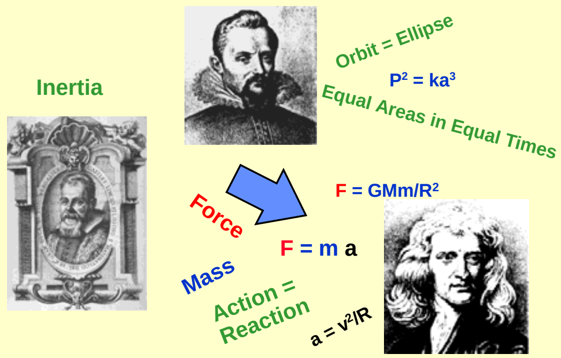
Lecture 5

Galileo & Kepler to Newton
Universal Laws of Classical Mechanics

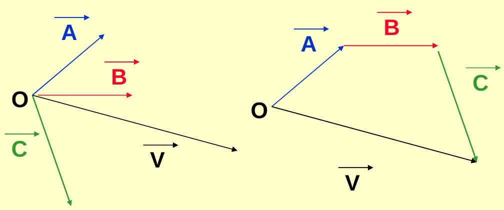


Today

- Newton puts it together: Generally regarded as the greatest scientific achievement of all time
- Newton's Laws
- Position, Velocity, Acceleration, Momentum as Vectors
- Key concepts: Space, Time, Mass, Force
  The new concept
- Next Time
  - Homework 2 due
  - Conservation Laws: Energy, Momentum
  - Read March, ch. 5; Lightman Ch 1

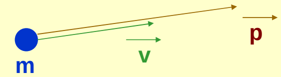
Vectors: Magnitude and Direction

- Nice Web site with java program that illustrates adding vectors
- http://home.a-city.de/walter.fendt/physengl/physengl.htm
- Example: $\vec{A} + \vec{B} + \vec{C} = \vec{V}$



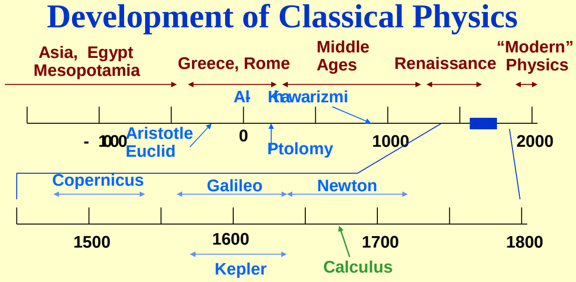
Vectors: Velocity, Acceleration, Momentum

- Momentum was known to Galileo & Descartes: Measure of "quantity of motion"
- Momentum Vector $\vec{p} = m\,\vec{v}$



- Note: m = mass is a scalar (a value, NOT a vector)
- Momentum has same direction as velocity
- Magnitude: p = m v
- (More on vectors later)

Development of Classical Physics



- Newton puts it together: Generally regarded as the greatest scientific achievement of all time
- One of the most influential developments of all time
- Invented calculus along the way!

Isaac Newton (1642 - 1727)

Born the year Galileo died

at Woolsthorpe, near Grantham in Lincolnshire, into a poor farming family.

Terrible farmer, sent to Cambridge University in 1661 to become a preacher. Instead, he studied mathematics.

Forced to leave Cambridge from 1665 to 1667 because of the great plague. Newton called this period the "Height of his Creative Power".

Greatest works were accomplished while he was 24- 26 years old!

One of the most influential people who ever lived
Newton's Paradigm - now called "classical physics"
- dominated Western thought for more than two centuries

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"In the beginning of the year 1665, I found the method of approximating series and the Rule for reducing any dignity of any Binomial into such a series. The same year in … November had the direct method of Fluxions, and in January had the Theory of Colours, and in May following I had entrance into the inverse method of Fluxions. And the same year I began to think of the orb of the Moon … from Kepler's Rule of the periodical times of the Planets … I deduced the forces which keep the Planets in their orbs must be reciprocally as the squares of their distances from the centres about which they revolve … All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded Mathematics and Philosophy more than any time since."

Isaac Newton (1642 - 1727) continued

- Suffered a mental breakdown in 1675.
- In 1679 (responding to a letter from Hooke) suggested that particles, when released, would spiral toward the center of the earth. Hooke wrote back claiming the path would be an ellipse.
- Hating to be publicly contradicted, Newton began to work out the mathematics of orbits.
- Urged by Halley to publish his calculations and results, Newton released Principia in 1687. This became one of the most important and influential works on physics of all times

Calculus – Newton vs. Leibnitz

- First known steps – ancient Greece
  - Zeno's paradox; Archimedes
- Newton wrote a tract (circulated among mathematicians) in 1666
  - First clear statement of the fundamental theorem of calculus
- Gottfried Wilhelm Leibnitz (1646 - 1716)
  - From a poor family
    Child Prodigy
  - Famous German Mathematician and Philsopher
  - Invented Calculus 1674-5; published 1684 – Controversial whether he had seen Newton's work

Newton's Three Laws

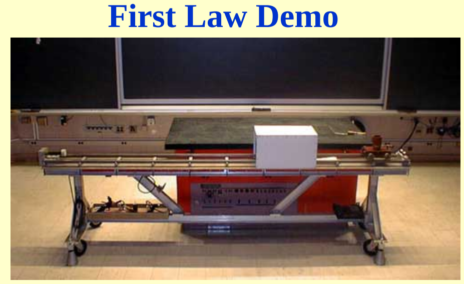
- Inertia:
  "Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by a force impressed on it."
- Force, Mass, Acceleration (F=ma):
  - "The change in motion [rate of change of momentum] is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."
- "Action = Reaction":
  - "To every action [change of momentum] there is always opposed an equal reaction; or, the mutual actions of two bodies are always equal, and directed to contrary parts."

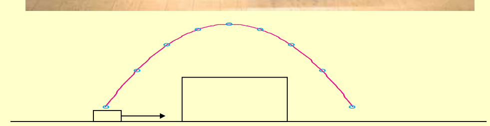
Newton's First Law

"Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by a force impressed on it."

- Same as Galileo's law of inertia.
  - If a body moves with constant velocity in a straight line, then there is NO net Force acting on the body.
  - If the body is moving in any other way (i.e. accelerating), then there MUST be a Force acting on the body.
- Galilean Relativity revisited:
  - "Rest" and "Uniform Motion" really are the same! No net force on the object
  - As Galileo argued, no experiment in a steadily moving ship will show that is is moving. Only by looking outside can you detect relative motion.

First Law Demo





Cart with constant velocity

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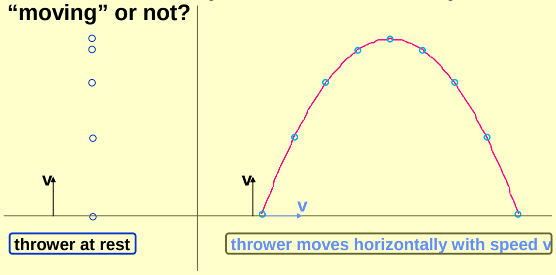
First Law Demo

- In what direction should you throw a ball if you want it to return to you? Does it matter if you are "moving" or not?



What does the trajectory look like if the thrower is "moving"?
The ball returns to the thrower. Both move so ball is always above the thrower. The laws of physics are the same whether or not the thrower is moving relative to the observer!

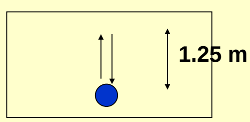
Exercise

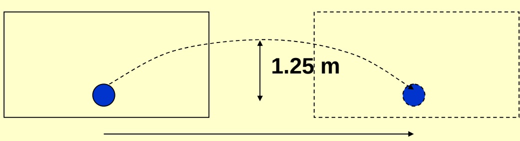
- Suppose you are on an airplane travelling at constant velocity with a speed of 500 miles per hour (roughly 200 m/s)
- If you throw a ball straight up, does it return to you?
- How does it appear to you?
- How does the path of the ball look to an observer on the ground?

- Can you think of any experiment done inside the airplane that would detect the motion of the airplane at constant velocity?

Exercise - Solution

To person on airplane
Time = 1 sec





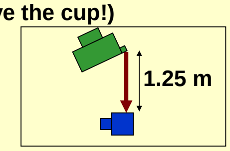
200 m

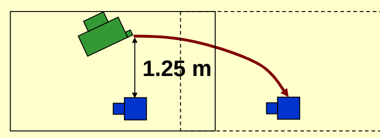
To person on ground - Time =1 sec

What about pouring coffee?

(We exaggerate and assume the coffee is poured 1.25 meters above the cup!)

To person on airplane
Time = 1/2 sec





To person on ground
Time =1/2 sec

100 m

Newton's Second Law

- "The change in motion [rate of change of momentum] is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.
- Equation:
  Force = mass x acceleration $\qquad \vec{F} = m\,\vec{a}$
- In terms of momentum: $\vec{p} = m\,\vec{v}$
  - $\vec{F} = m\,\vec{a} = m\,\Delta\vec{v}/\Delta t = \Delta\vec{p}/\Delta t$
  - Thus Force = rate of change of momentum
- Quantitative Concepts: Force and Mass

Mass

- What is this thing called Mass?
- Mass is a property of an object. In Newton's theory it is always constant for a given object.
- Mass is not weight, not volume, ...
- Mass is a quantitative measure of how hard it is to accelerate the object.
  - Mass of objects can be calibrated by measuring their acceleration by the same force
  - Tested experimentally-- found to be true that different measurements with different forces give consistent values of the mass

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Force

- What is a force?
- Force is the tendency to cause acceleration.
- Operationally defined by measuring accelerations.
- Is this just a circular definition?
- No! Forces can be related to physical systems. Compressed springs, gravitational forces, …. This is the basis for the predictive power of Newton's equations.

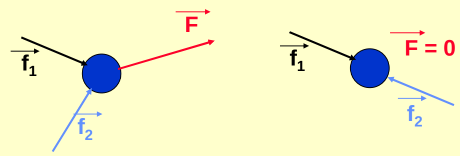
- More later on Forces

This is the new idea not present in Galileo's work

Force is a Vector

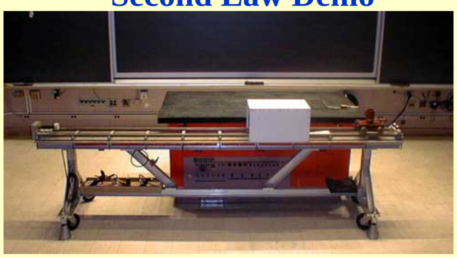
- The "Net Force" or "Total Force" on an object is the vector sum of all the forces on it due to other objects
- This what goes in Newton's Equation

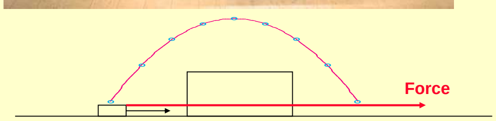
Force = mass x acceleration $\vec{F} = m\,\vec{a}$



Net Force F is the vector sum of the three applied forces

Second Law Demo





Cart with horizontal acceleration – caused by a horizontal force
Does the ball land in the cup?

Newton's Third Law

"To every action [change of momentum] there is always opposed an equal reaction; or, the mutual actions of two bodies are always equal, and directed to contrary parts."

- Consider collision of m_1 with m_2:
  - Newton's Second Law says that the force acting on m_2 (= F_{12}) during a time Δt results in a change in the momentum of m_2 (=Δp_2) equal to the force times the time ($\Delta p_2 = F_{12}\,\Delta t$). Similarly the change in momentum of m_1 is given by: $\Delta p_1 = F_{21}\,\Delta t$
  - Newton's Third Law says that the force m_1 exerts on m_2 (= F_{12}) must be equal in magnitude, but in the opposite direction of the force m_2 exerts on m_1 (= F_{21}), *i.e.,* $F_{12} = - F_{21}$
  - Therefore, the change in momentum of m_1 (= Δp_1) is equal in magnitude, but in the opposite direction of the change in momentum of m_2 (= Δp_2).

$$\vec{\Delta p}_1 = \vec{F}_{21}\,\Delta t = - \vec{F}_{12}\,\Delta t = - \vec{\Delta p}_2$$

- THE TOTAL MOMENTUM DOES NOT CHANGE!

Demonstration: Newton's Third Law: Action/Reaction

- Examples of equal and opposite forces
  - Does not matter which body "caused" the force

- Person pushing on a table

- How does a rocket accelerate?

- Rocket Cart! ---- DEMONSTRATION!

- Note that the total momentum does not change (We will come back to this as an example of a "conservation law" -- momentum is "conserved")

Exercise: Action/Reaction

- Suppose a tennis ball (m= 0.1 kg) moving at a velocity v = 40 m/sec collides head-on with a truck (M = 500 kg) which is moving with velocity V = 10 m/sec.
  - During the collision, the tennis ball exerts a force on the truck which is smaller than the force which the truck exerts on the tennis ball. TRUE or FALSE ?

  - The tennis ball will suffer a larger acceleration during the collision than will the truck. TRUE or FALSE ?

  - Suppose the tennis ball bounces away from the truck after the collision. How fast is the truck moving after the collision?
    < 10 m/sec = 10 m/sec > 10 m/sec ?

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Exercise: Action/Reaction solution

- Suppose a tennis ball (m= 0.1 kg) moving at a velocity v = 40 m/sec collides head-on with a truck (M = 500 kg) which is moving with velocity V = 10 m/sec.
  - During the collision, the tennis ball exerts a force on the truck which is smaller than the force which the truck exerts on the tennis ball. TRUE or [FALSE] ?

  Equal and opposite forces!

  The tennis ball will suffer a larger acceleration during the collision than will the truck. [TRUE] or FALSE ?

  Acceleration = Force / mass

  - Suppose the tennis ball bounces away from the truck after the collision. How fast is the truck moving after the collision?

    [< 10 m/sec] = 10 m/sec > 10 m/sec ?

  To conserve total momentum, the truck's speed must decrease since the tennis ball moves in the opposite direction after the collision.

Summary – to this point

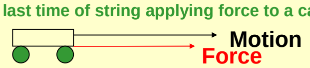
- Definitions: displacement, velocity, acceleration, momentum are vectors that describe motion
- Newton's three laws:
  - 1. A body moves with constant velocity unless acted upon by a force -- equivalent to principle of inertia
  - 2. F=ma
  - 3. Equal and opposite forces -- action/reaction (equivalent to conservation of momentum – more later)
- Concept of Force, Mass
  - Mass is a scalar measure of "inertia" or resistance to acceleration"
  - Force is a vector - tends to cause acceleration
  - The force in Newton's equation is the "Net Force" -- the vector sum of all forces on a body
- Demonstrations of Laws

Curved Motion & Circular Motion

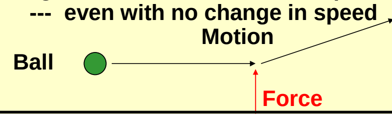
- Curved motion is accelerated motion!

Force is required to change the Magnitude or Direction of Velocity

- From First law motion continues in straight line at constant velocity unless there is a force
- Change of speed in the same direction requires a force in that direction
  - Car speeding up- positive acceleration
  - Car slowing down- braking- negative acceleration
  - Demonstration last time of string applying force to a cart on wheels

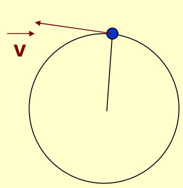


- Change of direction of motion requires a force --- even with no change in speed

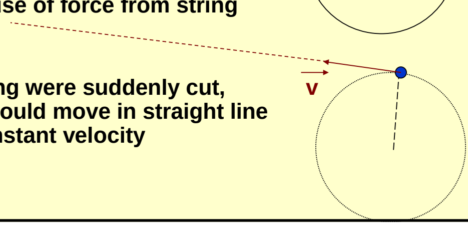


Force is required to change the Direction of Velocity

- Example: Circular Motion
- Accelerates even though speed does not change!
- Object moves in circle because of force from string



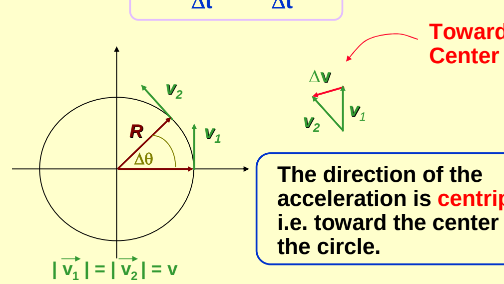
- If string were suddenly cut, ball would move in straight line at constant velocity



Acceleration & Circular Motion

- Acceleration is the change in velocity per unit time.
- Velocity is a vector (magnitude & direction).

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$



The direction of the acceleration is centripetal, i.e. toward the center of the circle.

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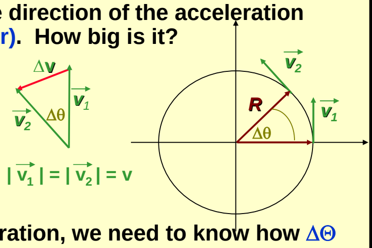
Acceleration & Circular Motion

- We now know the direction of the acceleration (toward the center). How big is it?

For small angles $\Delta\theta$, measured in radians:

$\Delta v = v\Delta\theta$



$|\vec{v}_1| = |\vec{v}_2| = v$

- To find the acceleration, we need to know how $\Delta\Theta$ is related to Δt :
  - For one revolution, the angular displacement is: $\Delta\theta = 2\pi$ (radians)
  - The time required for one revolution (period) is: $\Delta t = 2\pi R / v$
  - Therefore, $\Delta\theta / \Delta t = v / R$
  - Combining these equations: $a = \frac{\Delta v}{\Delta t} = v \frac{\Delta\theta}{\Delta t} = \frac{v^2}{R}$

Circular Motion

- Centripetal Force must be provided by something!
- $F = m v^2 / R$
- Force is toward the center, perpendicular to direction of motion

---

- How does an automobile go around a curve?
- How does a rocket is space change direction?
- What makes the moon circle the earth?
  HOMEWORK

Newton's theory of gravity

- Builds upon the idea that ALL curved motion is due to some FORCE
- Planets?
- All objects in the universe?

Kepler's Third Law Provides a Key

- Kepler's 3rd Law: $P^2 = k R^3$
- But, period = $P = 2\pi R / v \Rightarrow 4\pi^2 R^2 / v^2 = k R^3$
- Therefore, $v^2 = 4\pi^2 / k R$
- Substituting this form for v^2 into Newton's 2nd Law:
- Uniform Circular Motion: $a = v^2 / R$
- Newton's 2nd Law: $F = ma = mv^2 / R$

$$F = \frac{4\pi^2}{k} \frac{m}{R^2}$$

- This is the force that the Sun must exert on a planet of mass m , orbital radius R, in order that the planet obey Kepler's Laws in the circular motion approximation.

Toward a Universal Theory of Gravitation

- We have shown that Kepler's Laws follow from Newton's 2nd Law if the force F on a planet is:

$$F = \frac{4\pi^2}{k} \frac{m}{R^2}$$

- Question: What more do we have to do to turn this into a "Universal" Law of Gravitation?
- Consider Newton's 3rd Law:
  - If this is the force on the planet due to the Sun, then the planet must also exert an equal force on the Sun, but in the opposite direction.
  - There is no mention of the Sun in this equation, but there must be if this force describes the force on the Sun due to the planet.
  - Therefore, Kepler's constant k is not really a universal constant! It must depend on the mass of the Sun!!

Universal Law of Gravitation

- The only form of the law that is symmetric in the two masses (mass of sun and mass of planet) is:

$$F = G \frac{Mm}{R^2}$$

Where M and m are the masses of any two bodies, R is the distance between them and G is a universal constant!

- This form of the law is universal.
- Newton's law of gravity: There is an attractive force obeying the above law between every pair of masses in the universe. The constant G is universal and applies to all masses in the universe.

Lecture 5

Newton Has Said More than Kepler!

- Kepler's Laws describe the motion of a planet about the Sun.
- Newton uses same laws that apply to all motion!
- Newton's framework (forces & masses) allows him to generalize from the Sun-planet system to all bodies in the universe! This is "universal" gravitation!
- Newton's Third Law implies that each body exerts equal and opposite forces on the other. Depends upon both masses.
  - Describes the moon orbiting the earth
  - The moons of Jupiter, and much more!
  - Totally different from Kepler's approach.

Exercise: Kepler's Laws

- Suppose you know that the radius of Saturn's orbit is about 9 AU. (the radius of the Earth's orbit = 1AU).
  - Can you predict the average speed of Saturn in its orbit in terms of the average speed of the Earth in its orbit?
    - If you can, do it; if you can't, what other information would you need?
  - Can you predict the acceleration of Saturn in its orbit in terms of the acceleration of the Earth in its orbit?
    - If you can, do it; if you can't, what other information would you need?
  - Can you predict the force that the Sun exerts on Saturn in terms of the force that the Sun exerts on the Earth?
    - If you can, do it; if you can't, what other information would you need?

The Apple and the Moon I

- Is Newton's Gravitation Force Law really "universal"? Does the same force law describe apples falling to the Earth and the Moon's orbit about the Earth? Can we predict the acceleration due to gravity on the surface of the Earth from the Period & Radius of the Moon's orbit?
  - Acceleration of the moon: $a_{moon} = v^2 / R = 4\pi^2 R / P^2$
  - If due to gravitation, then also: $a_{moon} = F / m_{moon} = GM_{earth} / R^2$

If we know
P (period of moon = 1 lunar month),
R (distance to the moon) ,
G (unversal constant measured much later)

We can determine the mass of the earth!

But Newton Did not know the value of G!

The Apple and the Moon II

- Newton showed that the total force the Earth exerts on an object near its surface can be calculated by taking all the mass of the Earth to be concentrated at its center.
- Therefore, $F_g = ma_g = mg = GM_{earth}\, m / R_{earth}^2$
- The acceleration due to gravity at the surface of the earth is: $g = GM_{earth} / R_{earth}^2$
- The acceleration of the moon is:

$$a_{moon} = GM_{earth} / R_{moon}^2$$

$$g_{pred} = a_{moon} \frac{R_{moon}^2}{R_{earth}^2}$$

Putting in numbers: $g_{pred} = 9.76\ m/sec^2$
Observed: $g = 9.78\ m/sec^2$

IT WORKS !!

Effects of gravity

- Seen everywhere around us
  - Falling objects
- Planets, Moons orbiting larger bodies
- Double star systems rotating around each other
- Galaxies - millions of stars clustered due to gravitational forces
- See Feyman, Chapter 5

Gravity is a VERY Tiny force

- Force between two objects each 1 Kg at a distance of 1 meter is

$$F = G M_1 M_2 / R^2$$

$$= 6.67 \times 10^{-11}\ N$$

- 1 N is about the weight of one apple on the earth
- The reason the effects of gravity are so large is that the masses of the earth, sun, stars, …. are so large -- and gravity extends so far in space

Additional Comments

- Newton's Theory of gravitation contains one deeply unsatisfying aspect
  - Newton recognized the problem
- The law $f = G M m / r^2$ means "action at a distance"
- Instantaneous force due on one object due to another object no matter how far they are away from one another
- What should a scientist do?

Summary

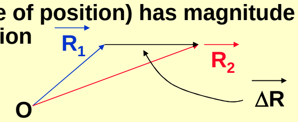
- Newton's 3 laws
- Circular Motion
  - Centripetal (toward center) accel. $a = v^2/r$
  - Centripetal force $f = ma$
  - Example: Ball on a string moving in a circle
- Kepler's Laws explained by gravitational force in Newton's laws
- Universal law of gravitation: $f = G M m / r^2$
  - The falling Apple and the Moon: each accelerates toward the earth obeying the same law!
  - Motion on Earth and in the heavens obeying the same simple laws!
- Force of gravity – extremely weak – large effects for large objects

Next Time

- Conservation Laws
- MORE important than Newton's Equations! - still valid in modern physics even though Newton's laws are not !
- The most useful conclusions without solving any equations!
  - Conservation of momentum: Follows from Newton's third law. (Chapt. 2 in Text)
  - Conservation of energy: The most important and useful law. (Chapt. 5 in Text, Chapter 4 in Feynman)

Extra - Position, Velocity, Acceleration are Vectors

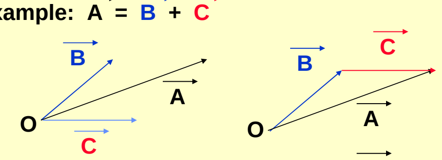
- A vector describes both magnitude and direction.
- Position (and change of position) has magnitude (distance) and direction



- Velocity is change of position vector per unit time. $\vec{v} = \dfrac{\Delta\vec{R}}{\Delta t} = \dfrac{\vec{R}_2 - \vec{R}_1}{\Delta t}$
- Acceleration is change of velocity vector per unit time. $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

Extra - Addition of Vectors

- Since a vector describes both magnitude and direction, adding vectors must take into account the direction
- Add vectors "head to tail" to get resultant vector
- Example: $\vec{A} = \vec{B} + \vec{C}$



- Subtraction is just adding the negative $\vec{C} = \vec{A} - \vec{B}$