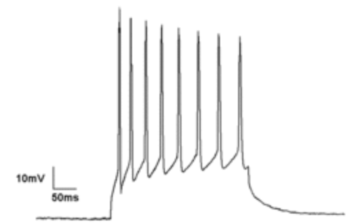
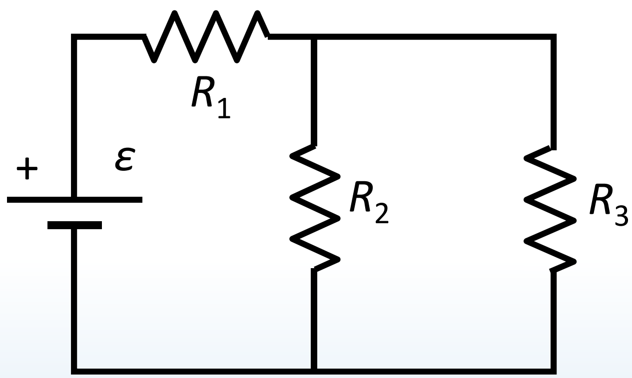


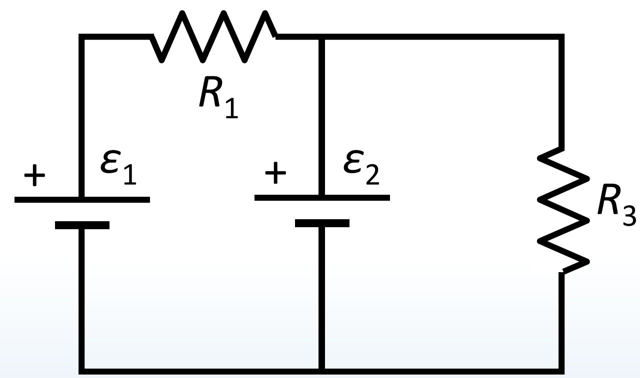
Phys 102 – Lecture 9

RC circuits

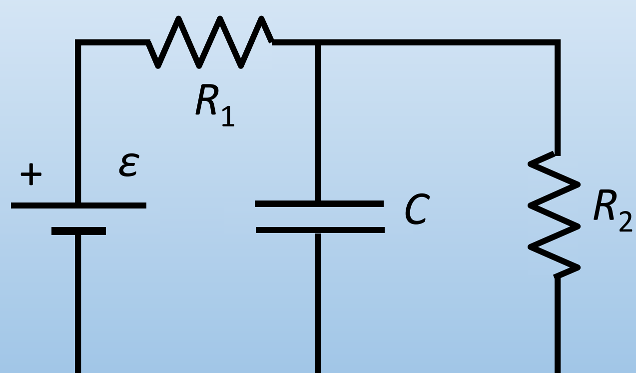
Recall from last time...

We solved various circuits with resistors and batteries (also capacitors and batteries)





What about circuits that combine all three... ...RC circuits



?

RC circuits

Circuits that store and release energy controllably...

Camera flash

Defibrillator

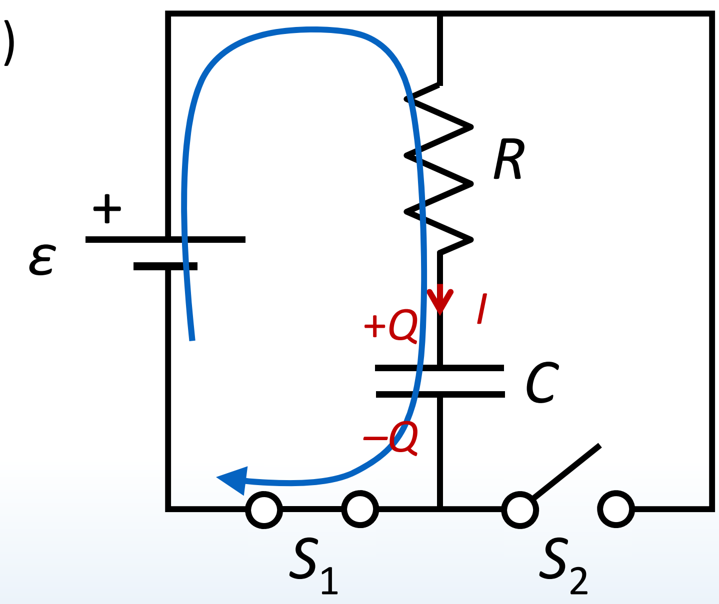
Nerve cells

Today we will...

- Learn about ***RC circuits***
  Charge on capacitors cannot change instantly,
  so behavior of RC circuit *depends on time*
- Analyze RC circuits under different situations
  Charging capacitors at short/long times
  Discharging capacitors at short/long times
  Time dependence
- Apply these concepts
  Nerve cells and nerve impulses (action potential)

Charging capacitor

Initially the capacitor is uncharged ($Q_0 = 0$)
At $t = 0$ we close switch S_1.

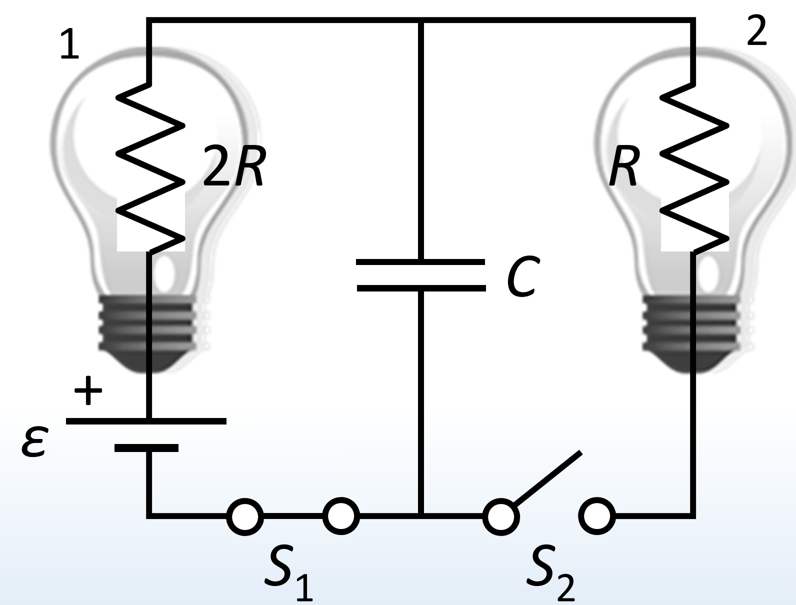


Immediately after:
Current I_0 flows around loop, through C
No charge on C ($Q_0 = 0$)

After a long time ($t = \infty$):
Charge on C builds until $V_C = \varepsilon$. Current decreases to zero ($I_\infty = 0$)

ACT: CheckPoint 1.1

Both switches are initially open and the capacitor is uncharged. What is the current through light bulb 1 ***right after*** switch S_1 is closed?



A. $I_b = 0$

B. $I_b = \varepsilon/3R$

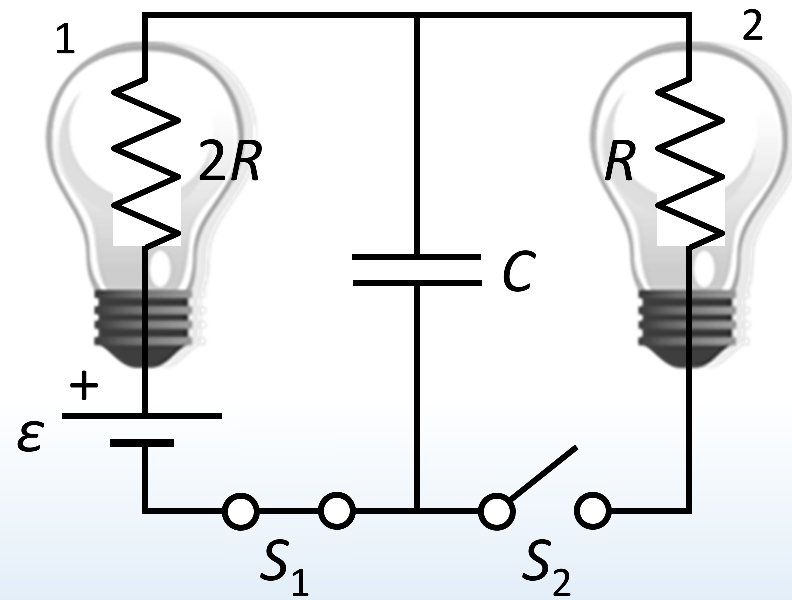
C. $I_b = \varepsilon/2R$

D. $I_b = \varepsilon/R$

DEMO

ACT: charging

Both switches are initially open and the capacitor is uncharged. What is the voltage across the capacitor <u>*a long time after*</u> switch S_1 is closed?



A. $V_C = 0$

B. $V_C = \varepsilon/2$

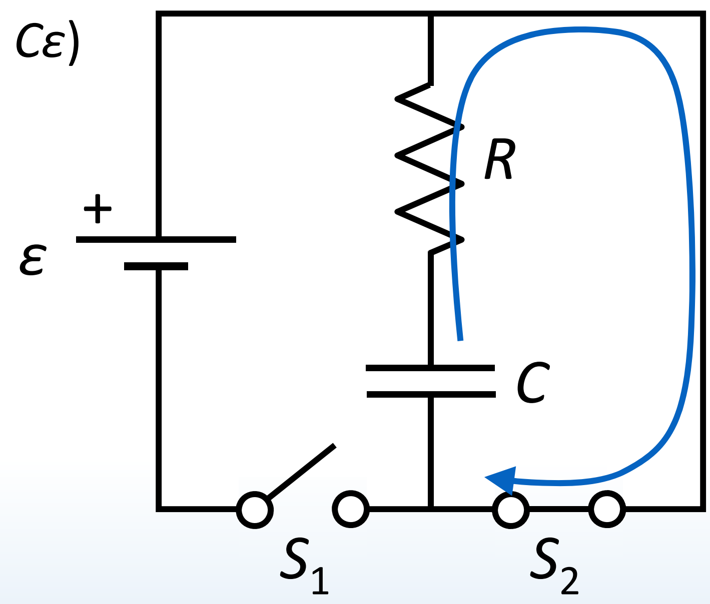
C. $V_C = \varepsilon$

D. $V_C = 2\varepsilon$

DEMO

Discharging capacitor

Initially the capacitor is fully charged ($Q_0 = C\varepsilon$)
At $t = 0$ we close switch S_2.

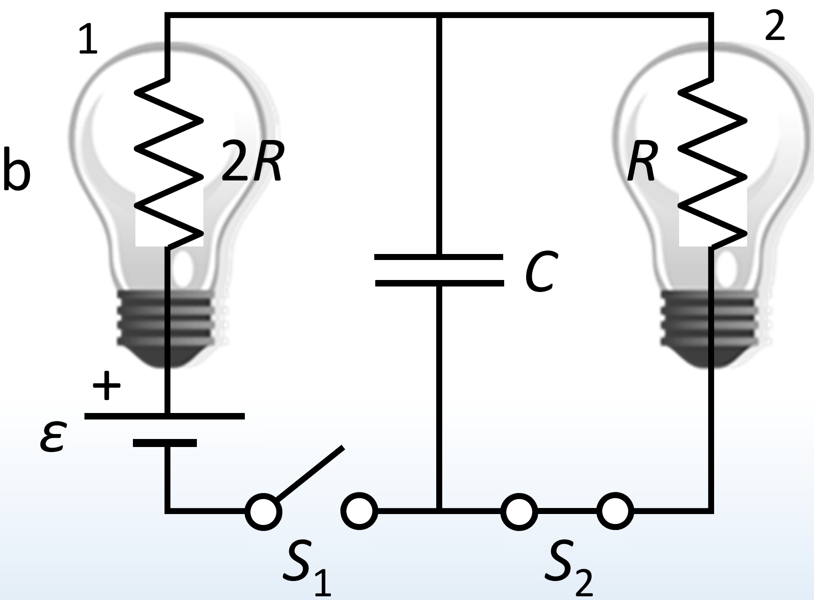


<u>*Immediately after*</u>:
Current I_0 driven around loop, through C
Charge on C from before ($Q_0 = C\varepsilon$)

<u>*After a long time*</u> ($t = \infty$):
Charge on C dissipates until $V_C = 0$. Current decreases to zero ($I_\infty = 0$)

ACT: CheckPoint 1.5

After S_1 has been closed for a long time, it is opened and S_2 is closed. What is the current through light bulb 2 *right after* S_2 is closed?



A. $I_b = 0$

B. $I_b = \varepsilon/3R$

C. $I_b = \varepsilon/2R$

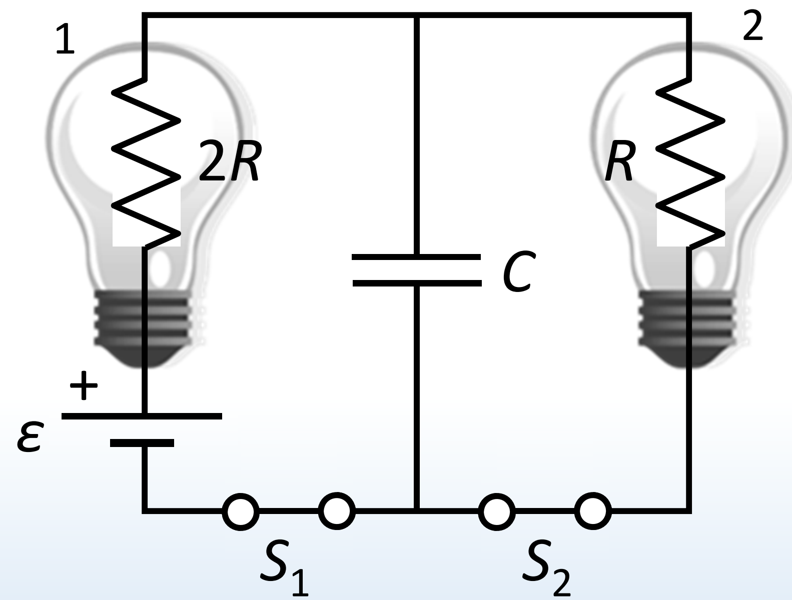
D. $I_b = \varepsilon/R$

DEMO

$$V_C - I_b R = 0$$

ACT: RC circuit practice

Now both S_1 *<u>and</u>* S_2 are closed. What is the current through light bulb 2 a *<u>long time after</u>* both switches are closed?



A. $I_b = 0$

B. $I_b = \varepsilon/3R$

C. $I_b = \varepsilon/2R$

D. $I_b = \varepsilon/R$

DEMO

Summary: charging & discharging

- Charge (and therefore voltage, since $V_C = Q/C$) on capacitors cannot change instantly
- *Short term* behavior of capacitor:

  If the capacitor is *charging*, current I drives charge onto it, and Q increases (acts like a wire)

  If the capacitor is *discharging*, current I drives charge off of it, and Q decreases (acts like a battery)

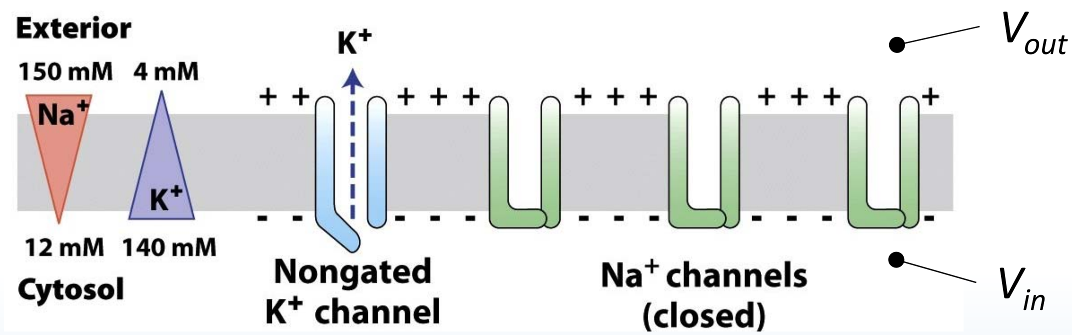
- *Long term* behavior of capacitor:

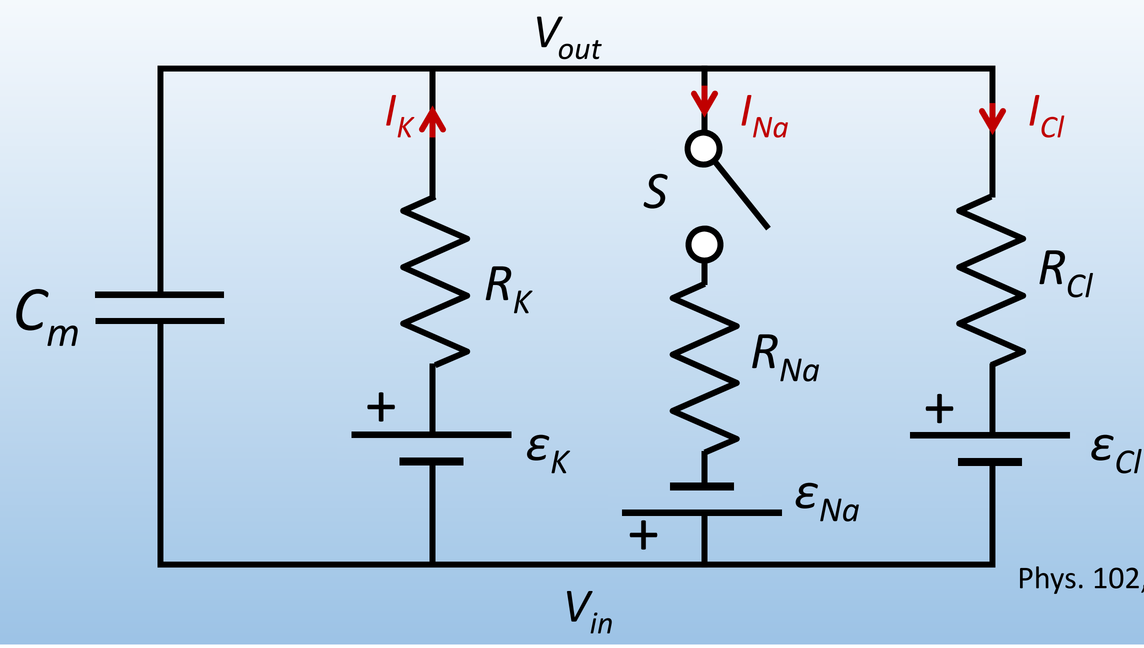
  If the capacitor is fully *charged*, $I = 0$ and Q is maximum (acts like an open circuit)

  If the capacitor is fully *discharged*, $I = 0$ and Q is minimum (acts like an open circuit)

Nerve cell equivalent circuit

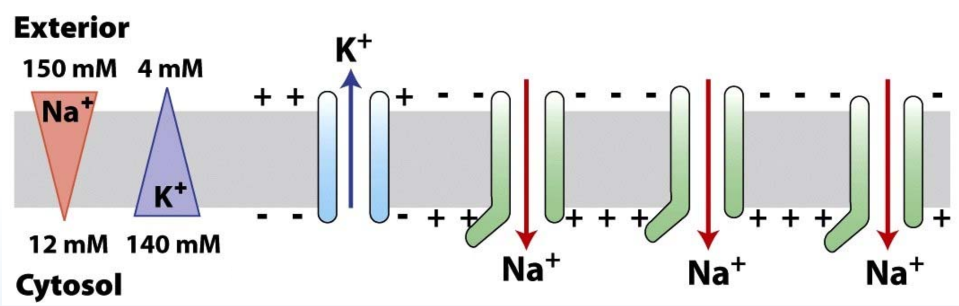
Neurons have ion channels (K^+,Na^+, and Cl^-) that pump current into and out of cell (it is *polarized*). Cell membrane also has capacitance

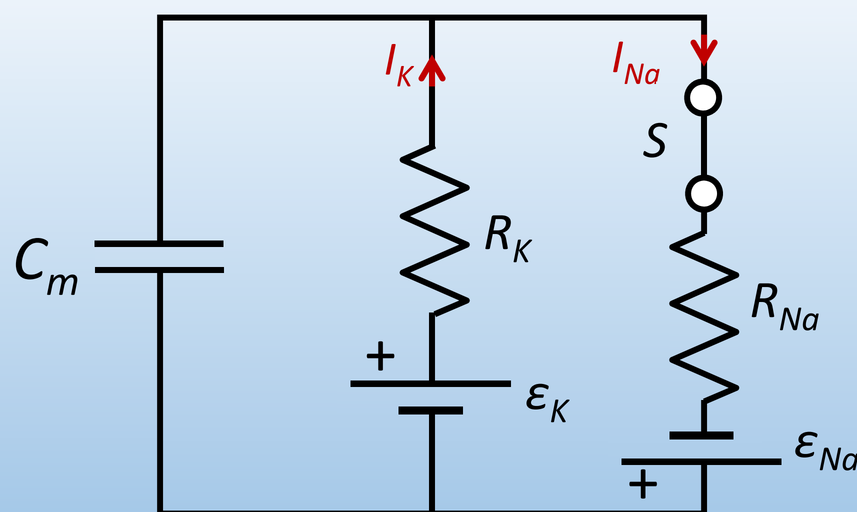


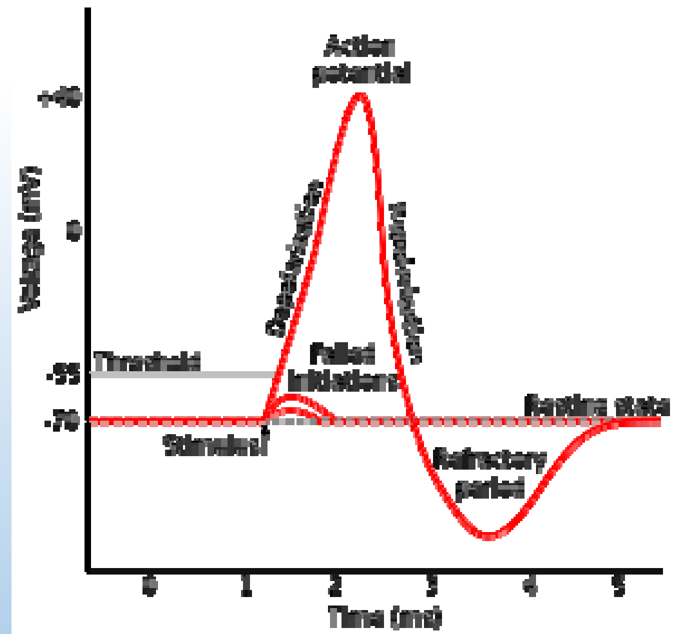


Action potential

At rest, Na^+ channels in cell are closed. When stimulated, the cell's voltage increases (*depolarization*). If a threshold is exceeded, the Na^+ channels open & trigger a nerve impulse (*action potential*)







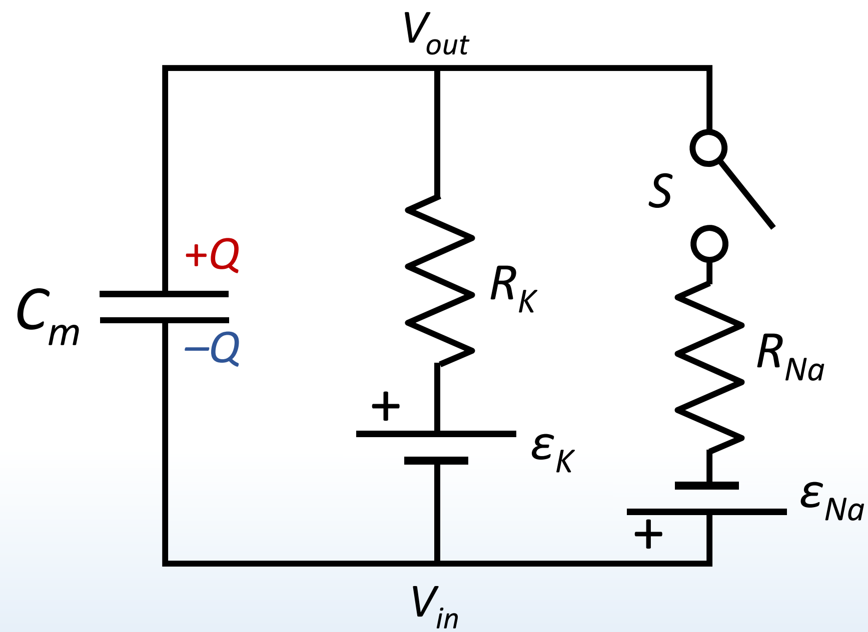
ACT: Resting state of neuron

The neuron has been in resting state for a long time. What is the voltage across the membrane capacitance?

A. $V_C > \varepsilon_K$

B. $V_C = \varepsilon_K$

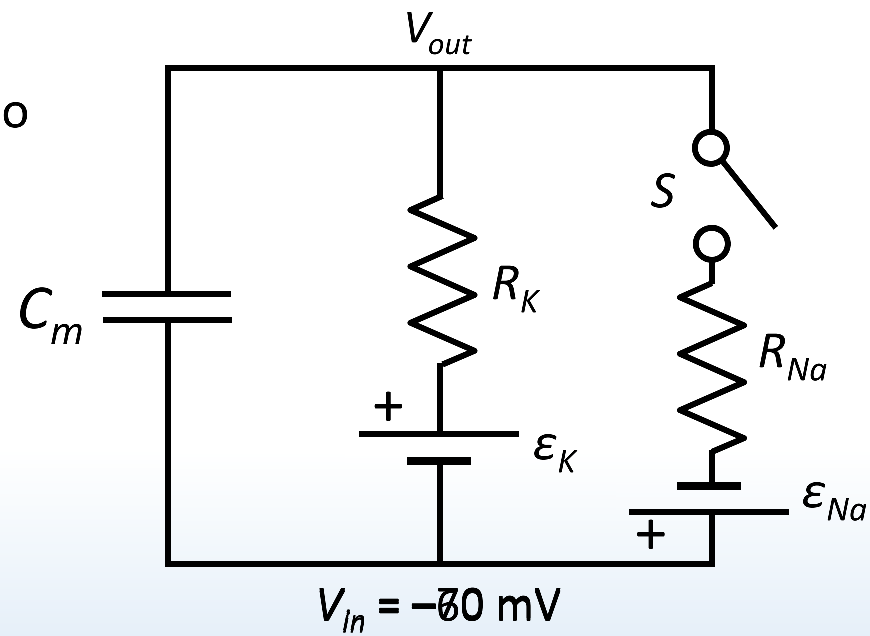
C. $V_C < \varepsilon_K$



ε_K = 70 mV, ε_{Na} = 60 mV,
R_K = 2 MΩ, R_{Na} = 0.4 MΩ, C_m = 300 pF

Calculation: action potential I

Some time ago, the cell was stimulated and depolarized to –60 mV, *less* than threshold to open Na^+ channels. What happens next?



ε_K = 70 mV, ε_{Na} = 60 mV,
R_K = 2 MΩ, R_{Na} = 0.4 MΩ, C_m = 300 pF

<u>*Immediately after*</u>:

No current through Na^+ channel

Current I_K driven by K^+ channel

Charge Q_0 on C_m from V_C = 60 mV

<u>*After a long time*</u>:

Current I_K decays to 0

Charge on C_m returns to rest value

RC circuit time dependence

Charging:

$$\varepsilon - I(t)R - \frac{Q(t)}{C} = 0$$

Charge builds up:

$$Q(t) = Q_\infty(1 - e^{-t/RC})$$

Current decays:

$$I(t) = I_0 e^{-t/RC}$$

Discharging:

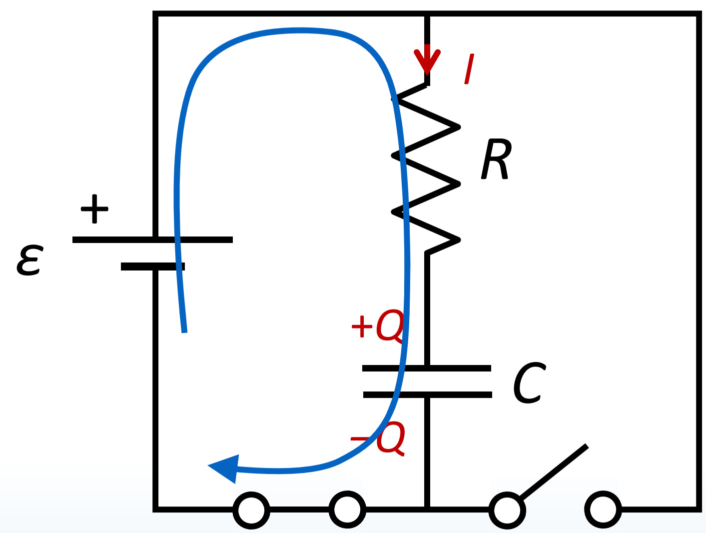
$$\frac{Q(t)}{C} - I(t)R = 0$$

Charge decays:

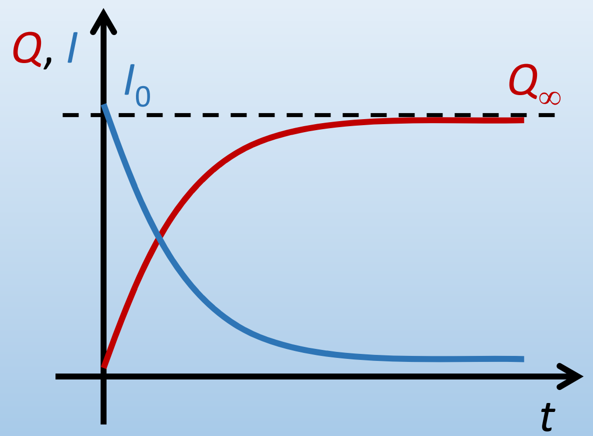
$$Q(t) = Q_0 e^{-t/RC}$$

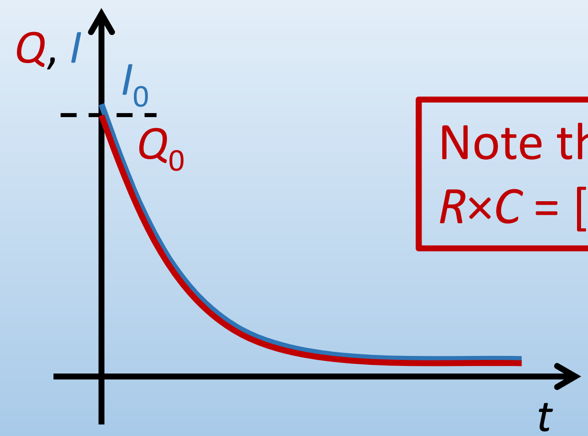
Current decays:

$$I(t) = I_0 e^{-t/RC}$$



DEMO

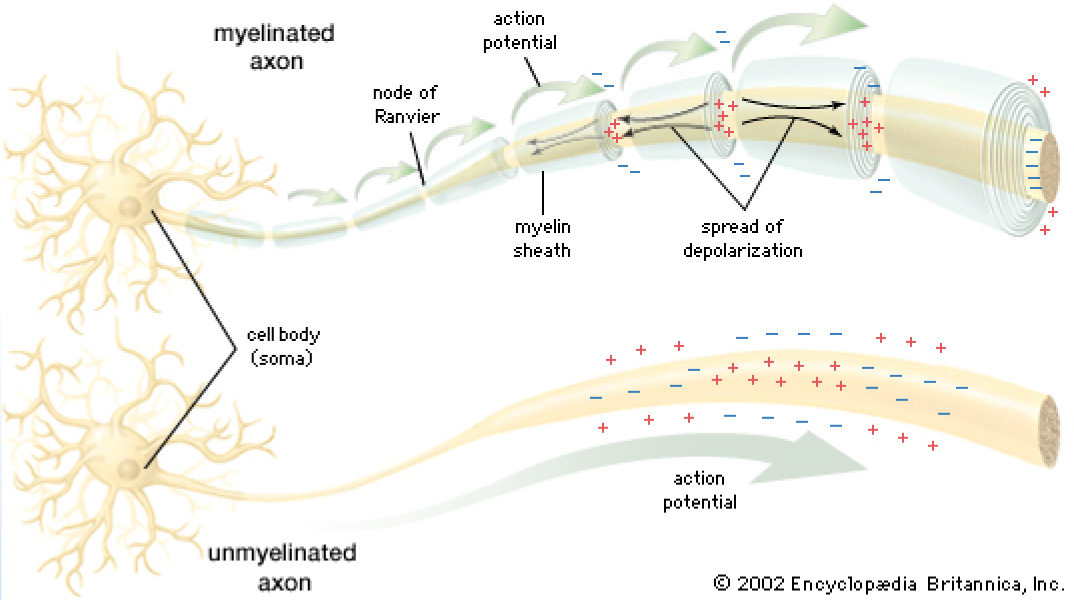




Note that *RC* has units of time!
R×*C* = [V]/[I] × [Q]/[V] = [Q]/[I] = [t]

Myelinated nerve cells

Action potentials propagate down nerve cell at rate determined by the cell's *RC* time constant.

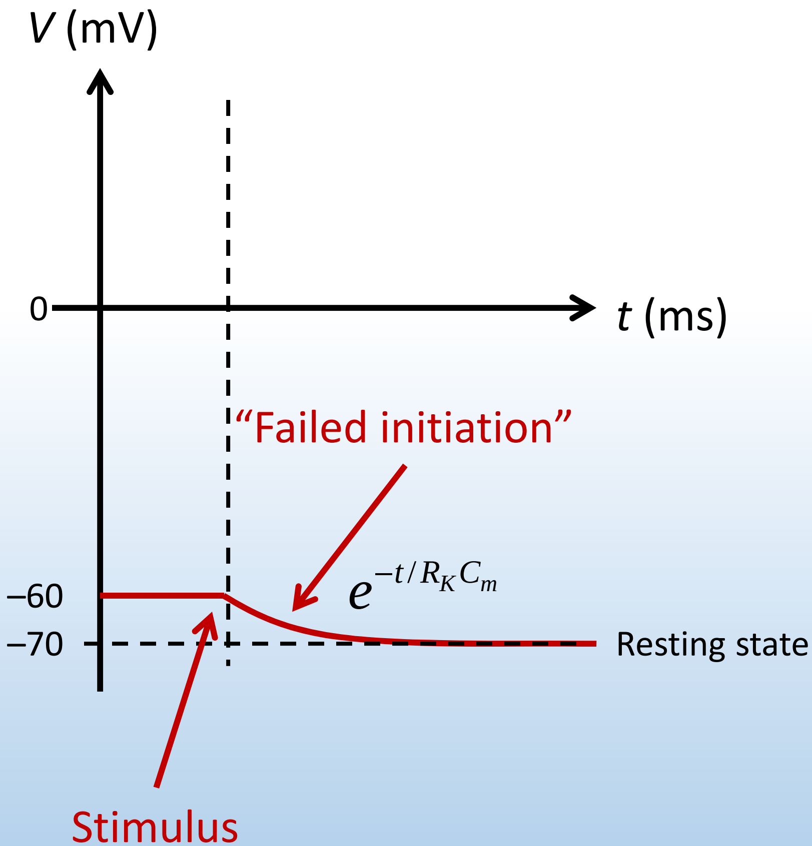


With very few exceptions (ex: C fibres) human neuron fibres are *myelinated.* Myelin reduces *C*, decreasing time constant & increasing propagation speed.

Many neurodegenerative diseases (ex: MS) cause progressive de-myelination.

Calculation: action potential I

How long does the cell take to return to 90% of its resting voltage?



Cell voltage $V_{in} - V_{out} = -V_C = -Q/C$:

$$\Delta V(t) = 10e^{-t/R_K C_m}$$

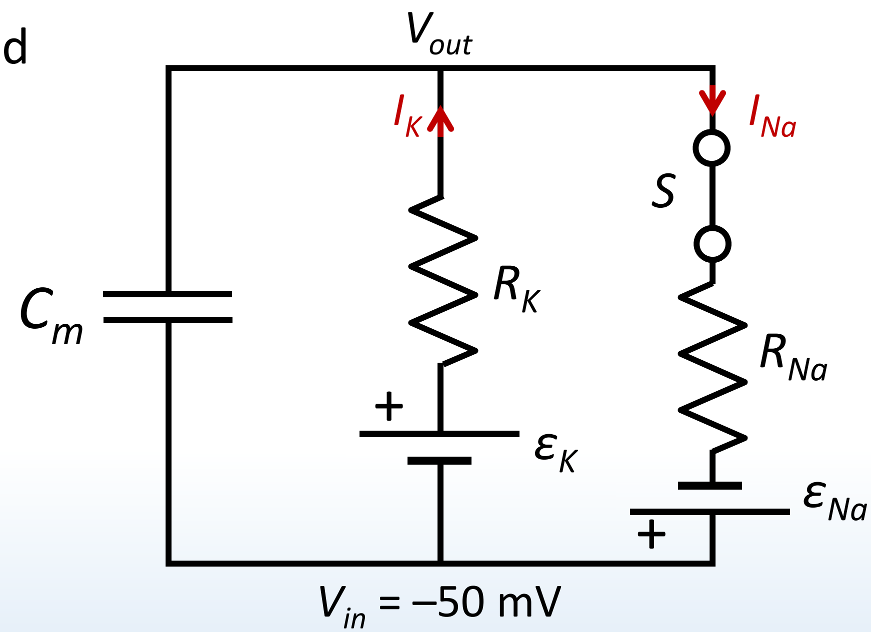
$$\Delta V(t_{90}) = 10(1-0.9) = 10e^{-t_{90}/R_K C_m}$$

Take natural log of both sides:

$$\begin{aligned} t_{90} &= -R_K C_m \ln(0.1) \\ &= -(2\times 10^9)(300\times 10^{-12})(-2.3) \\ &\approx 1.4\,\text{ms} \end{aligned}$$

Calculation: action potential II

Now, the cell was stimulated and depolarized to –50 mV, *over* the threshold to open Na^+ channels. What happens next?



<u>*Immediately after*</u>:

Current I_{Na} through Na^+ channel

Current I_K driven by K^+ channel

Charge Q_0 on C_m from before

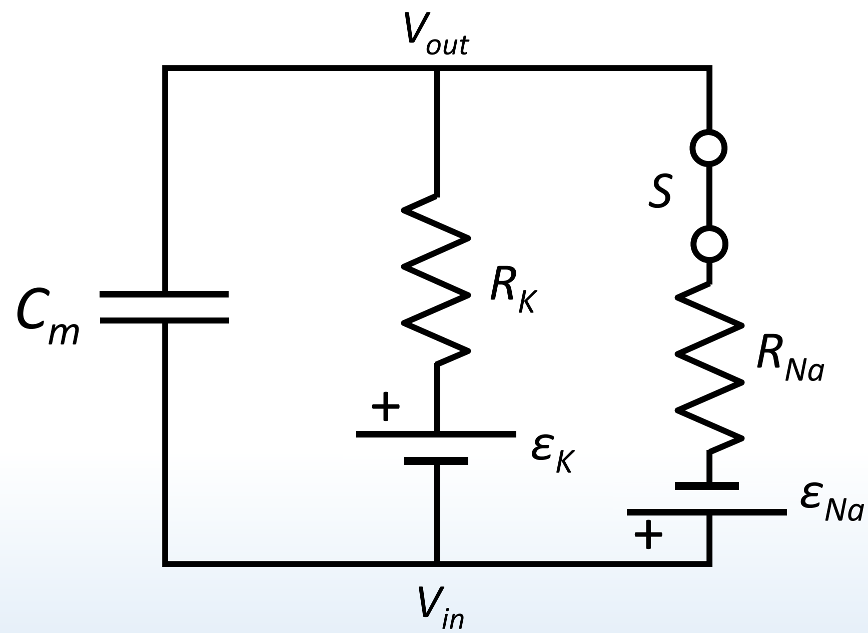
ε_K = 70 mV, ε_{Na} = 60 mV,

R_K = 2 MΩ, R_{Na} = 0.4 MΩ, C_m = 300 pF

ACT: action potential II

A long time after stimulating the cell, which statement below holds TRUE?

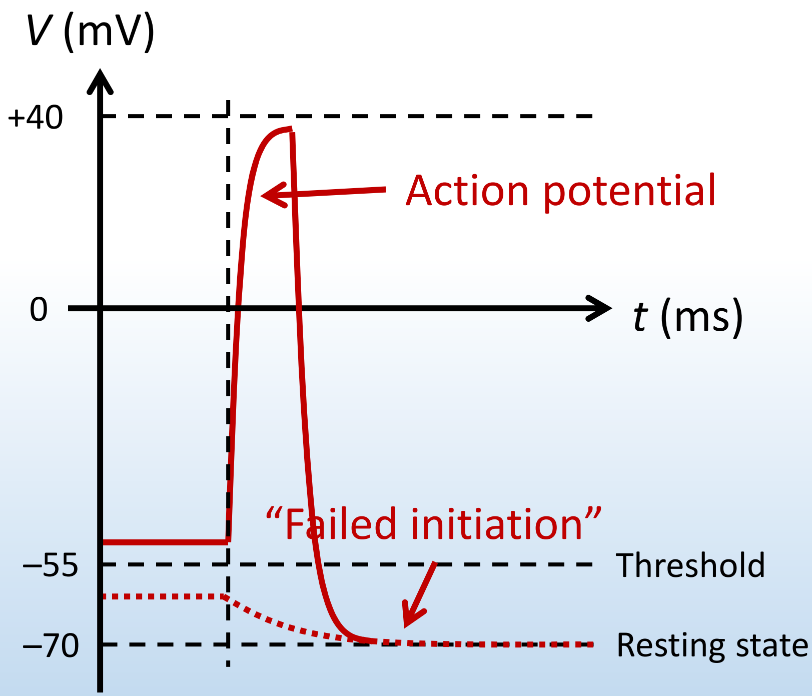
A. All currents are 0
B. The currents $I_K = I_{Na} \neq 0$
C. Voltage across C_m is 0

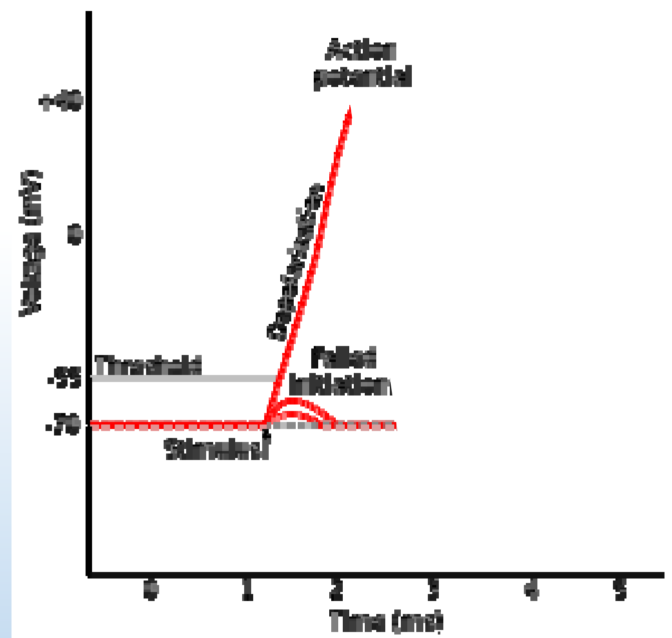


ε_K = 70 mV, ε_{Na} = 60 mV,
R_K = 2 MΩ, R_{Na} = 0.4 MΩ, C_m = 300 pF

Action potential summary

If the stimulus exceeds –55 mV, the Na^+ channels open, *depolarize* the cell & trigger an *action potential*.





Once a +40 mV potential is reached, the Na^+ channels close again & the cell *repolarizes* to its resting potential.

Summary of today's lecture

- RC circuits depend on time

  Charge on capacitors cannot change instantly

- Short/long times & charging/discharging

  $t = 0$: I flows Q onto/off of C, Q increases/decreases (charging/discharging)

  $t = \infty$: I through C decays to 0, Q reaches maximum/minimum (charging/discharging)

  $\tau = RC$: provides time to charge/discharge

Next week magnetism!