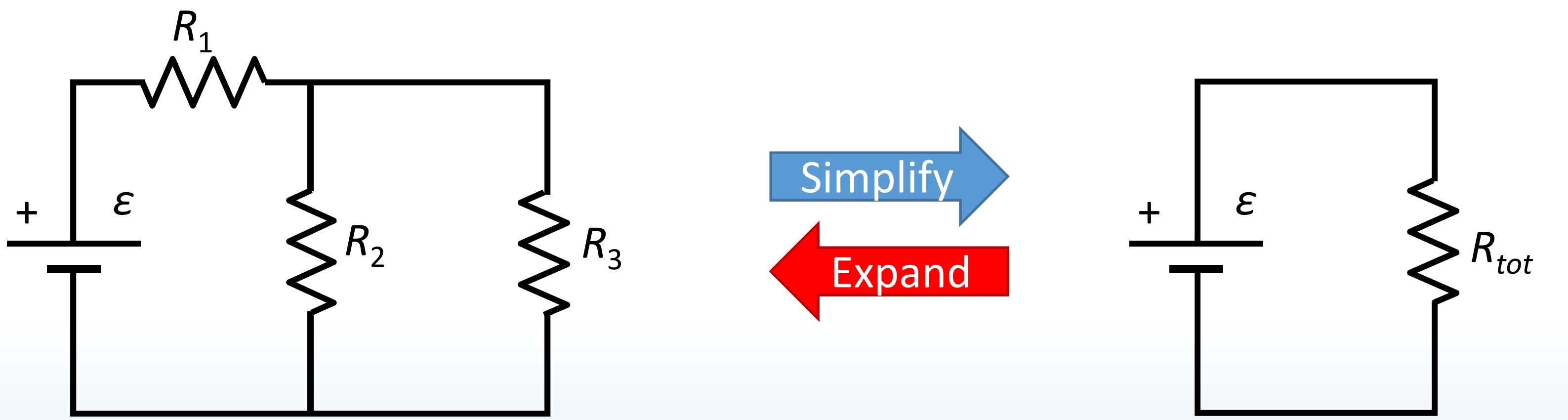
Phys 102 – Lecture 8

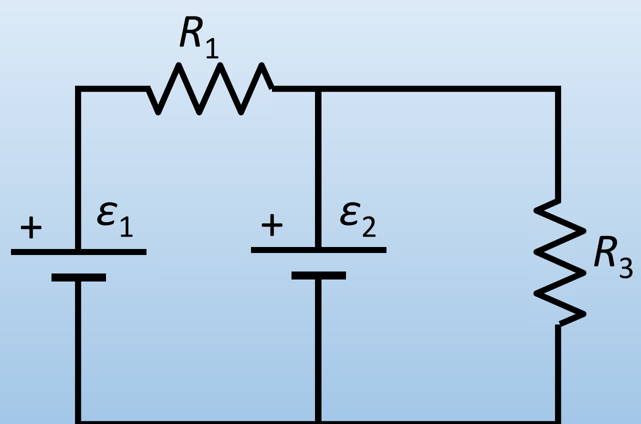
Circuit analysis and Kirchhoff's rules

Recall from last time...

We solved circuits like... by combining series & parallel components



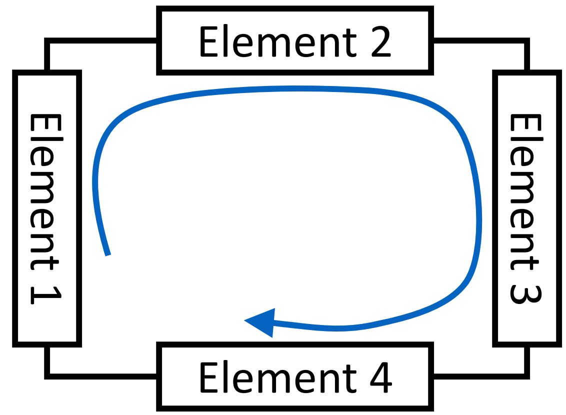
What about a circuit like...



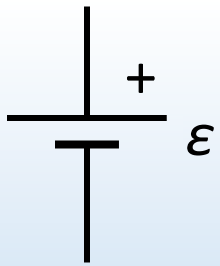
Kirchhoff's loop rule

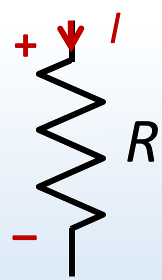
Voltages around a loop sum to zero

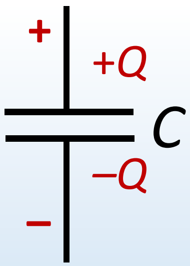
$$\sum \Delta V = 0$$



Is voltage positive or negative?







Batteries: + end is always at higher potential

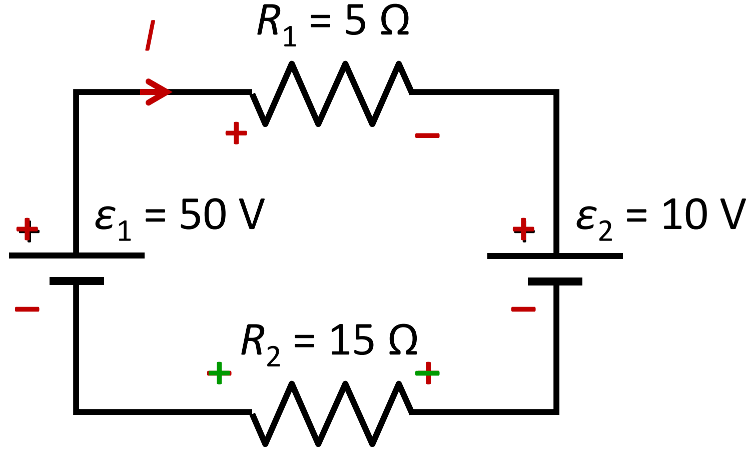
Resistors: higher/lower potential depends on current direction

Capacitors: higher/lower potential depends on which plate has $+Q/-Q$

Label +/– for higher/lower electric potential

Go around loop and write $+V_{element}$ if electric potential increases $-V_{element}$ if it decreases

Calculation: single loop practice



Calculate the current *I* in the circuit

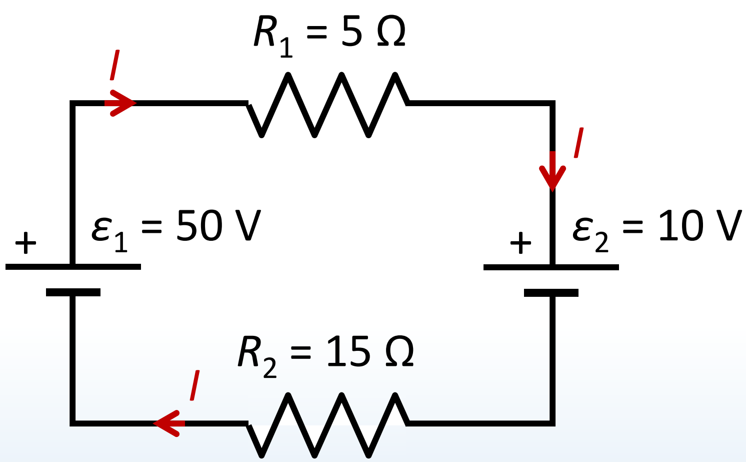
What if we go around the loop the "wrong" way?

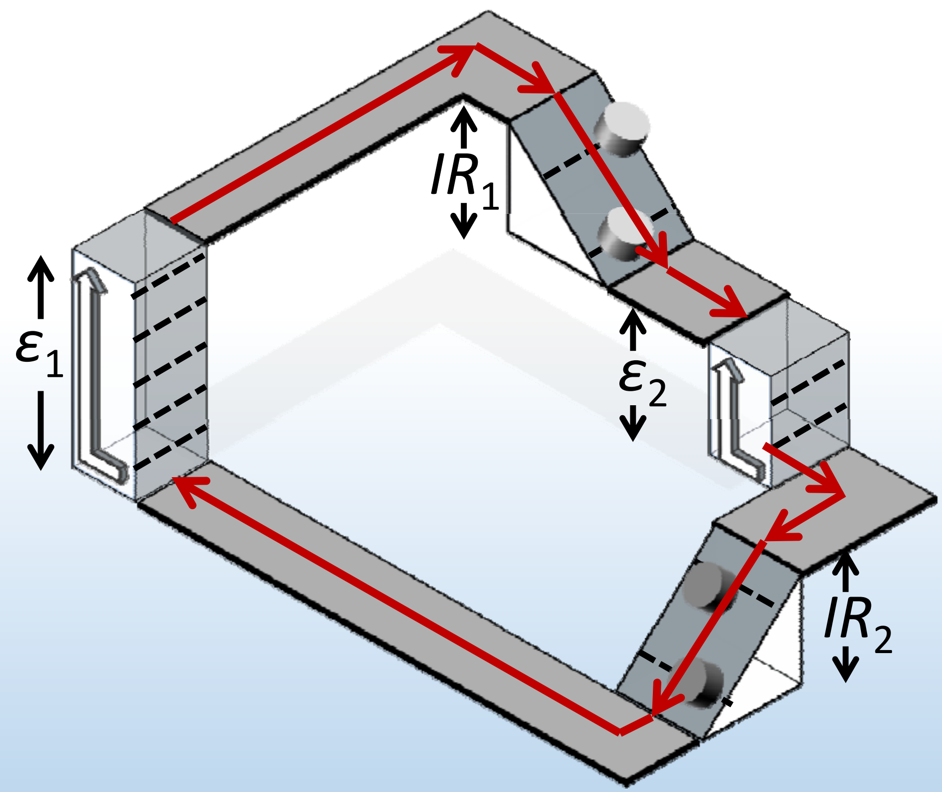
What if we're not given the current direction?

What if we pick the "wrong" direction?

Calculation: single loop practice

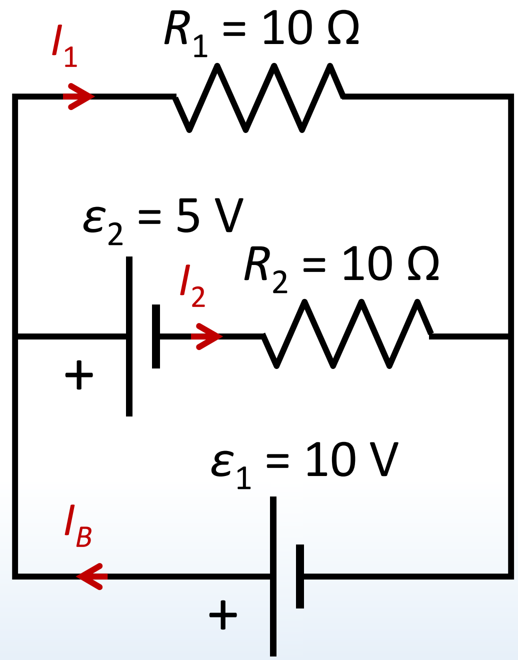
How can the current be driven opposite battery 2?





ACT: Checkpoint 1.1

Calculate the current through R_1.



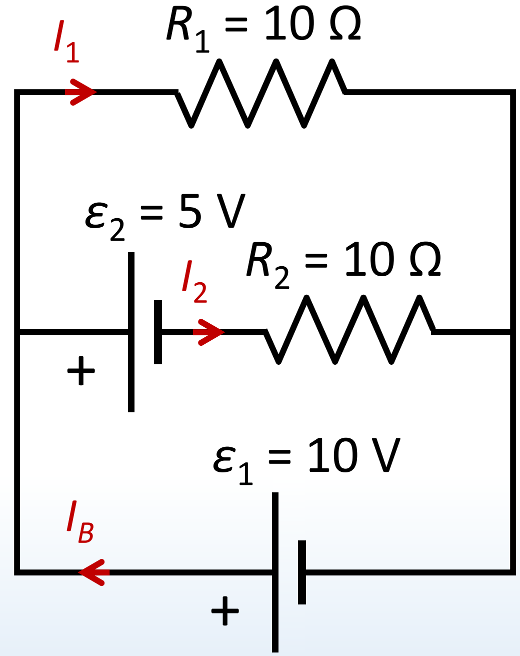
A. $I_1 = 0.5$ A

B. $I_1 = 1.0$ A

C. $I_1 = 1.5$ A

ACT: Checkpoint 1.2

Calculate the current through R_2.



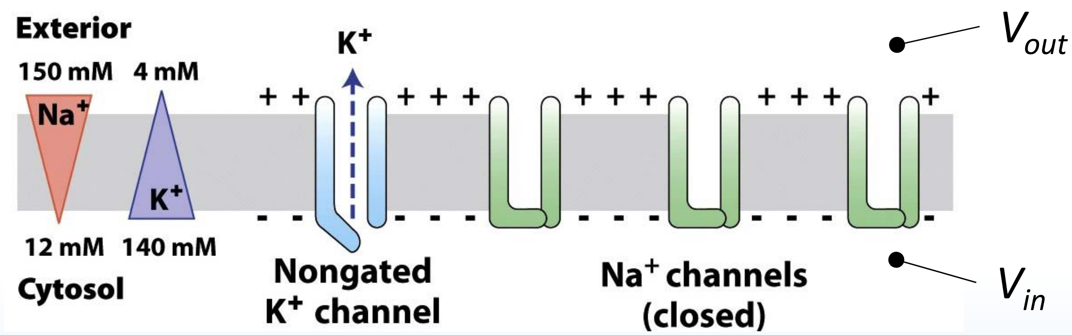
A. $I_2 = 0.5$ A

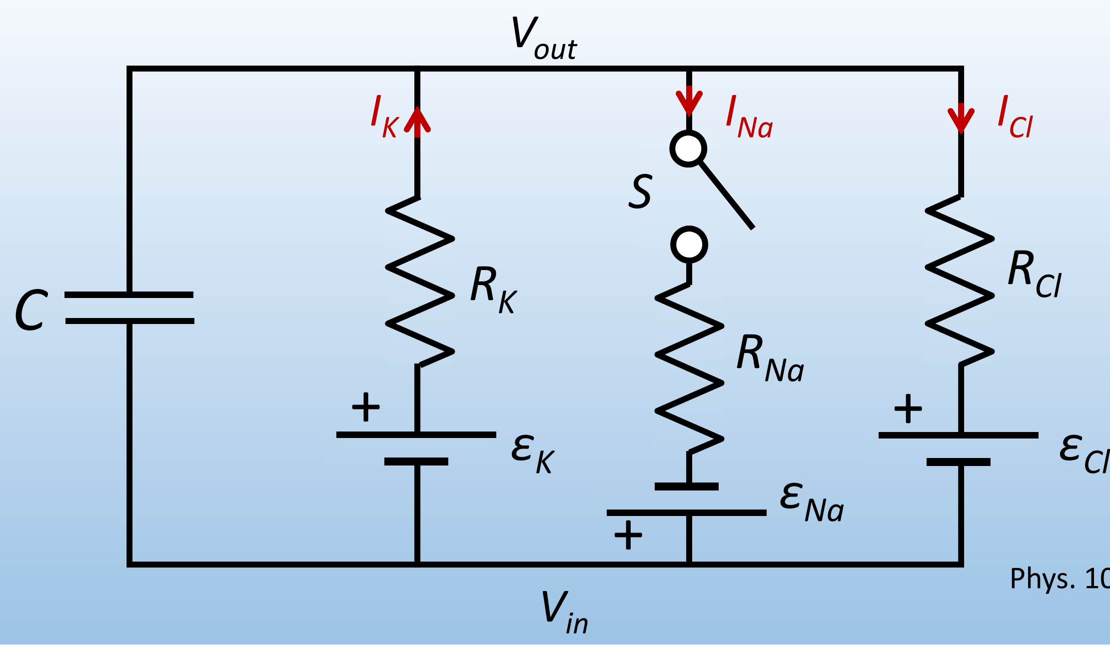
B. $I_2 = 1.0$ A

C. $I_2 = 1.5$ A

Nerve cell equivalent circuit

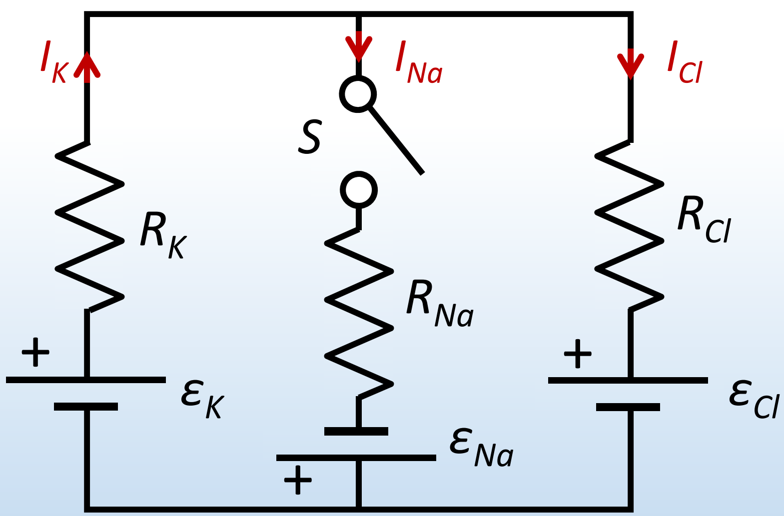
Neurons have different types of ion channels (K^+, Na^+, and Cl^-) that pump current into and out of cell – act like batteries!





ACT: loop

Na$^+$ channels have a "gate" (represented by the switch *S*) that allows or blocks ion flow. In its resting state, a Na$^+$ channel is shut (i.e. switch *S* is open). Which equation is correct?



A. $+\varepsilon_K - I_K R_K - I_K R_{Cl} - \varepsilon_{Cl} = 0$

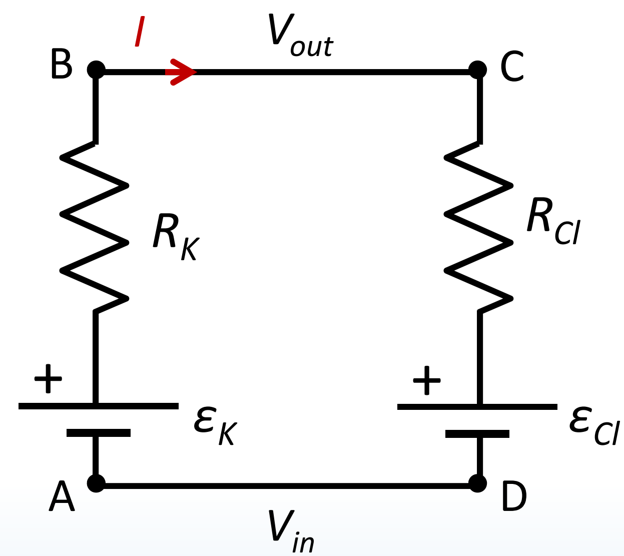
B. $+\varepsilon_K - I_K R_K - I_{Na} R_{Na} - \varepsilon_{Na} = 0$

C. $+\varepsilon_K + I_K R_K - I_{Cl} R_{Cl} - \varepsilon_{Cl} = 0$

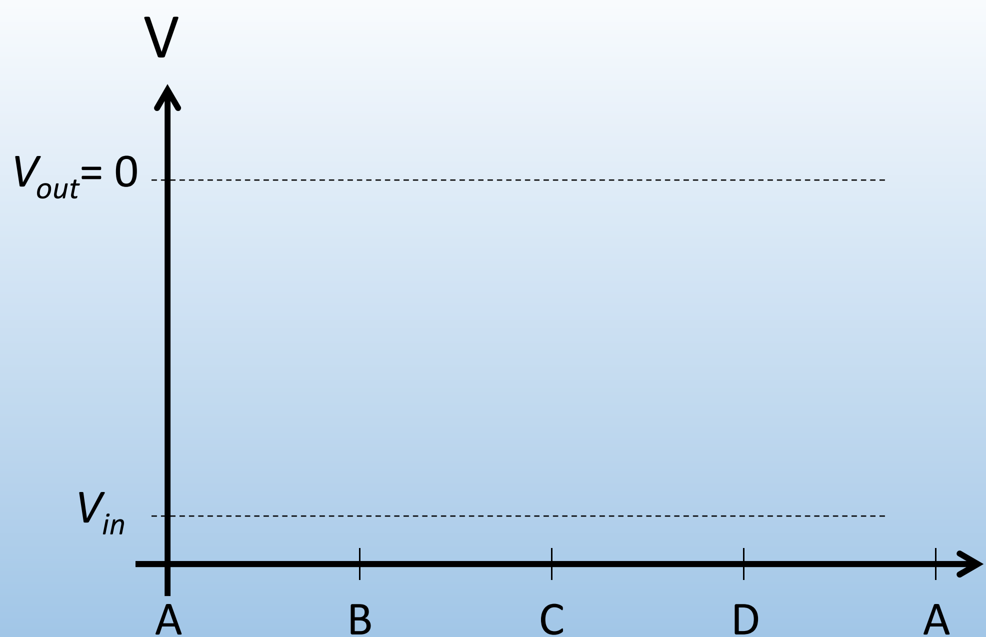
Calculation: electric potential

Find the electric potential difference across the cell $V_{in} - V_{out}$ (Assume $V_{out} = 0$ for reference)

$\varepsilon_K = 80$ mV, $\varepsilon_{Na} = 60$ mV, $\varepsilon_{Cl} = 50$ mV
$R_K = 2$ MΩ, $R_{Na} = 0.2$ MΩ, $R_{Cl} = 5$ MΩ



$$\varepsilon_K - IR_K - IR_{Cl} - \varepsilon_{Cl} = 0$$



$$V_{in} + \varepsilon_K - IR_K = V_{out}$$

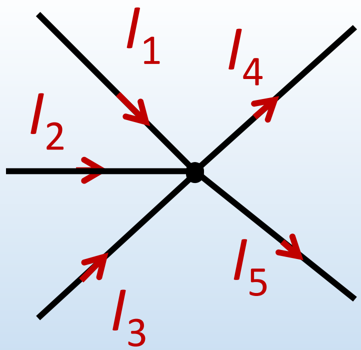
or: $$V_{out} - IR_{Cl} - \varepsilon_{Cl} = V_{in}$$

Kirchhoff's junction rule

The sum of currents into a junction equals the sum of currents out of a junction

$$\sum I_{in} = \sum I_{out}$$

Example:



$$I_1 + I_2 + I_3 = I_4 + I_5$$

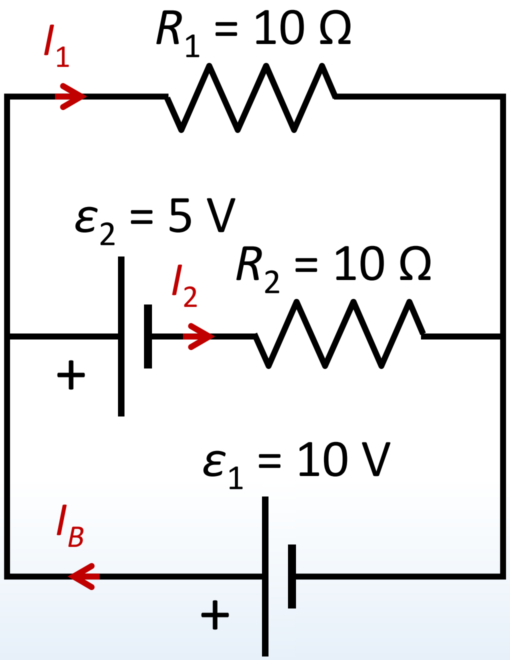
ACT: Checkpoint 1.3

Calculate the current through the battery I_B.

A. $I_B = 0.5$ A

B. $I_B = 1.0$ A

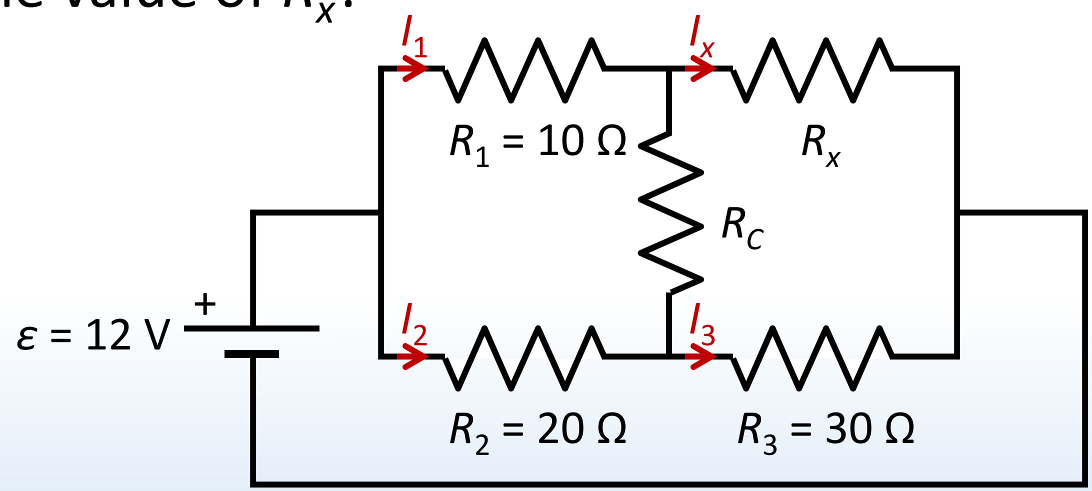
C. $I_B = 1.5$ A



Calculation: Kirchhoff's laws

In the circuit, the current through R_C is 0. What is the current through R_3 and the value of R_x?

From EX1 FA13

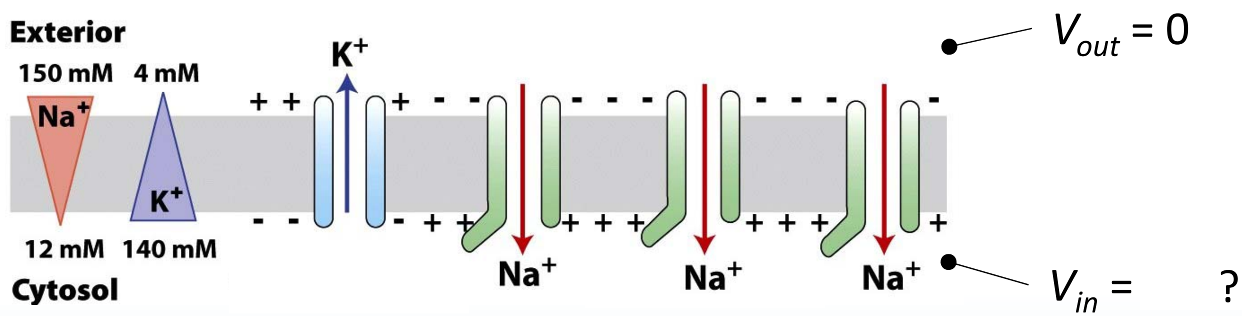


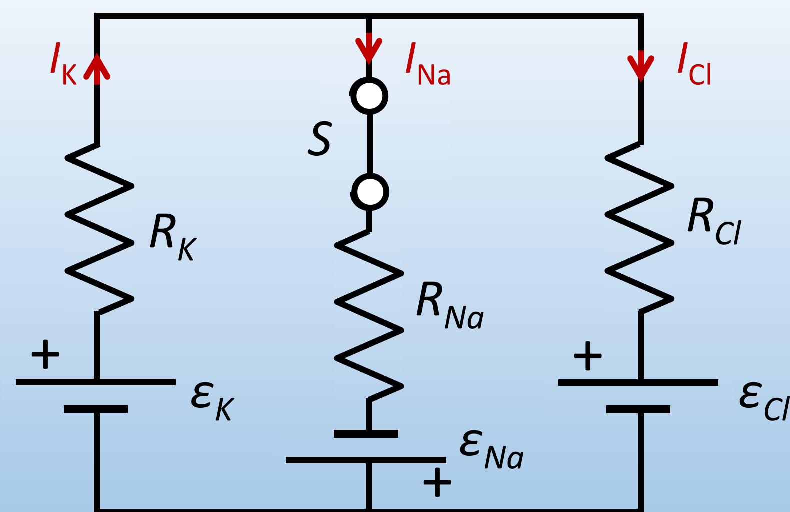
No current flows through R_C so $I_2 = I_3$ and $I_1 = I_x$

No current flows through R_C so $V_C = 0$

Nerve cell equivalent circuit

During nerve impulse, Na^+ channels open (i.e. switch *S* closes) and allow Na^+ to enter the cell

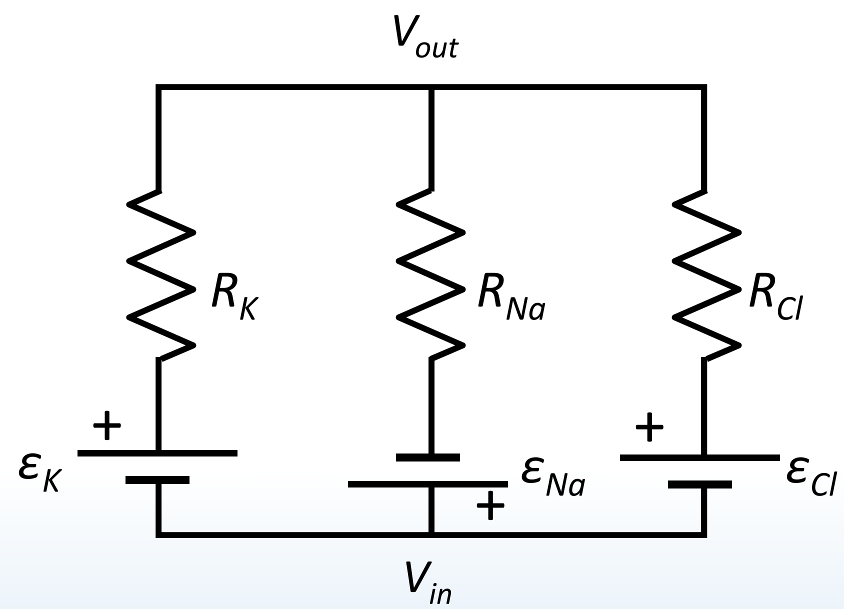




What happens to the currents through the channels and the potential in the cell?

Calculation: two loop circuit

Given the circuit to the right, find I_K, I_{Na} and I_{Cl} and $V_{in} - V_{out}$.

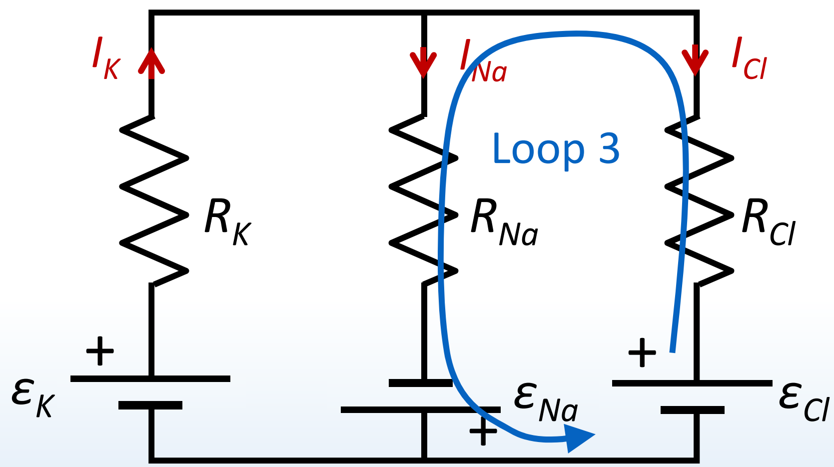


ε_K = 80 mV, ε_{Na} = 60 mV, ε_{Cl} = 50 mV
R_K = 2 MΩ, R_{Na} = 0.2 MΩ, R_{Cl} = 5 MΩ

1. Label all currents
2. Label +/– for all elements
3. Choose loop and direction
4. Write down voltage differences

ACT: Kirchhoff loop rule

What is the correct expression for “Loop 3” in the circuit below?



A. $+\varepsilon_{Cl} - I_{Cl}R_{Cl} - I_{Na}R_{Na} + \varepsilon_{Na} = 0$

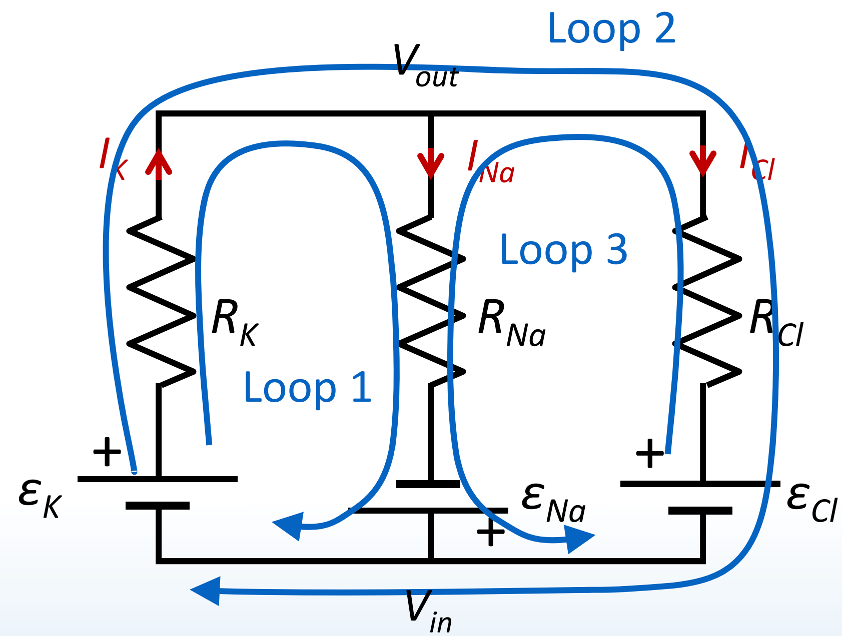
B. $+\varepsilon_{Cl} - I_{Cl}R_{Cl} + I_{Na}R_{Na} + \varepsilon_{Na} = 0$

C. $+\varepsilon_{Cl} + I_{Cl}R_{Cl} - I_{Na}R_{Na} + \varepsilon_{Na} = 0$

Calculation: two loop circuit

Given the circuit to the right, find I_K, I_{Na} and I_{Cl} and $V_{in} - V_{out}$.

ε_K = 80 mV, ε_{Na} = 60 mV, ε_{Cl} = 50 mV
R_K = 2 MΩ, R_{Na} = 0.2 MΩ, R_{Cl} = 5 MΩ



We have 3 unknowns, need 3 equations

Loop 1:

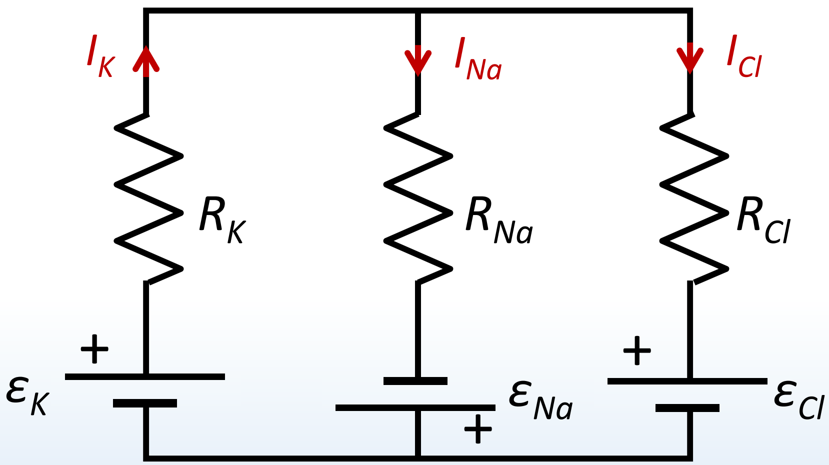
Loop 2:

Loop 3:

5. Write down junction rule

ACT: Kirchhoff junction rule

What is the correct expression for junction in the circuit?



A. $I_K + I_{Na} = I_{Cl}$

B. $I_{Na} + I_{Cl} = I_K$

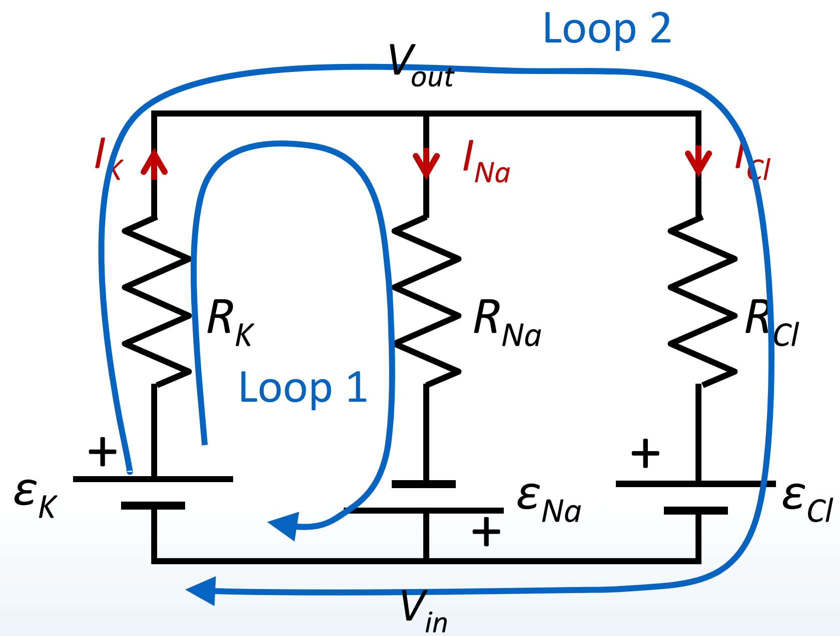
C. $I_{Cl} + I_K = I_{Na}$

Calculation: two loop circuit

Given the circuit to the right, find I_K, I_{Na} and I_{Cl} and $V_{in} - V_{out}$.

ε_K = 80 mV, ε_{Na} = 60 mV, ε_{Cl} = 50 mV
R_K = 2 MΩ, R_{Na} = 0.2 MΩ, R_{Cl} = 5 MΩ

3 equations, 3 unknowns, the rest is algebra!



(1) $+\varepsilon_K - I_K R_K - I_{Na} R_{Na} + \varepsilon_{Na} = 0$ $\quad +80 - 2I_K - 0.2I_{Na} + 60 = 0$

(2) $+\varepsilon_K - I_K R_K - I_{Cl} R_{Cl} - \varepsilon_{Cl} = 0$ $\quad +80 - 2I_K - 5I_{Cl} - 50 = 0$

(3) $I_{Na} + I_{Cl} = I_K$ $\quad +80 - 2I_K - 5(I_K - I_{Na}) - 50 = 0$

Substitute Eq. (3) into Eq. (2) and rearrange

(2’) $+30 - 7I_K + 5I_{Na} = 0$

Calculation: two loop circuits

Now 2 equations (1 and 2'), 2 unknowns (I_K and I_{Na})

(1) $+70 - I_K - 0.1 I_{Na} = 0$ $\qquad I_K = 70 - 0.1 I_{Na}$

(2') $+30 - 7 I_K + 5 I_{Na} = 0$ $\qquad +30 - 7(70 - 0.1 I_{Na}) + 5 I_{Na} = 0$

Substitute I_K in Eq. (1) into Eq. (2') and rearrange

$$-460 + 5.7 I_{Na} = 0 \qquad I_{Na} = \frac{460\,\mathrm{mV}}{5.7\,\mathrm{M\Omega}} = 81\,\mathrm{nA}$$

Plug solution into Eq. (2') to get I_K

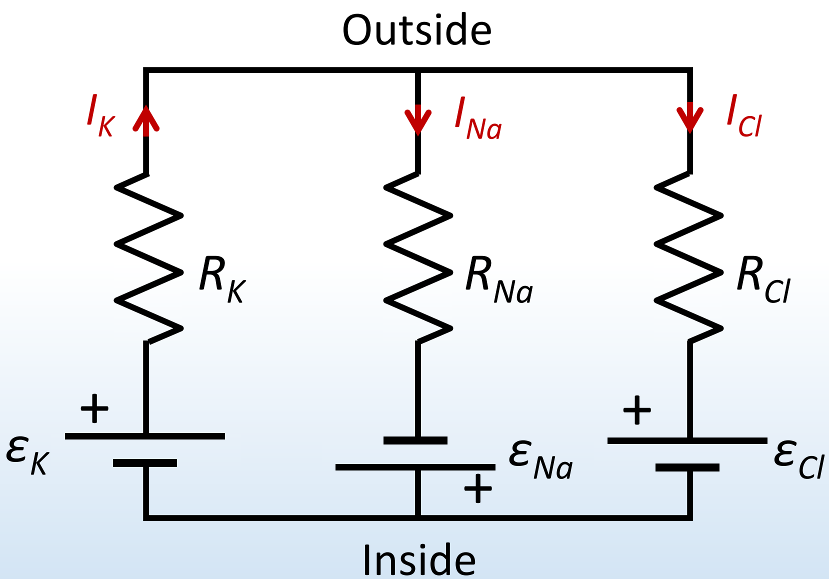
$$+30 - 7 I_K + 5 \cdot 81 = 0 \qquad I_K = \frac{435\,\mathrm{mV}}{7\,\mathrm{M\Omega}} = 62\,\mathrm{nA}$$

Use junction Eq. (3) to get I_{Cl}

$$I_{Cl} = 62 - 81 = -19\,\mathrm{nA}$$

ACT: Kirchhoff junction rule

We found that I_K = 62 nA, I_{Na} = 81 nA and I_{Cl} = –19 nA. Which of the following statements is FALSE?



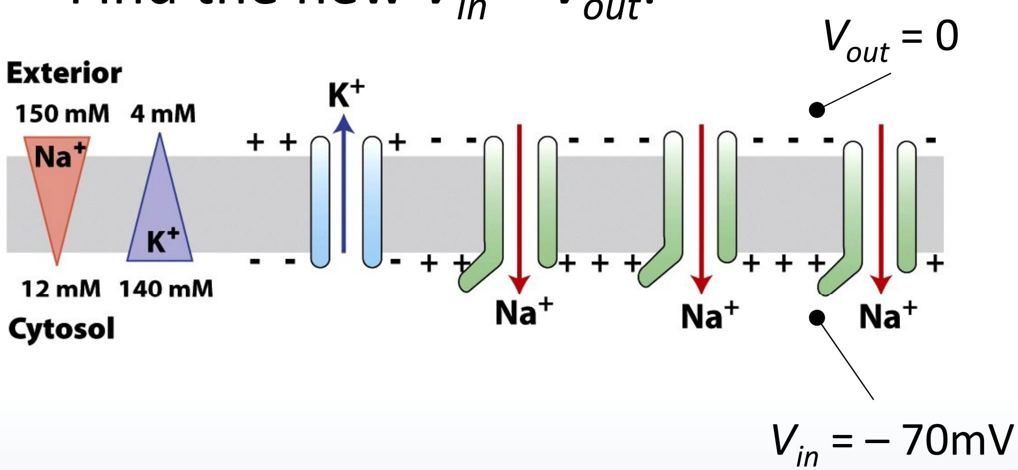
A. I_K is out of the cell

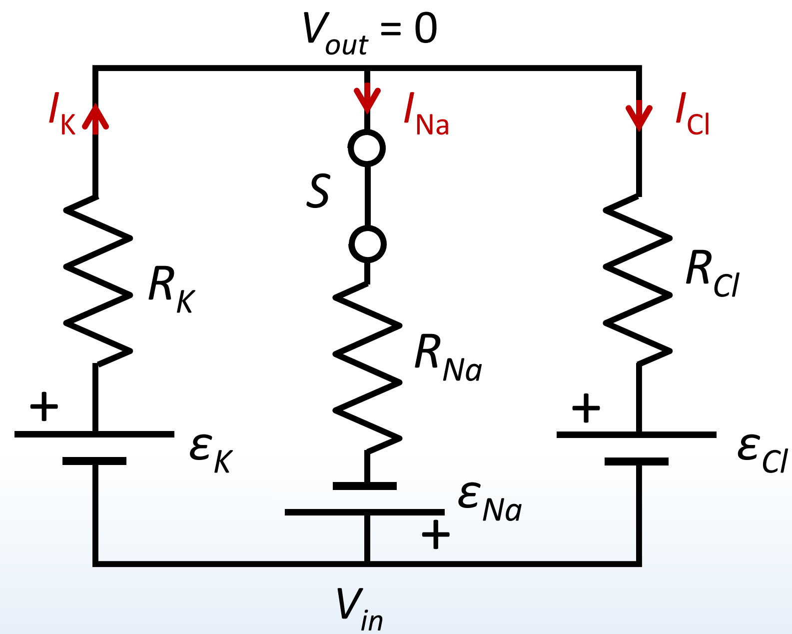
B. I_{Na} is into the cell

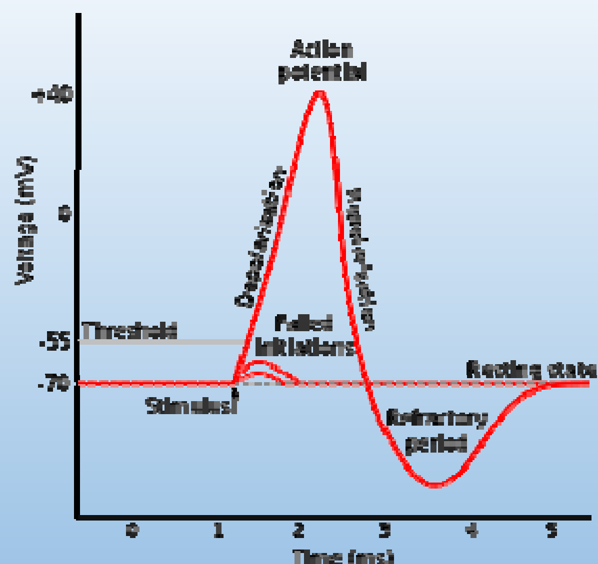
C. I_{Cl} is into the cell

Calculation: two loop circuit

Find the new $V_{in} - V_{out}$:







Summary of today's lecture

- Two basic principles:
- Kirchhoff loop rule

  Voltages around circuit loop sum to zero (based on conservation of energy)

$$\sum \Delta V = 0$$

- Kirchhoff junction rule

  Currents into a circuit branch equal currents out (based on conservation of charge)

$$\sum I_{in} = \sum I_{out}$$

Summary of today's lecture

- Basic approach to solving complex circuits:
  1. Label all currents
  2. Label +/– for all elements
  3. Choose loop(s) and direction(s)
  4. Write down voltage differences
  5. Write down junction rule

  The rest is algebra!