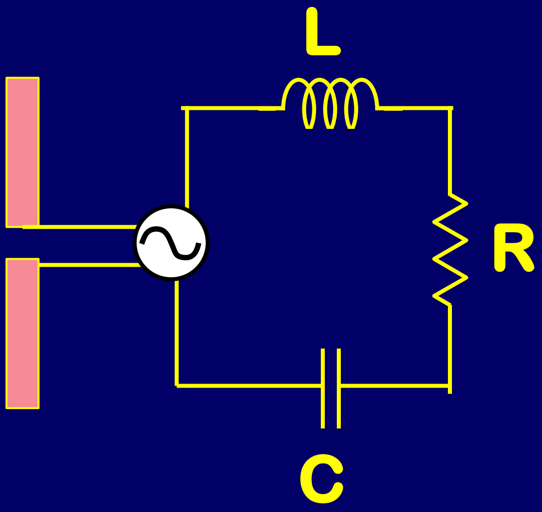
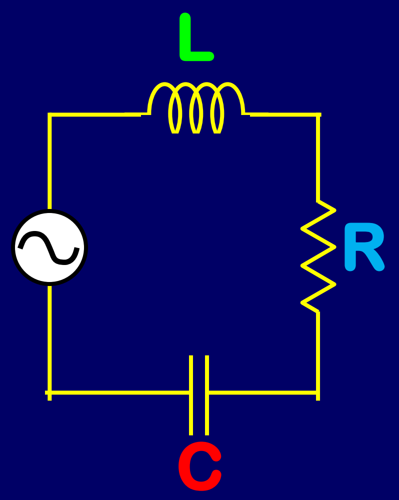
Physics 102: Lecture 13

RLC circuits & Resonance



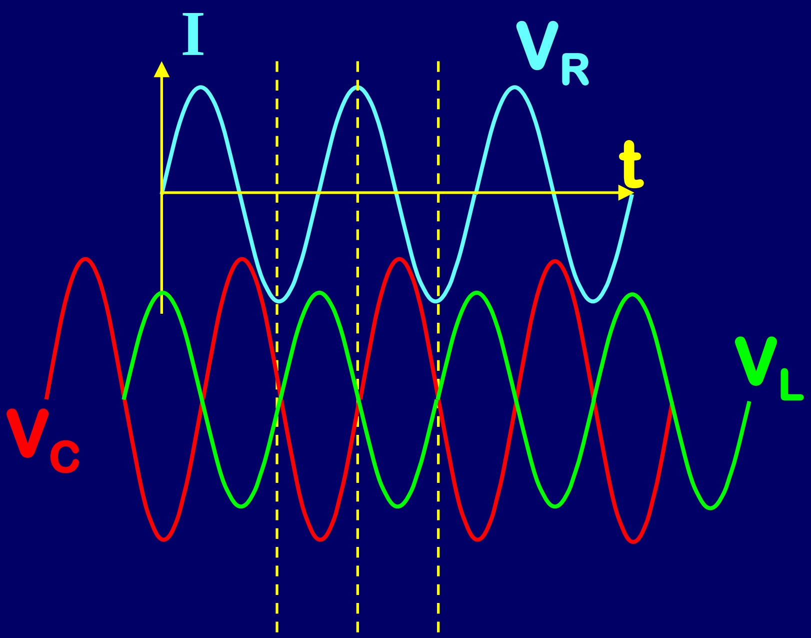
Review: AC Circuit



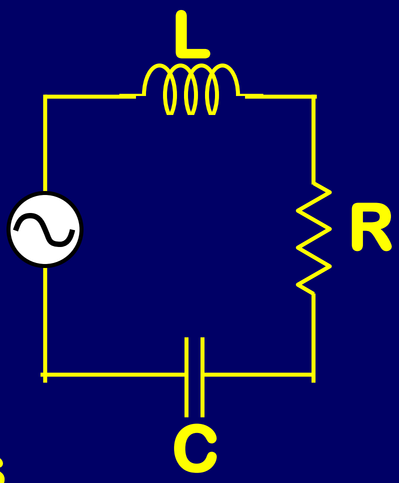
- $I = I_{max}\sin(2\pi ft)$
- $V_R = I_{max}R\sin(2\pi ft)$
  - V_R in phase with I

- $V_C = I_{max}X_C\sin(2\pi ft - \pi/2)$
  - V_C lags I **"ICE"**

- $V_L = I_{max}X_L\sin(2\pi ft + \pi/2)$
  - V_L leads I **"ELI"**



Peak & RMS values in AC Circuits (REVIEW)



When asking about RMS or Maximum values relatively simple expressions

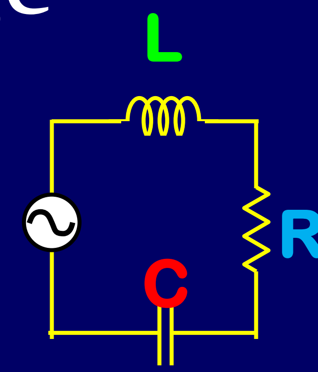
$$V_{R,\max} = I_{\max} R$$

$$V_{C,\max} = I_{\max} X_C \qquad X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

$$V_{L,\max} = I_{\max} X_L \qquad X_L = 2\pi f L = \omega L$$

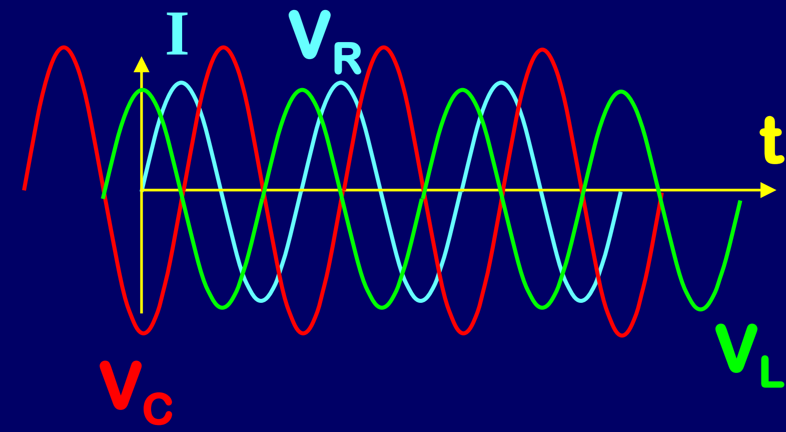
Kirchhoff: generator voltage

- Instantaneous voltage across generator (V_{gen}) must equal sum of voltage across all of the elements at all times:



$$V_{gen}(t) = V_R(t) + V_C(t) + V_L(t)$$

$$\mathbf{V_{gen,max} \neq V_{L,max} + V_{R,max} + V_{C,max}}$$



What is $V_{gen,max}$?

Define <u>impedance</u> Z: $V_{gen,max} \equiv I_{max} Z$

Like: V=IR

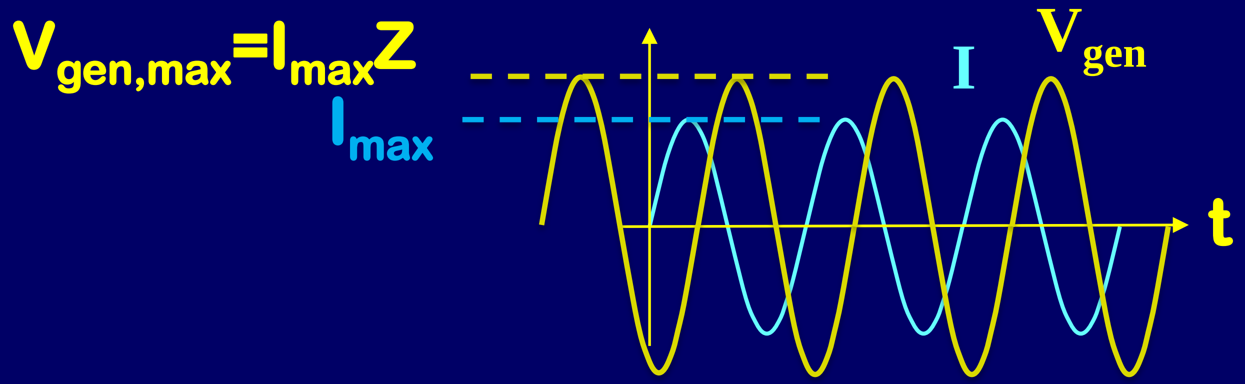
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

One last ingredient: is the generator voltage leading or lagging the current?

$$I = I_{max}\sin(2\pi ft)$$

$$V_{gen} = I_{max}Z\sin(2\pi ft + \phi)$$

Phase angle: $\tan(\phi) = \frac{(X_L - X_C)}{R}$



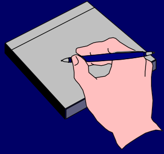
$\phi < 0$, voltage is lagging

Power in AC circuits

- The voltage generator supplies power.
  - Only resistor dissipates power.
  - Capacitor and Inductor store and release energy.
- $P(t) = I(t)V_R(t)$ oscillates so sometimes power loss is large, sometimes small.
- Average power dissipated by resistor:

$$\bar{P} = \tfrac{1}{2} I_{max} V_{R,max}$$
$$= \tfrac{1}{2} I_{max} V_{gen,max} \cos(\phi)$$
$$= I_{rms} V_{gen,rms} \cos(\phi)$$

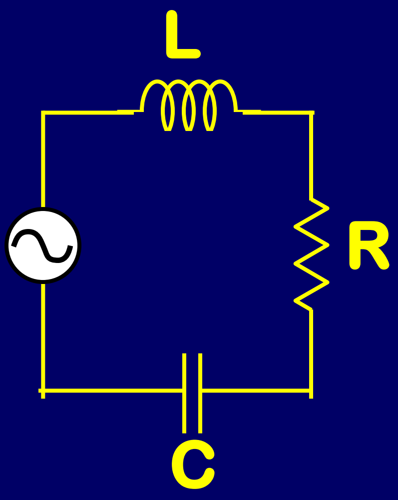
If there is only a resistor, $\phi = 0$



Example

Problem Time!

An AC circuit with R= 2 Ω, C = 15 mF, and L = 30 mH is driven by a generator with voltage V(t)=2.5 sin(8πt) Volts. Calculate the maximum current in the circuit, and the phase angle.



$I_{max} = V_{gen,max} / Z$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z =$$

$I_{max} =$ $\qquad\qquad =$ $\qquad$ Amps

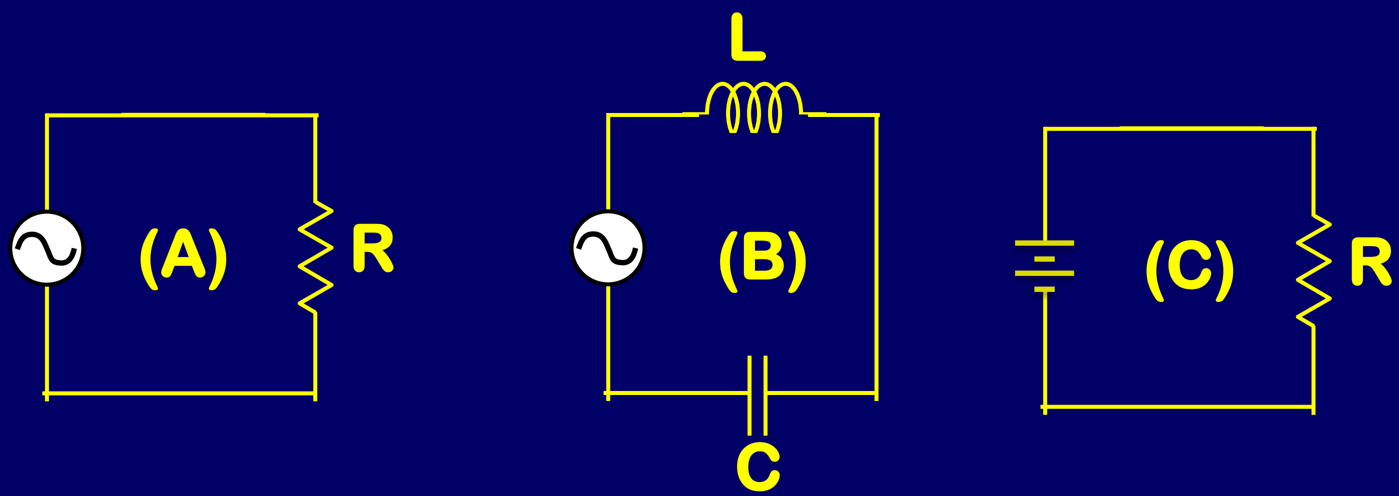
$$\tan(\phi) = \frac{X_L - X_C}{R} =$$

$$\phi =$$

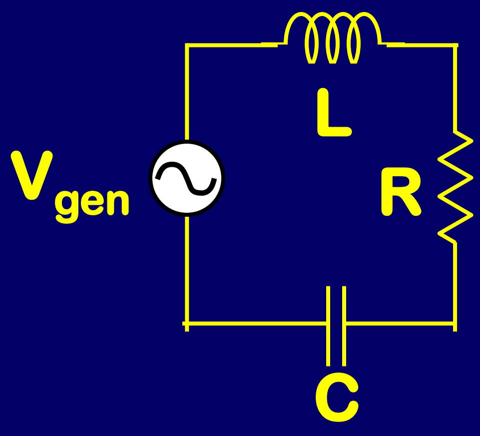
ACT: Power dissipation

Which one of these circuits dissipates the most power?

$V_{gen,max}$=10 V $V_{gen,max}$=100 V ε=1 V



Kirchhoff: generator voltage



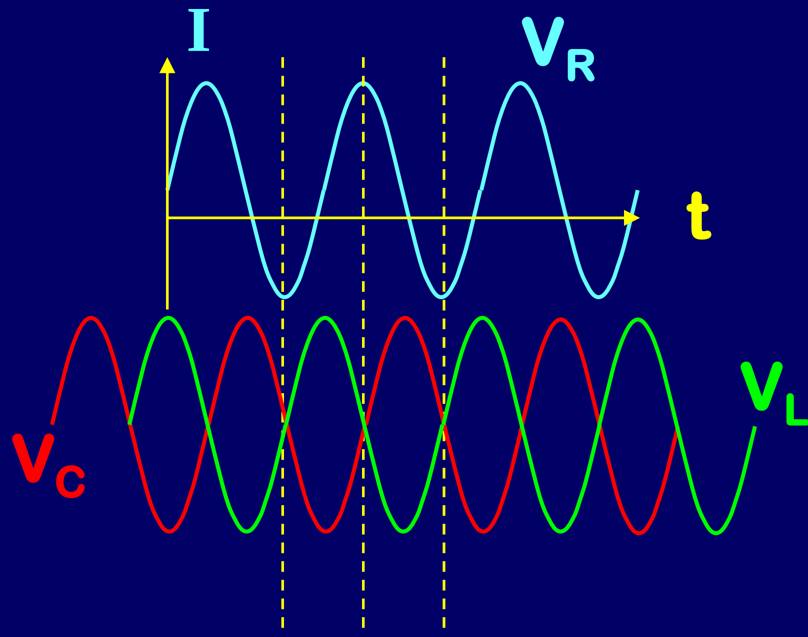
Write down Kirchhoff's Loop Equation:

$V_{gen}(t) = V_L(t) + V_R(t) + V_C(t)$ at every instant of time

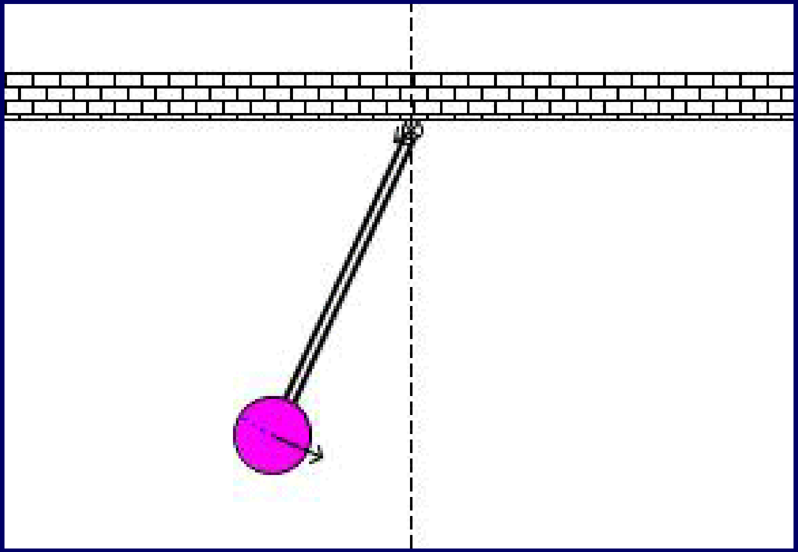
If the frequency is just right...

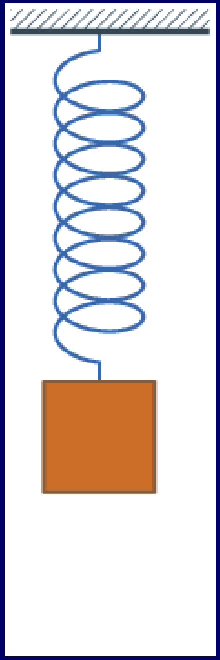
$$V_{gen}(t) = V_R(t)$$

Resonance!



Examples of mechanical resonance





Pendulum:

$$\omega_0 = \sqrt{g/l}$$

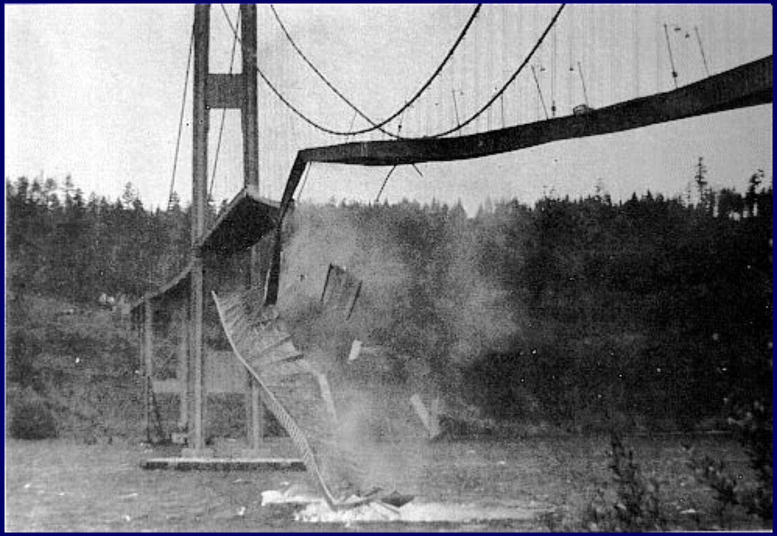
Mass on a spring:

$$\omega_0 = \sqrt{k/m}$$

Common features:

- **Energy converts between two forms (kinetic & potential)**
- **Weak input leads to a big response!**

Resonance!

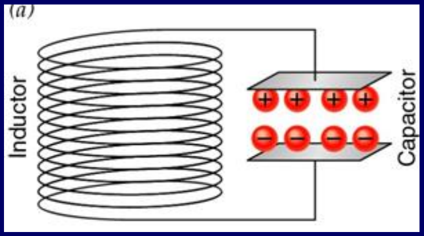


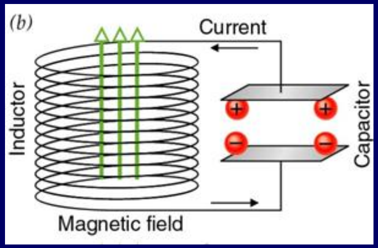
Tacoma Narrows Bridge

Millennium Bridge

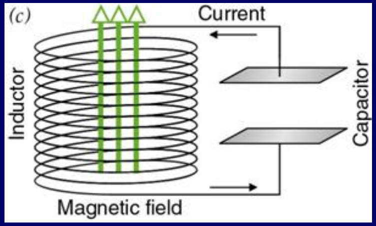
RLC circuit: Electrical resonance

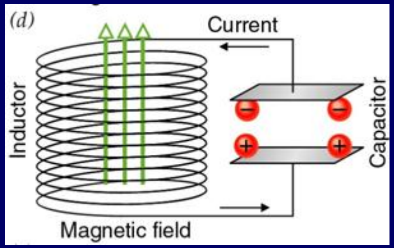
Electric energy

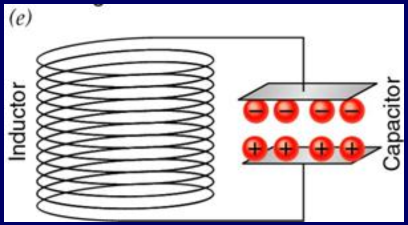




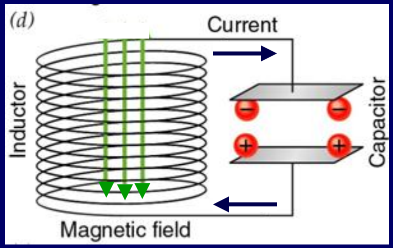
Magnetic energy

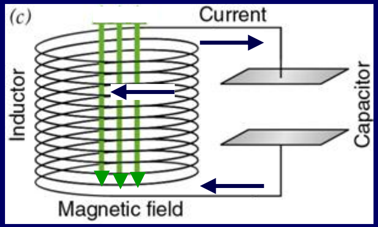




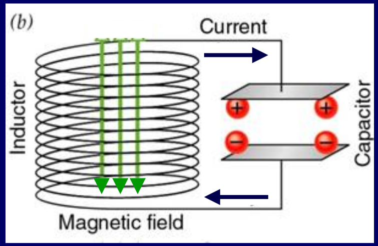


Electric energy

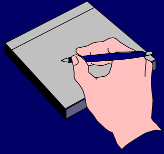




Magnetic energy



Resonance



R is independent of f

X_L increases with f

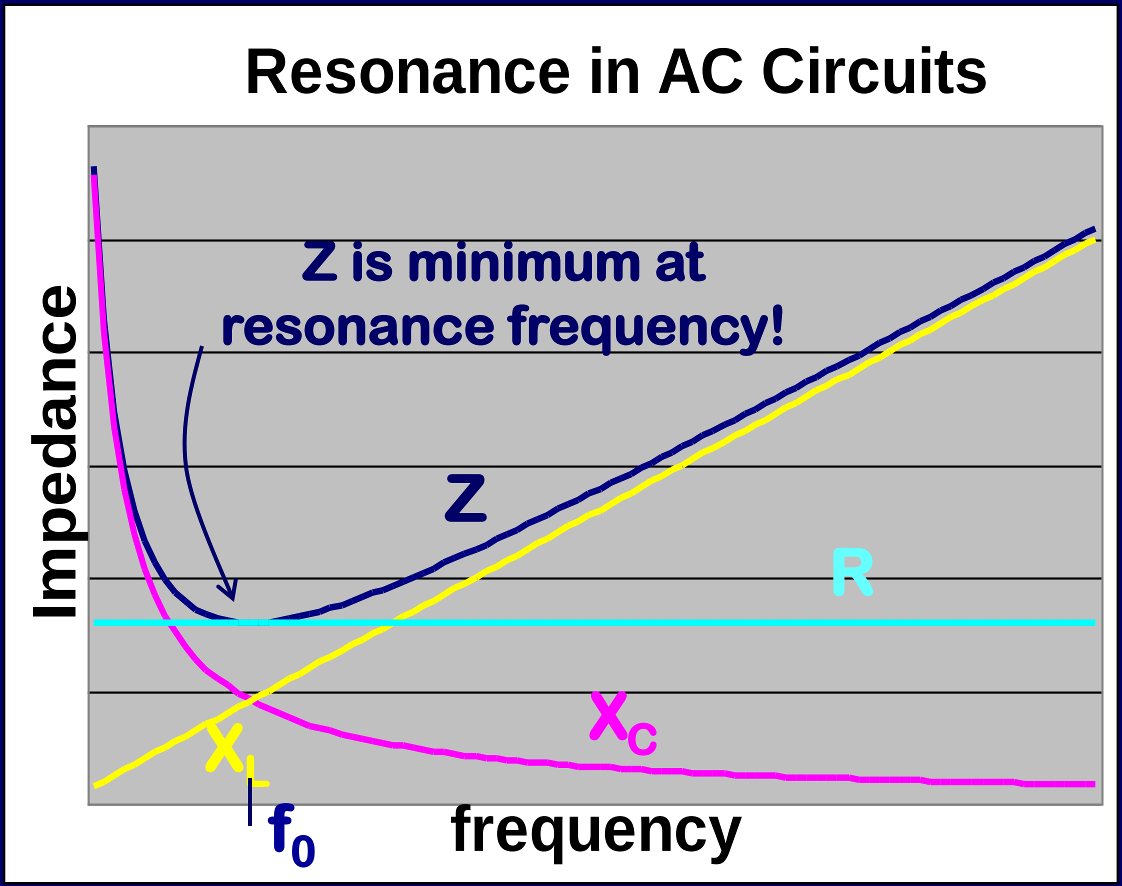
$$X_L = 2\pi f L$$

X_C decreases with f

$$X_C = 1/(2\pi f C)$$

Z: X_L and X_C subtract

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



Resonance: $X_L = X_C$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Resonance

R is independent of f

X_L increases with f

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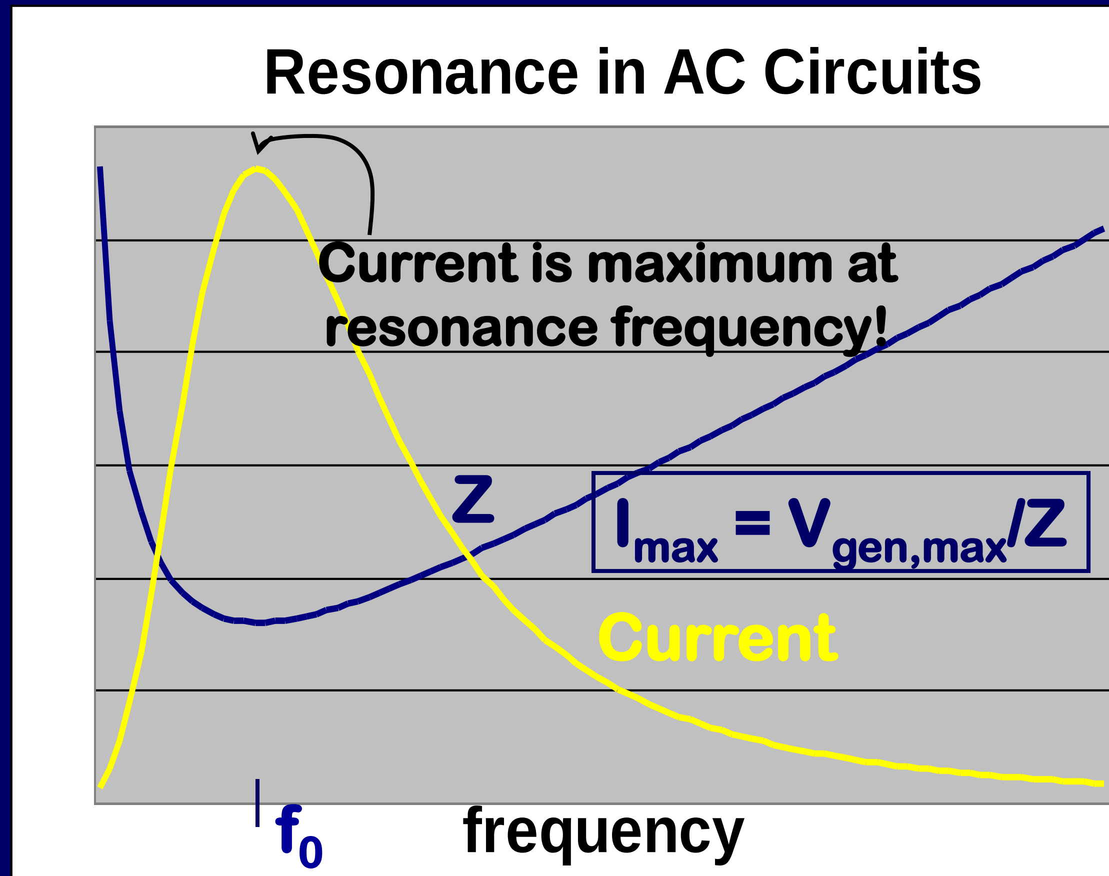
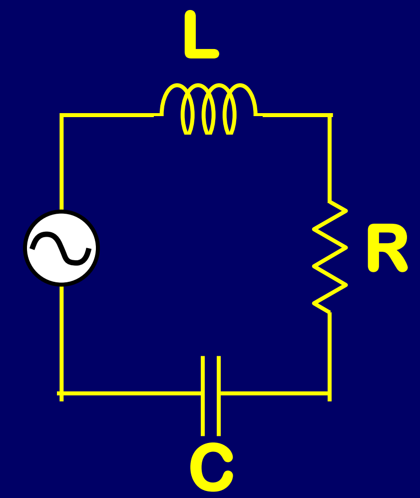


Resonance: $X_L = X_C$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

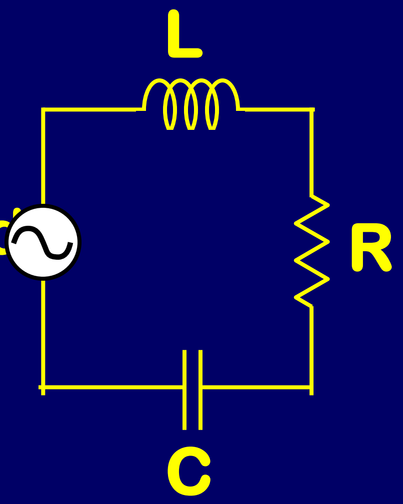
ACT: Resonance



The AC circuit to the right is being driven at its resonance frequency. Compare the maximum voltage across the capacitor with the maximum voltage across the inductor.

1) $V_{C,max} > V_{L,max}$
2) $V_{C,max} = V_{L,max}$
3) $V_{C,max} < V_{L,max}$
4) Depends on R

CheckPoint 14.1



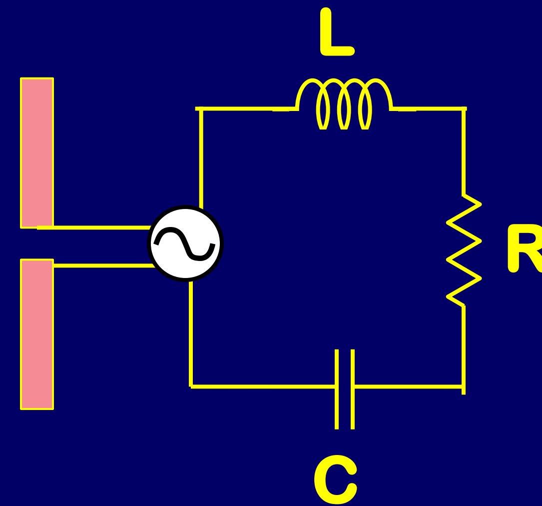
As the frequency of the circuit is either raised above or lowered below the resonant frequency, the impedance of the circuit:

Always increases

Only increases for lowering the frequency

Only increases for raising the frequency

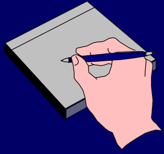
What is it good for?



- Current through circuit depends on frequency (maximum at resonance frequency f_o)
  - Radio receiver
  - Stereo equalizer
  - NMR/MRI

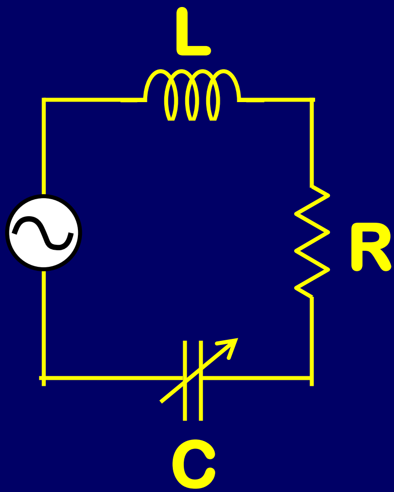
  - Picks out radio station freq f_o
  - Picks out music freq f_o
  - Picks out signal from protons at f_o

Example



Resonance in Radios

An AC circuit with R= 2 Ω, L = 0.30 μH and variable capacitance is connected to an antenna to receive radio signals at the resonance frequency. If you want to listen to music broadcasted at 96.1 MHz, what value of C should be used?



$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

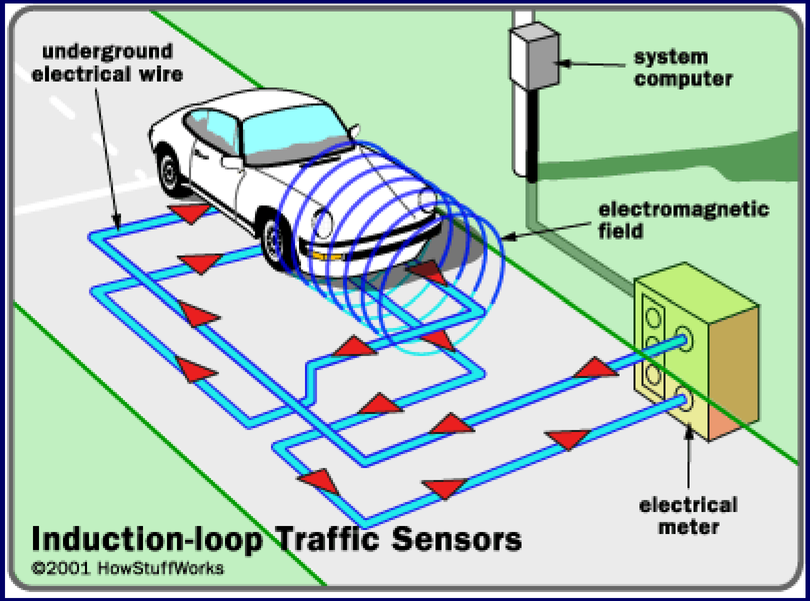
$$\boldsymbol{C} = \qquad = \qquad = \qquad \boldsymbol{F}$$

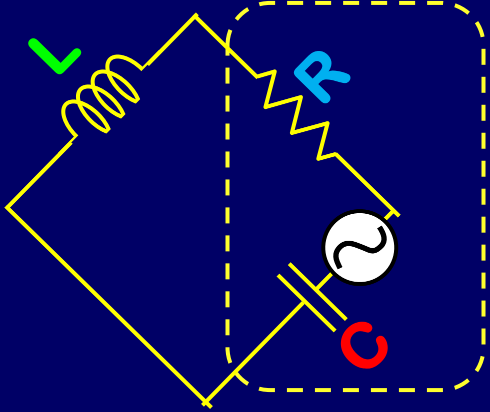
ACT: Radios

Your radio is tuned to FM 96.1 MHz and want to change it to FM 105.9 MHz, which of the following will work.

1. Increase Capacitance
2. Decrease Capacitance
3. Neither, you need to change R

Another use for RLC circuits: traffic sensors





AC Summary

Resistors: $V_{R,max} = I_{max} R$

In phase with I

Capacitors: $V_{C,max} = I_{max} X_C$ $X_c = 1/(2\pi f C)$

Lags I

Inductors: $V_{L,max} = I_{max} X_L$ $X_L = 2\pi f L$

Leads I

Generator: $V_{gen,max} = I_{max} Z$ $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Can lead or lag I $\tan(\phi) = (X_L - X_C)/R$

Power is only dissipated in resistor:

$$\bar{P} = \tfrac{1}{2} I_{max} V_{gen,max} \cos(\phi)$$