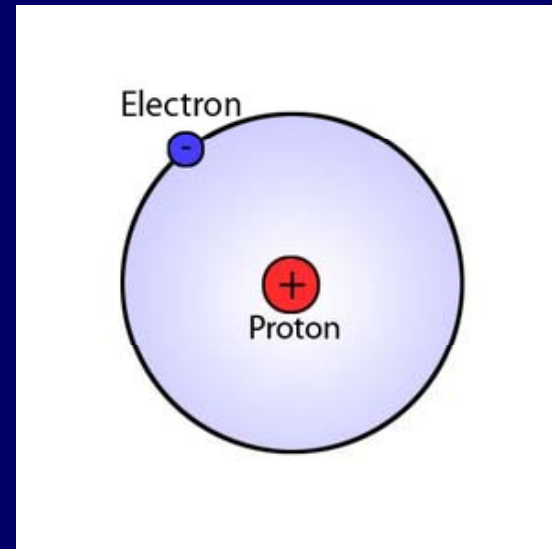


# Physics 102: Lecture 24

## Heisenberg Uncertainty Principle & Bohr Model of Atom



# Heisenberg Uncertainty Principle

Recall: Quantum Mechanics tells us outcomes of individual measurements are uncertain



$$\Delta p_y \Delta y \geq \frac{\hbar}{2}$$

$$\hbar = h/2\pi$$

Uncertainty in  
momentum (along y)

Uncertainty in  
position (along y)

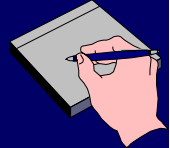


Rough idea: if we know momentum very precisely, we lose knowledge of location, and vice versa.

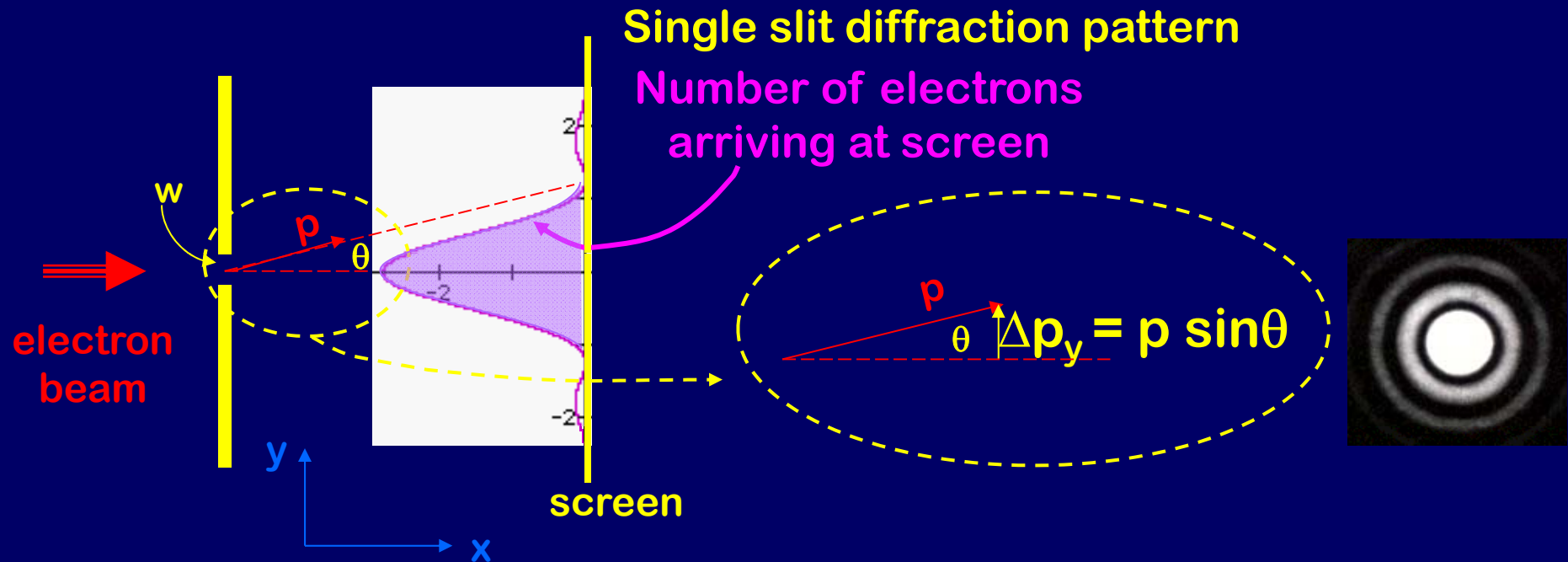
This “uncertainty” is fundamental: it arises because quantum particles behave like waves!

Example

# Electron diffraction



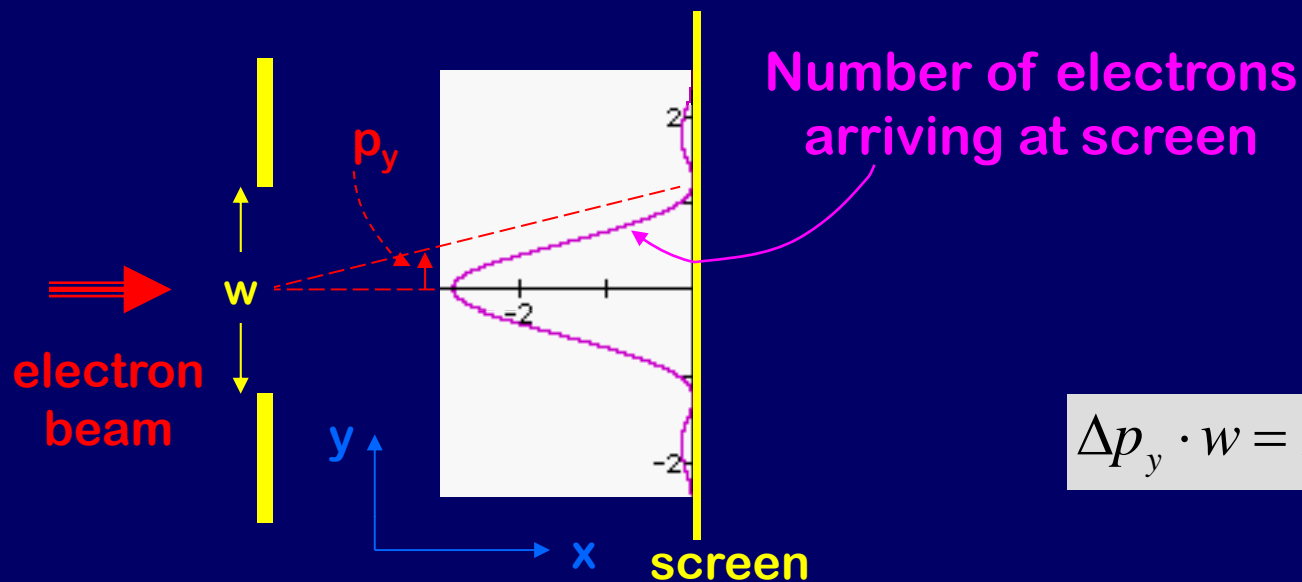
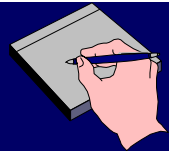
Electron beam traveling through slit will diffract



Recall single-slit diffraction 1<sup>st</sup> minimum:

$$\sin \theta = \lambda / w \quad w = \lambda / \sin \theta = \Delta y$$

$$\Delta p_y \Delta y = p \sin \theta \frac{\lambda}{\sin \theta} = \lambda p = h \quad \leftarrow \text{Using de Broglie } \lambda = h/p$$

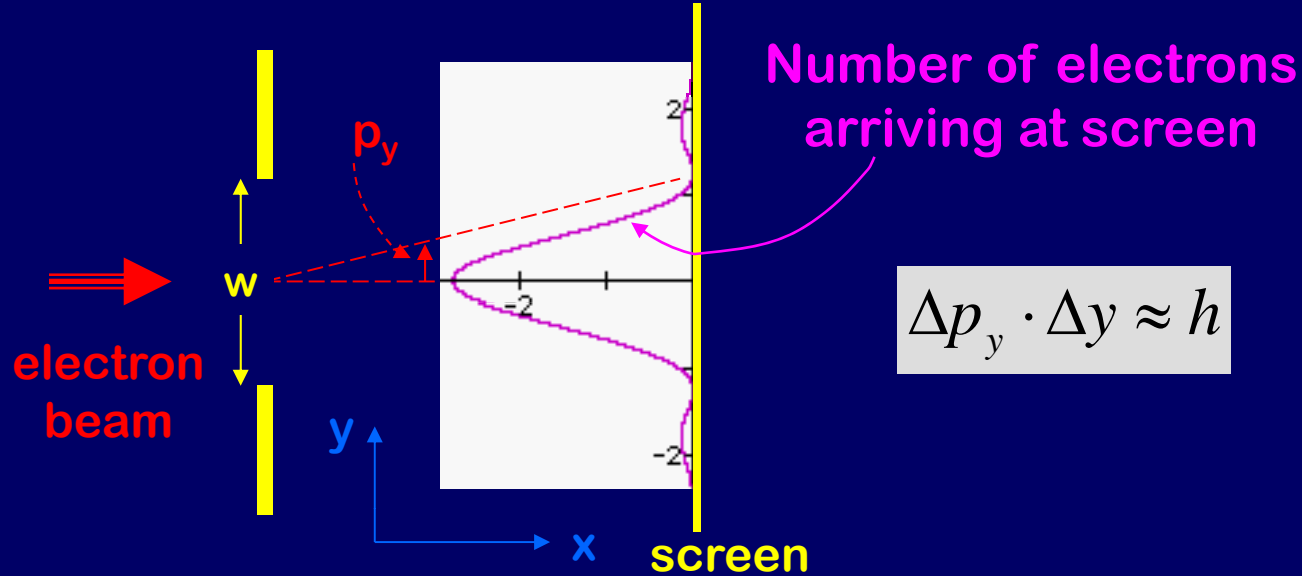


Electron entered slit with momentum along x direction and no momentum in the y direction. When it is diffracted it acquires a  $p_y$  which can be as big as  $h/w$ .

The “Uncertainty in  $p_y$ ” is  $\Delta p_y \approx h/w$ .

An electron passed through the slit somewhere along the y direction. The “Uncertainty in y” is  $\Delta y \approx w$ .

$$\Delta p_y \cdot \Delta y \approx h$$



If we make the slit narrower (decrease  $w = \Delta y$ ) the diffraction peak gets broader ( $\Delta p_y$  increases).

$$\sin\theta = \lambda/\Delta y$$

$$\Delta p_y \approx h/\Delta y$$

“If we know location very precisely, we lose knowledge of momentum, and vice versa.”



to be precise...

$$\Delta p_y \Delta y \geq \frac{\hbar}{2}$$

Of course if we try to locate the position of the particle along the x axis to  $\Delta x$  we will not know its x component of momentum better than  $\Delta p_x$ , where

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

and the same for z.

## Checkpoint 1

According to the H.U.P., if we know the x-position of a particle, we cannot know its:

- |                |                |
|----------------|----------------|
| (1) y-position | (2) x-momentum |
| (3) y-momentum | (4) Energy     |

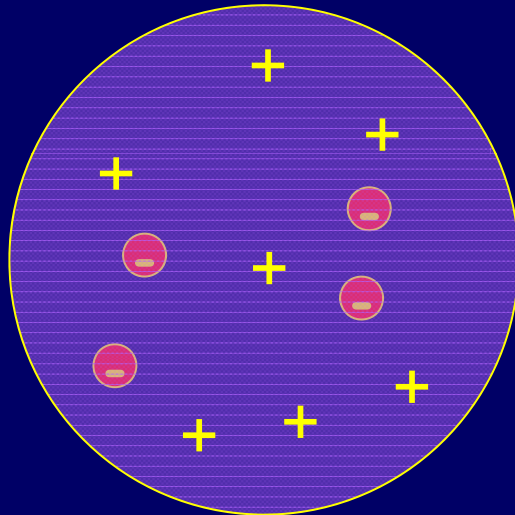
# Atoms

- Evidence for the nuclear atom
  - Bohr model of the atom
  - Spectroscopy of atoms
  - Quantum atom
- } Today
- } Next lecture

# Early Model for Atom

- **Plum Pudding**

- negative charges (electrons) in uniformly distributed cloud of positive charge like plums in pudding



**But how can you look inside an atom  $10^{-10}$  m across?**

Light (visible)

$$\lambda = 10^{-7} \text{ m}$$

Electron (1 eV)

$$\lambda = 10^{-9} \text{ m}$$

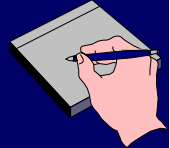
Helium atom

$$\lambda = 10^{-11} \text{ m}$$



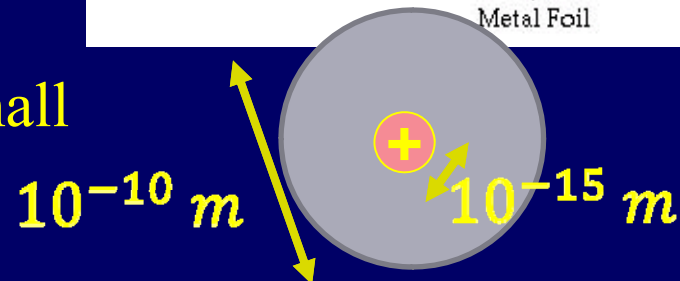
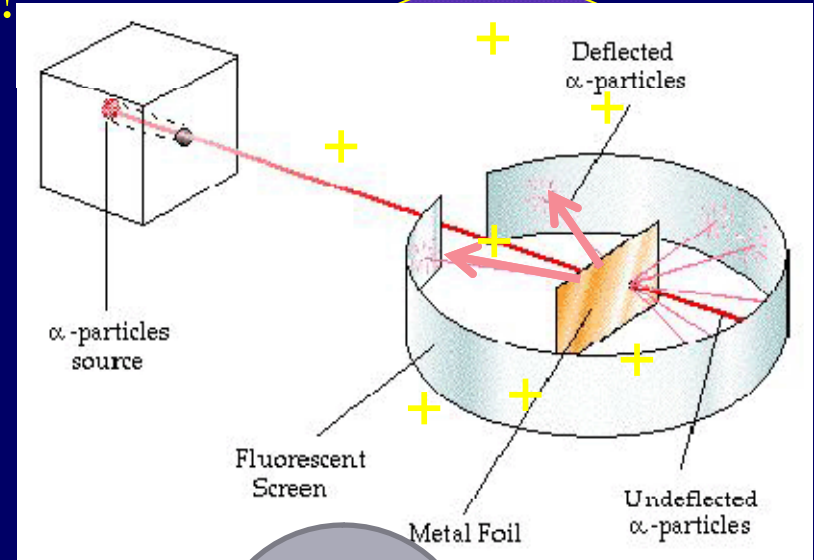
# Rutherford Scattering

1911: Scattering  $\text{He}^{++}$  (an “alpha particle”) atoms off of gold.  
Mostly go through, some scattered back!



Plum pudding theory:  
– electrons in “cloud” of uniformly distributed + charge  
→ electric field felt by alpha never gets too large

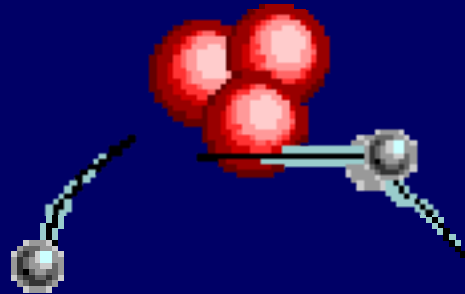
To scatter at large angles, need positive charge concentrated in small region (the nucleus)



Atom is mostly empty space with a small ( $r = 10^{-15} \text{ m}$ ) positively charged nucleus surrounded by “cloud” of electrons ( $r = 10^{-10} \text{ m}$ )

# Nuclear Atom (Rutherford)

Large angle scattering  Nuclear atom



**Classic nuclear atom is not stable!**

Electrons orbit nucleus  $\rightarrow$  constant centripetal acceleration

Accelerated charges radiate energy

Electrons will radiate and spiral into nucleus!

} Need  
quantum  
theory

**Early “quantum” model: Bohr**

# Bohr Model is Science fiction

The Bohr model is complete nonsense.

Electrons do not circle the nucleus in little planet-like orbits.

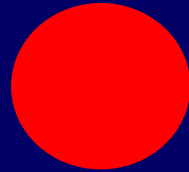
The assumptions injected into the Bohr model have no basis in physical reality.

**BUT** the model does get some of the numbers right for SIMPLE atoms...



# Hydrogen-Like Atoms

single electron with charge  $-e$

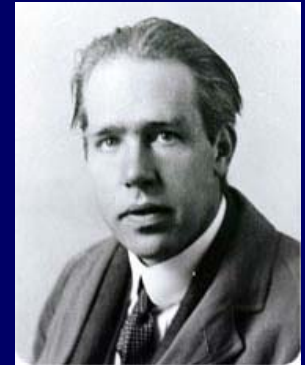


nucleus with charge  $+Ze$   
( $Z$  protons)

$$e = 1.6 \times 10^{-19} \text{ C}$$

Ex: H ( $Z=1$ ), He ( $Z=2$ ), Li ( $Z=3$ ), etc

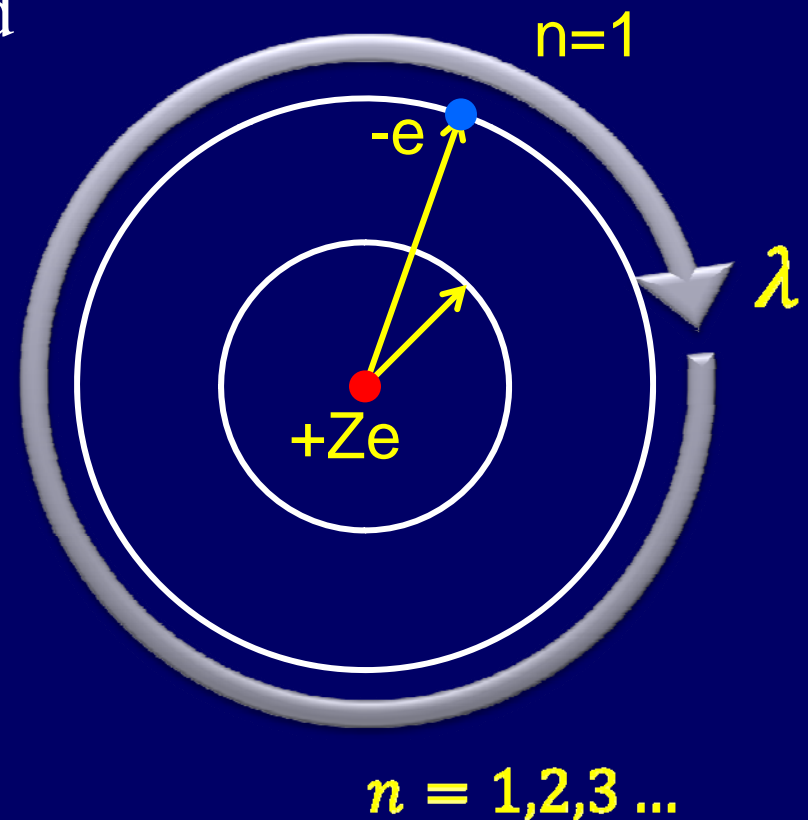
# The Bohr Model



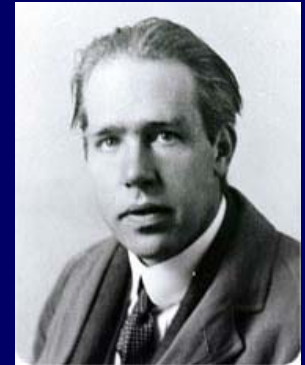
Electrons circle the nucleus in orbits

Only certain orbits are allowed

$$2\pi r = n\lambda$$



# The Bohr Model



Electrons circle the nucleus in orbits

Only certain orbits are allowed

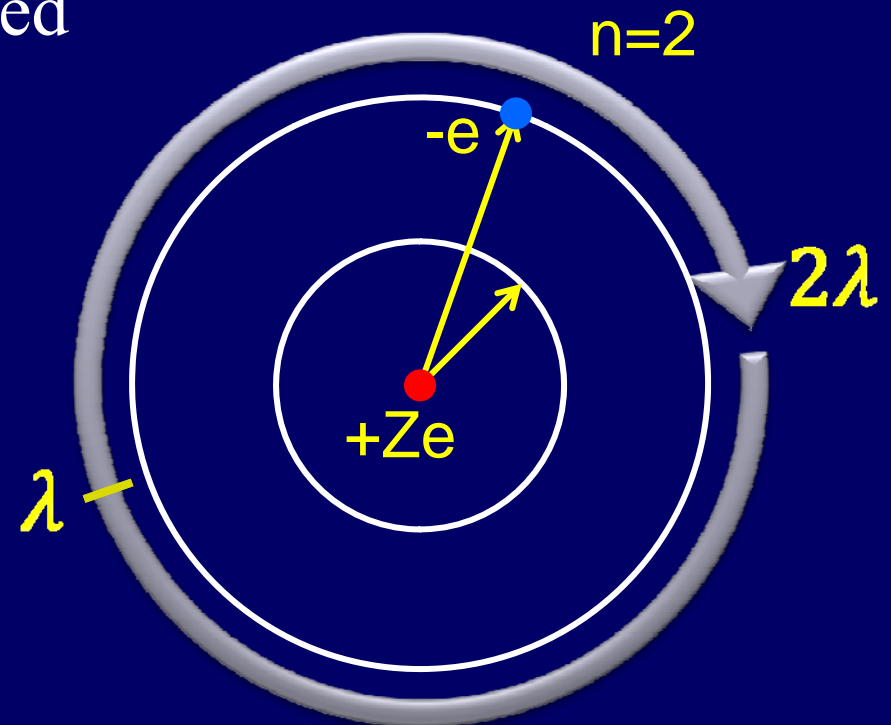
$$2\pi r = n\lambda$$
$$= nh/p$$

$$L = pr = nh/2\pi = n\hbar$$

Angular momentum is quantized

Energy is quantized:  $E = -13.6 \text{ eV } Z^2/n^2$   $n = 1, 2, 3 \dots$

$v$  is also quantized in the Bohr model!

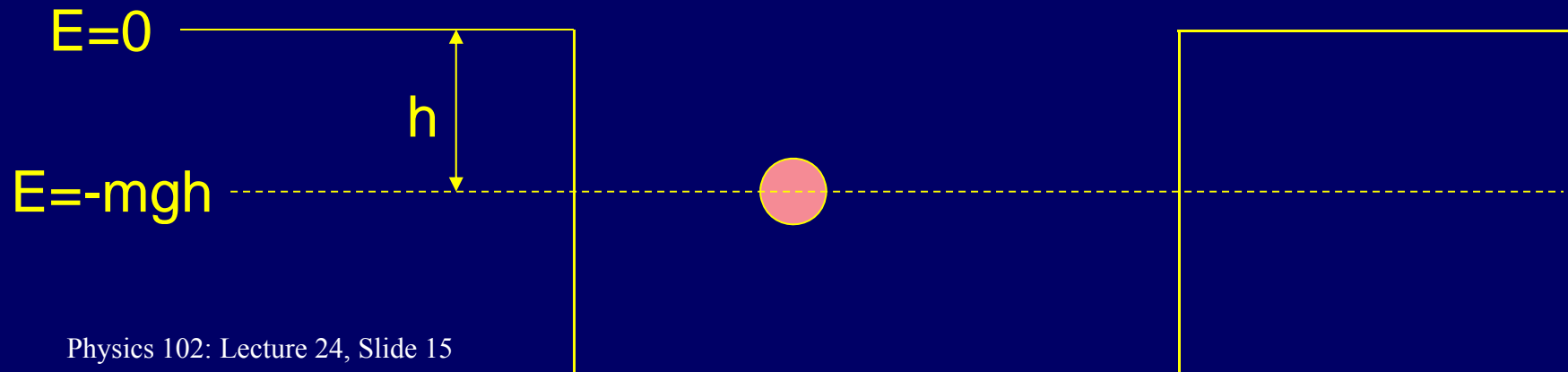


# An analogy: Particle in Hole

- Let's define energy so that a free particle has

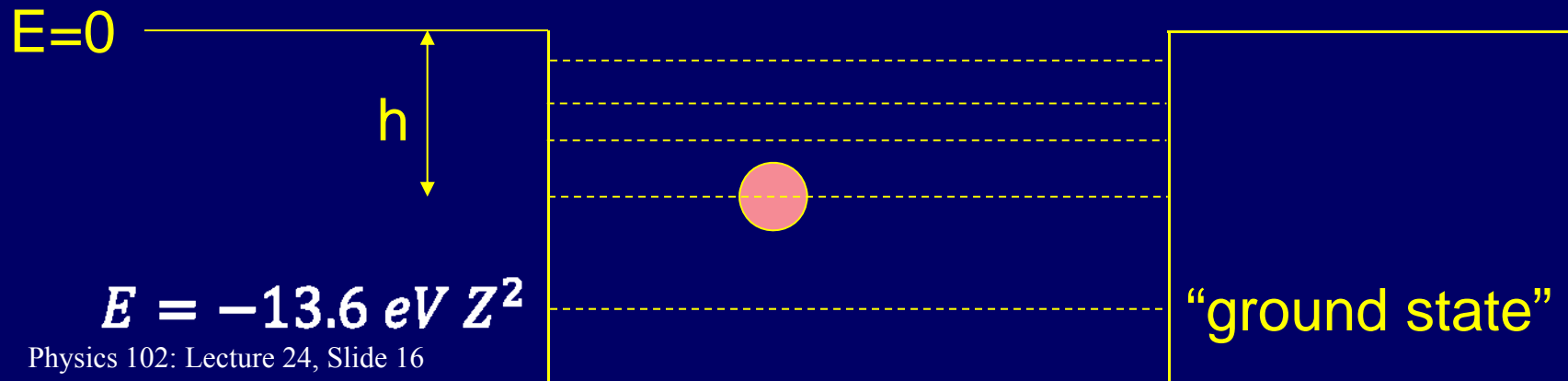
$$E = 0$$

- Consider a particle trapped in a hole
- To free the particle, need to provide energy  $mgh$
- Relative to the surface, energy =  $-mgh$ 
  - a particle that is “just free” has 0 energy



# An analogy: Particle in Hole

- Quantized Energy: only fixed discrete heights of particle allowed
- Lowest energy (deepest hole) state is called the “ground state”





# For Hydrogen-like atoms:

Energy levels (relative to a “just free”  $E=0$  electron):

$$E_n = -\frac{mk^2e^4}{2\hbar^2} \frac{Z^2}{n^2} \approx -\frac{13.6 \cdot Z^2}{n^2} \text{ eV} \quad (\text{where } \hbar \equiv h/2\pi)$$

Radius of orbit:

$$r_n = \left(\frac{h}{2\pi}\right)^2 \frac{1}{mke^2} \frac{n^2}{Z} = (0.0529 \text{ nm}) \frac{n^2}{Z}$$

# Checkpoint 2

$$r_n = \left(\frac{h}{2\pi}\right)^2 \frac{1}{mke^2} \frac{n^2}{Z} = \underbrace{(0.0529nm)}_{\text{Bohr radius}} \frac{n^2}{Z}$$

Bohr radius

If the electron in the hydrogen atom was 207 times heavier (a muon), the Bohr radius would be

- 1) 207 Times Larger
- 2) Same Size
- 3) 207 Times Smaller

$$\text{Bohr Radius} = \left(\frac{h}{2\pi}\right)^2 \frac{1}{mke^2}$$

This "m" is electron mass!



# ACT/Checkpoint 3

A single electron is orbiting around a nucleus with charge +3. What is its ground state ( $n=1$ ) energy? (Recall for charge +1,  $E = -13.6 \text{ eV}$ )

1)  $E = 9 (-13.6 \text{ eV})$

2)  $E = 3 (-13.6 \text{ eV})$

3)  $E = 1 (-13.6 \text{ eV})$

$3^2/1 = 9$

$$E_n = -13.6 \text{ eV} \frac{Z^2}{n^2}$$

**Note: This is LOWER energy since negative!**



# ACT: What about the radius?

$Z=3, n=1$

1. larger than H atom
2. same as H atom
3. smaller than H atom

$$r_n = \left(\frac{h}{2\pi}\right)^2 \frac{1}{mke^2} \frac{n^2}{Z} = (0.0529nm) \frac{n^2}{Z}$$

# Summary

- Bohr's Model gives accurate values for electron energy levels...
- But Quantum Mechanics is needed to describe electrons in atom.
- Next time: electrons jump between states by emitting or absorbing photons of the appropriate energy.

# Some (more) numerology

- 1 eV = kinetic energy of an electron that has been accelerated through a potential difference of 1 V

$$1 \text{ eV} = q\Delta V = 1.6 \times 10^{-19} \text{ J}$$

- $h$  (Planck's constant) =  $6.63 \times 10^{-34} \text{ J}\cdot\text{s}$

$$hc = 1240 \text{ eV}\cdot\text{nm}$$

- $m$  = mass of electron =  $9.1 \times 10^{-31} \text{ kg}$

$$mc^2 = 511,000 \text{ eV}$$

- $U = ke^2/r$ , so  $ke^2$  has units  $\text{eV}\cdot\text{nm}$  (like  $hc$ )

$$2\pi ke^2/(hc) = 1/137 \quad (\text{dimensionless})$$

“fine structure constant”