: AC power dissipation

When your hair dryer is plugged in and running, it uses 1200 W of average power. If the max voltage delivered by the wall outlet is 120V, what is the max current delivered to the hair dryer?

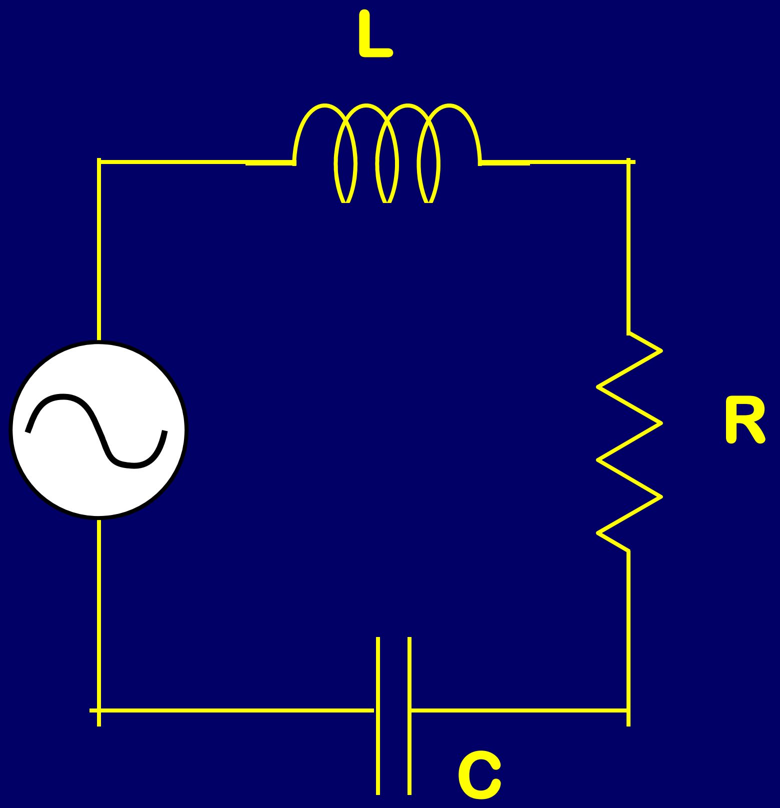
(A) 0.05 A (B) 10 A (C) 20 A

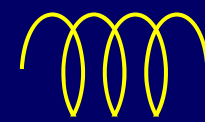
$$P_{avg} = \frac{1}{2} I_{max} V_{max} \qquad I_{max} = \frac{2P_{avg}}{V_{max}} = \frac{2400}{120} = 20\ A$$

Inductors 

Inductors: a solenoid used as a circuit element

Inductors enable circuits to have a resonance…

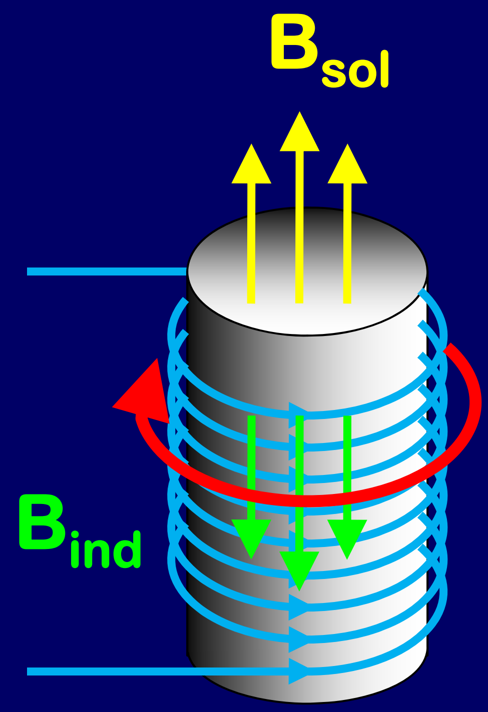


Self-Inductance 

Recall the solenoid cannon

- Changing current
- Changing B_{sol} field
- Changing Φ through itself! $\Phi \propto B_{sol} \propto I$
  - Φ proportional to I: $\Phi = LI$

"Inductance"



- Induced EMF (voltage)
  - Recall Faraday's law:

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t} = -L\frac{\Delta I}{\Delta t} = -L\frac{I_f - I_i}{t_f - t_i}$$

- Direction
  - Given by Lenz's Law
  - Opposes change in current!

Units: L = ε t / I
1 H = 1V-sec/amp

Physical Inductor

tries to maintain a constant I

$$L \equiv \frac{\Phi}{I}$$

Recall: $\Phi = NBA \cos\phi$

$$L = \frac{NBA}{I}$$

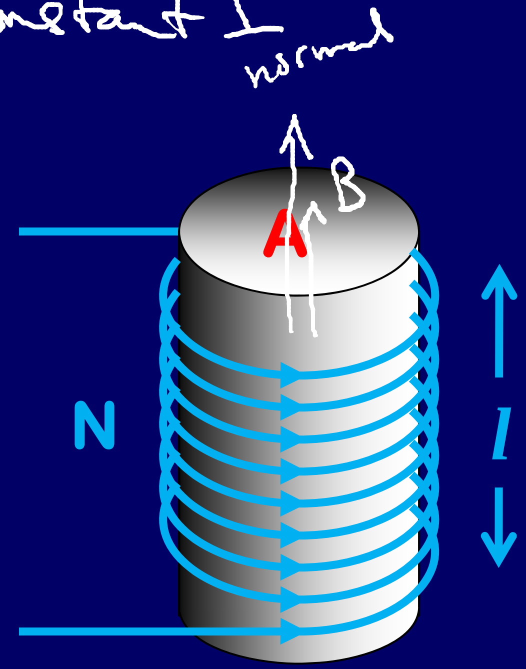
Recall: $B = \mu_o n I$

$$L = \frac{N\mu_0 n I A}{I}$$

$$L = N\mu_0 n A$$

$$L = \mu_0 n^2 l A$$

$$= \mu_0 \frac{N^2}{l} A$$

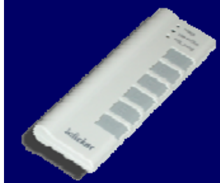


(# turns) = (# turns/meter) x (# meters)

$$N = n\, l$$

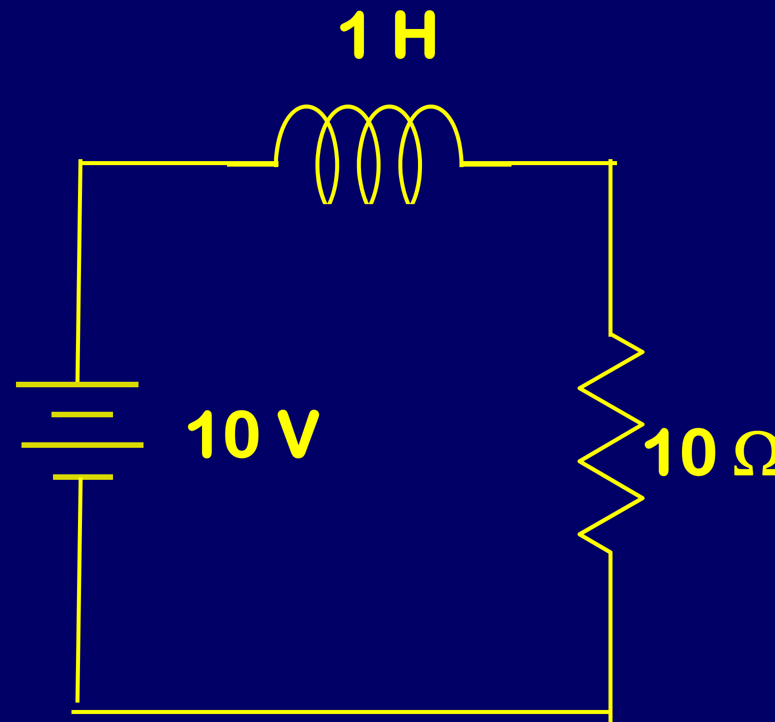
Energy stored:

$$U = \tfrac{1}{2} L I^2$$



ACT: Inductors

A 10 Ω resistor is wired is series with a 10 V battery and a 1 H inductor. What is the voltage across the inductor?



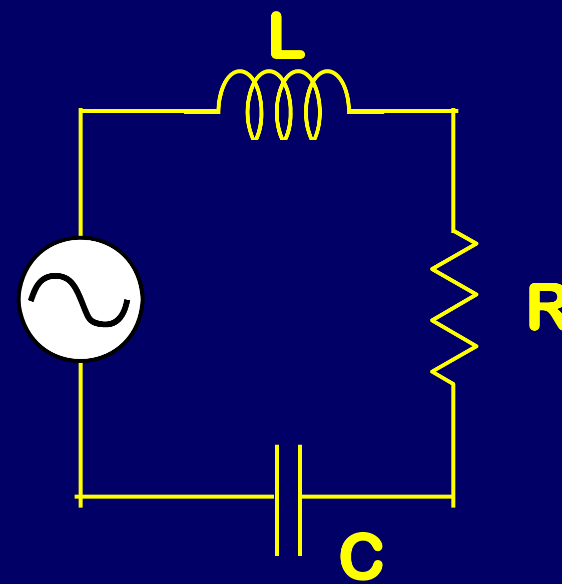
(A) 0 V **(B) 0.1V** **(C) 10 V**

$$|V_L| = L\frac{\Delta I}{\Delta t} = 0$$

RLC circuits

A circuit with an inductor, resistor, and capacitor in series!

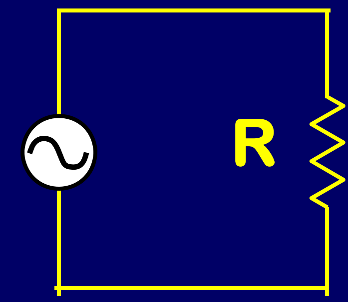
Used in:
- **Cell phones / radios**
- **Computers**
- **Watches / clocks**



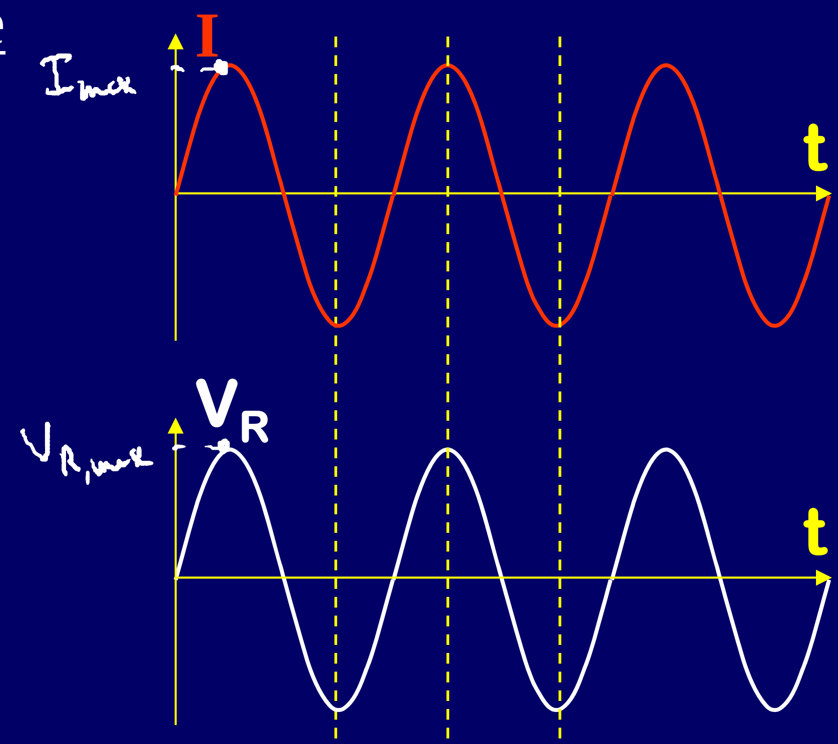
First: Understand each element individually

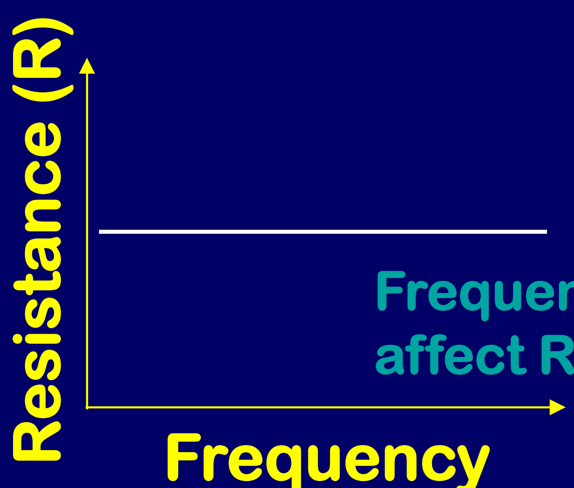
Resistors in AC circuit

$V_R = I\,R$ always true – Ohm's Law



- $V_{R,max} = I_{max}R$ $\quad V_{R,rms} = I_{rms}R$
- Voltage across resistor is "IN PHASE" with current.
  - V_R goes up and down <u>at the same times</u> as I does.

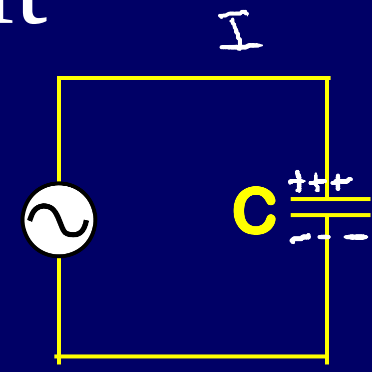




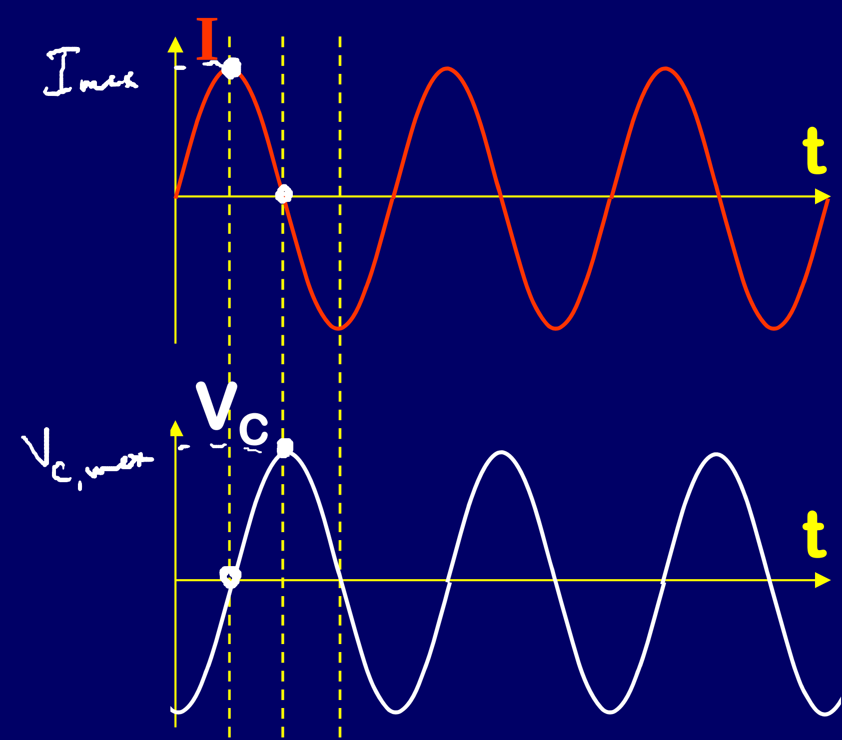
Frequency <u>does not</u> affect Resistance!

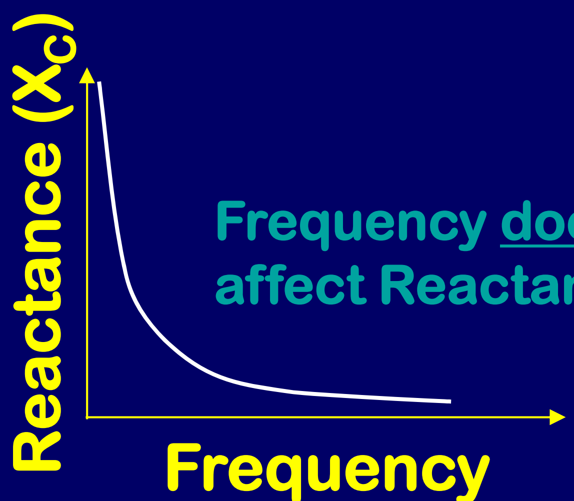
Capacitors in AC circuit

$V_C = Q/C$ always true



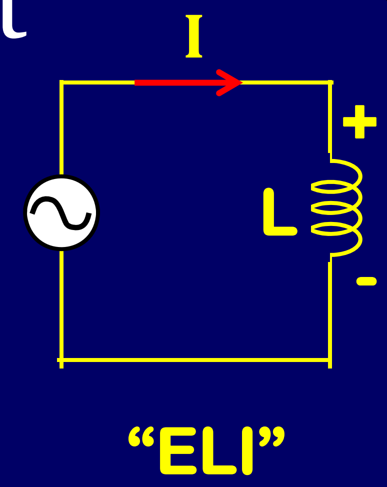
- $V_{C,max} = I_{max} X_C$
- Capacitive Reactance: $X_C = 1/(2\pi f C)$
- Voltage across capacitor "LAGS" current.
  - V_C goes up and down <u>just after</u> I does.





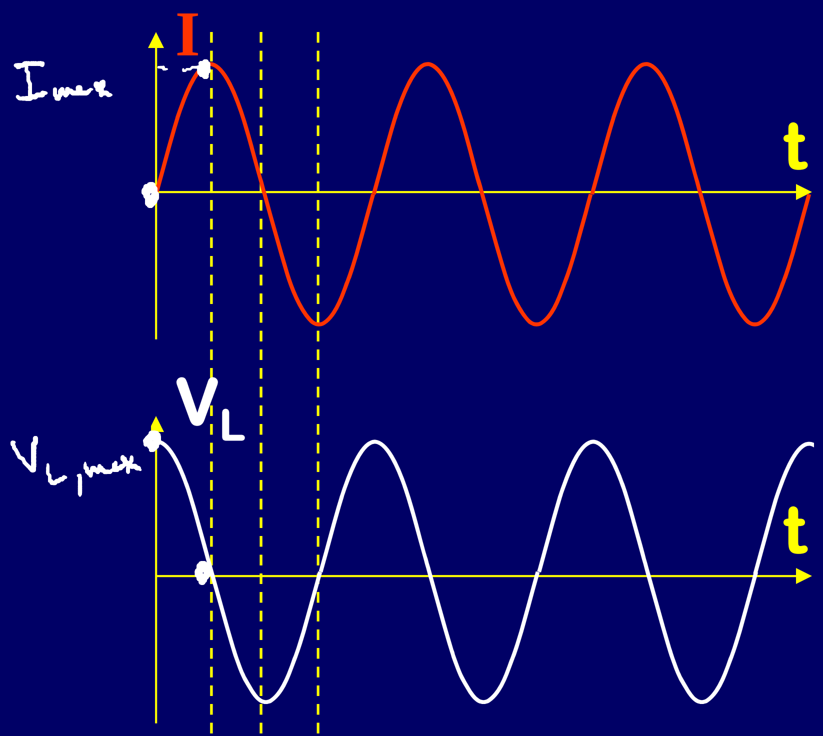
Frequency does affect Reactance!

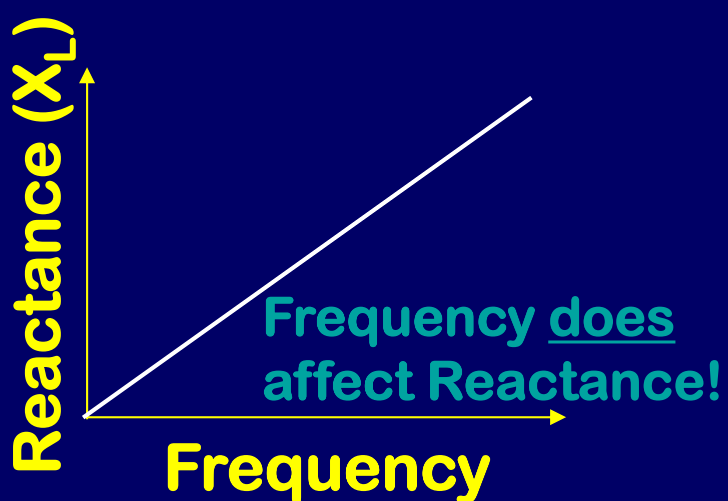
Inductors in AC circuit



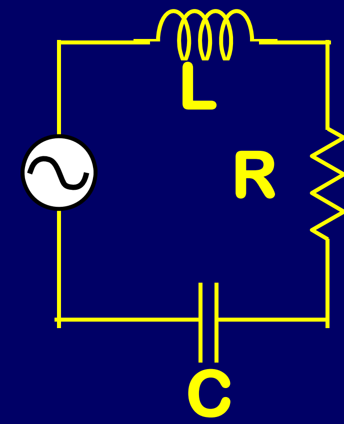
$V_L = +L(\Delta I)/(\Delta t)$ always true

- $V_{L,max} = I_{max} X_L$
- Inductive Reactance: $X_L = 2\pi f L$
- Voltage across inductor "LEADS" current.
  - V_L goes up and down just before I does.





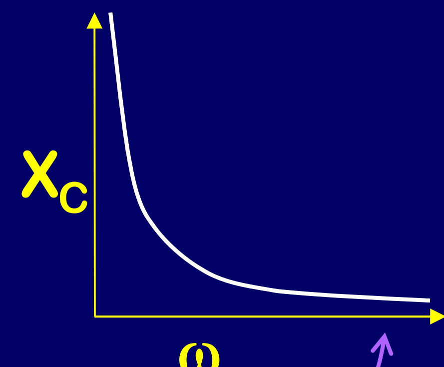
ACT/CheckPoints 3.1, 3.2



The capacitor can be ignored when…

(a) frequency is very large

(b) frequency is very small

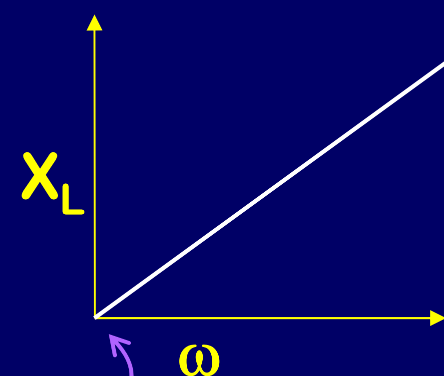


very large ω gives very small X_C

The inductor can be ignored when…

(a) frequency is very large

(b) frequency is very small



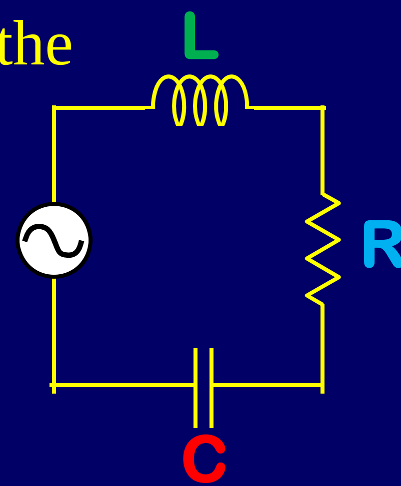
very small ω gives very small X_L

Example

AC Circuit Voltages

An AC circuit with R = 2 Ω, C = 15 mF, and L = 30 mH has a current $I(t) = 0.5 \sin(8\pi t)$ amps. Calculate the maximum voltage across R, C, and L.

2πF



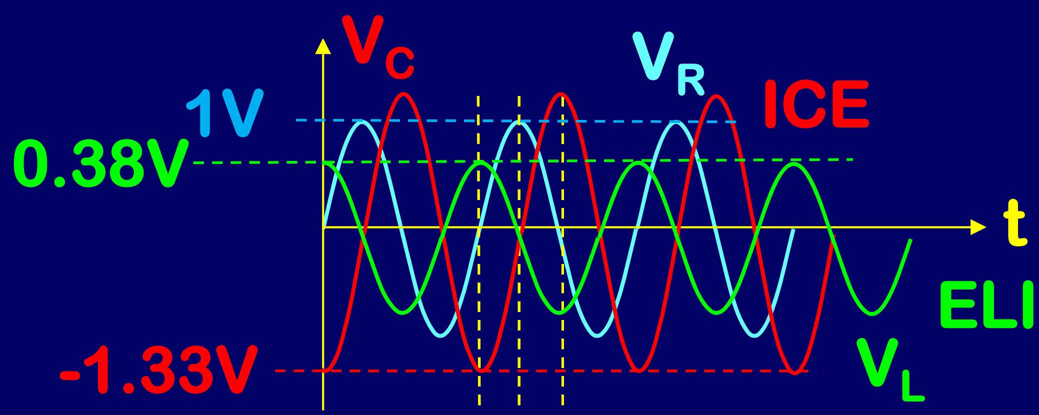
$V_{R,max} = I_{max} R = 0.5 \times 2 = 1$ Volt

$V_{C,max} = I_{max} X_C = 0.5 \times 1/(8\pi\times0.015) = 1.33$ Volts

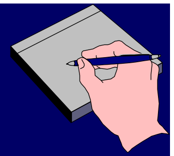
$V_{L,max} = I_{max} X_L = 0.5 \times 8\pi\times0.03 = 0.38$ Volts

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

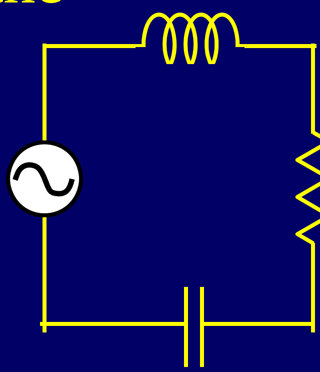
$$X_L = 2\pi f L = \omega L$$



ACT: AC Circuit Voltages



An AC circuit with R = 2 Ω, C = 15 mF, and L = 30 mH has a current I(t) = 0.5 sin(8π t) amps. Calculate the maximum voltage across R, C, and L.



Now the frequency is increased so I(t) = 0.5 sin(16π t). Which element's maximum voltage decreases?

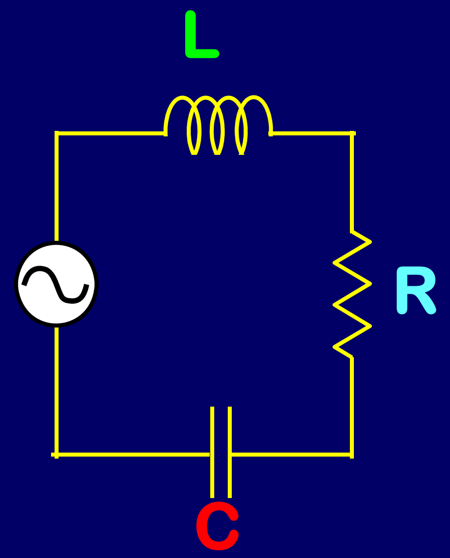
1) $V_{R,max}$ **Stays same: R doesn't depend on f**

2) $V_{C,max}$ **Decreases: $X_C = 1/(2\pi fC)$**

3) $V_{L,max}$ **Increases: $X_L = 2\pi f L$**

Summary so far…

- $I = I_{max}\sin(2\pi ft)$
- $V_R = I_{max}R\sin(2\pi ft)$
  - V_R in phase with I



- $V_C = I_{max}X_C\sin(2\pi ft - \pi/2)$
  - V_C lags I $\quad X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{\omega C}$

"ICE"

- $V_L = I_{max}X_L\sin(2\pi ft + \pi/2)$
  - V_L leads I $\quad X_L = 2\pi f L = \omega L$

"ELI"