

Lecture 6

Conservation Laws: The Most Powerful Laws of Physics

Momentum $p = m_1 v_1 + m_2 v_2 + \dots$

Energy $E = PE + KE + \dots$

Potential Energy mgh Kinetic Energy $\frac{1}{2}mv^2$

Other forms of energy

Announcements

- **Mon., Sept. 22:** Second Law of Thermodynamics
Give out Homework 4
- **Wed., Sept. 24:** Waves
Homework 3 due
- **Mon., Sept 29:** Review before Exam I
Homework 4 due
- **Wed., Oct. 1: Exam I**
Covers material through the Review
Chapters 1 – 5, 7 of March; Ch. 11-2 of Lightman

Introduction

- **Last Time:**
Newton's 3 Laws & Gravitational Forces
 - Newton's 3 laws tell us how to predict the motion of any body if we know the forces that act on it
 - **The examples we used were the simplest cases:**
Constant acceleration (which means constant force)
Uniform circular motion
Examples of the effects of gravitational forces
 - Very complicated to apply in most cases!
- **Today: Conservation Laws**
 - The most useful conclusions without solving equations!
 - **Conservation of momentum:** Follows from Newton's third law. (Chapt. 2 in March)
 - **Conservation of energy:** The most important and useful law. (Chapt. 5 in March)
 - MORE important than Newton's Equations! - still valid in modern physics even though Newton's laws are not!

Conservation Laws Why they are so powerful

- **Newton's Laws** show how to describe the motion of every object
 - Determined by the **FORCES** acting on each object at a each time t .
 - Newton's 2nd Law gives the **ACCELERATION** at time t .
 - **Acceleration** determines how the **velocity** and position of the object will change at time t .
 - **VERY complicated** to apply to most problems!
- **What can be known without finding all the details?**
- **Can any predictions of future behavior be made?**
 - **Yes.. conservation laws allow us to make important conclusions without knowing any details!**

Conservation Laws Examples outside physics (May be not be exact like physical laws)

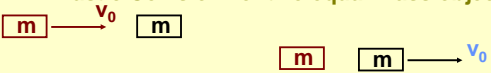
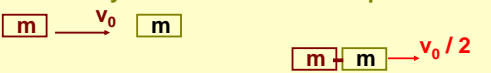
- **Example in Lightman - Child's blocks**
 - Even if blocks disappear, there may be ways to detect them and show the number does not change
- **Money in your pockets**
 - Conserved if you do not add or subtract
 - Change in total can be related directly to income minus outgo
- **Other**
 -

Momentum and Kinetic Energy Two Different Measures of Motion

- **Momentum (vector)**
 - **Momentum for one particle** $\vec{p} = m \vec{v}$
 - **Momentum for many particles** $\vec{p} = \sum_i m_i \vec{v}_i$
- **Energy (scalar - no direction)** $KE = \frac{1}{2} m |v|^2$
- **Kinetic Energy for a particle**
- **Kinetic Energy for many particles** $KE = \sum_i \frac{1}{2} m_i |v_i|^2$

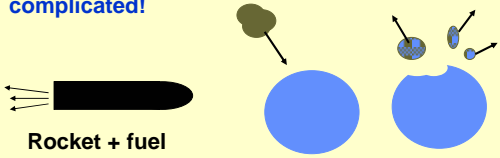
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Conservation of Momentum

- As discussed previously, Newton's 3rd Law (in conjunction with the 2nd Law) implies that the total momentum of two interacting objects is conserved (ie does not change in time).
- Air Track Demos:**
 - "Elastic Collision" of two equal mass objects:
 
 - "Totally Inelastic Collision" - equal mass objects:
 

Momentum is conserved in both these cases, but the final motions are quite different. How do we understand the origin of this difference?

Conservation of Momentum

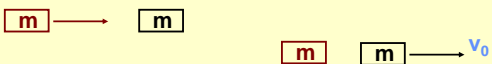
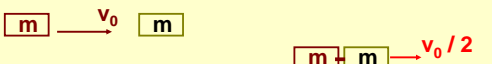
- Momentum is conserved in both cases, even though in the both cases complicated things are going on the causes the cars to bounce or to stick together.
- For an isolated system (no external forces) momentum is conserved no matter how complicated!
 

Momentum is conserved in all these cases.

Conservation of Momentum

- Exercise - List examples**
- For an isolated system (no external forces) momentum is conserved no matter how complicated!
- Put list on board

What about Energy?

- "Elastic Collision" of two equal mass objects:
 
- Kinetic Energy:**
Before: $(1/2)mv_0^2$ same as After: $(1/2)mv_0^2$
- "Totally Inelastic Collision" - equal mass objects:
 
- Kinetic Energy:**
Before: $(1/2)mv_0^2$
After: $(1/2)(2m)(v_0/2)^2 = (1/4)mv_0^2$
Kinetic Energy NOT the same after collision!

Conservation of Energy

First Law of Thermodynamics

- Total energy is conserved** - this is even more basic than Newton's laws
- Holistic Law**
- Energy comes in many forms. One form can be converted into another, but the total never changes!
- Kinetic energy:** energy of motion
Potential energy: Stored energy (due to gravity, compressed springs, batteries, chemical reactions, . . .)
- Heat:** Hotter objects contain more energy
Other: Nuclear, . . .
- (Later we will see that Einstein showed a different interpretation of this idea, but nevertheless the conservation law still applies!)

Conservation of Energy

- Exercise - List examples**
- For an isolated system (no exchange with the rest of the world) energy is conserved no matter how complicated!
- Types of energy**
- Put list on board

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Conservation of energy (continued)

- The conservation of energy is one of the most important laws in physics - **One of the most important for society as well!**
- Energy is the engine of modern society
- Conversion of energy from one form to another is the "infrastructure" of nations
 - Oil to kinetic energy
 - Gravity to lights in your home
 - Sun's energy to food
- All uses of energy have some loss - to friction - that wind up as heat
- Reducing losses (for example by thermal insulation, efficient motors, . . .) is a key goal for the future

Gravitational Potential Energy

- How do we describe freely falling bodies in terms of energy?
 - Initially, if released from rest, there is **NO** kinetic energy.
 - When the body falls, the kinetic energy increases.
 - Where does this energy come from? What has changed? **Only the position of the body with respect to the surface of the Earth!**
- Define the **gravitational potential energy** of mass **m** near the surface of the Earth as:

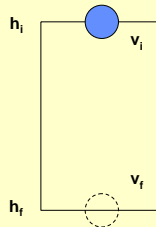
Potential Energy = mgh

where **h** = height of mass above some reference point (e.g. floor)
- As the mass falls, its potential energy is converted to kinetic energy. **This energy can be recovered!**
- Works for complex problems - like roller coasters

Gravitational Potential Energy & Kinetic Energy

- Derivation of formulas for conservation of energy
 - Assume the energy is all gravitational or kinetic energy.
- We know

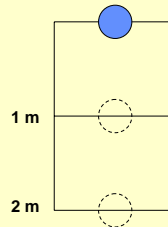
$$v_f - v_i = g(t_f - t_i) \quad \text{and} \quad h_i - h_f = \frac{1}{2} (v_i + v_f)(t_f - t_i)$$
- Thus $h_i - h_f = \frac{1}{2} (v_i + v_f)(v_f - v_i)/g = (v_f^2 - v_i^2)/2g$
 And $mg(h_i - h_f) = \frac{1}{2} m (v_f^2 - v_i^2)$
- Then **$E = mgh_i + \frac{1}{2} m v_i^2$**
 $= mgh_f + \frac{1}{2} m v_f^2$



- **PE + KE is conserved**

Gravitational Potential Energy

- Example of conservation of energy
 - Assume the energy is all gravitational or kinetic energy.
 - That is we assume there is no input from an engine, no loss to heat or other conversion of energy to other forms
- Use conservation of **$E = mgh + \frac{1}{2} m v^2$**
- If $v = 0$ at $h = h_{\text{top}}$, what is v at $h = h_{\text{top}} - 1 \text{ m}$?
- What is v at $h = h_{\text{top}} - 2 \text{ m}$?



Other types of Potential Energy

- A compressed (or extended) spring
For a high quality spring essentially all the energy required to deform it can be recovered - i.e., it is useful potential energy
- Twisted rubber band
- Bending of the bow which transfers energy to the arrow

Work

- **Work** is the **transfer of energy** by force acting on an object that is displaced
- Work is a form of energy: conservation of energy means that the energy of a system increases by the amount of work done on it
- Example: it takes work to raise a body and increase its potential energy
 - Work is needed to raise a roller coaster to the top
- Formula: **$W = F \times \cos(\theta)$**
 - $W = 0$ for force perpendicular to displacement (such as the effect of a centripetal force on a body moving in a circle)
 - $W = Fx$ for force parallel to displacement

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Work

- Formula:** $W = F \times \cos(\theta)$
 - $W = 0$ for force perpendicular to displacement (such as the effect of a centripetal force on a body moving in a circle)
 - $W = Fx$ for force parallel to displacement

Work to raise ball 2 m is $W = mg(2m)$

Work = 0 to keep ball moving in circle at constant v

Solving problems using Cons. of Energy

- The Lever Principle (Lightman)**

Heat

- Heat is a form of energy – internal energy of a material made up of atoms in motion (Atoms? More about them later)**

- Heat is due to motions of atoms in random directions - **cannot be completely channeled into useful work**
- Why?** This brings in new concepts and the second law of thermodynamics – Next time.
- Friction** causes conversion of mechanical energy to heat.

First law of thermodynamics

- Conservation of Energy is The First Law**
- Heat was very important in generalizing the conservation law to ALL forms of energy

- Heat is not obviously visible like mechanical motion of a large object
- Julius Meyer** is credited with formulating the law as conservation of all forms of energy

Conservation of Energy: Roller Coaster

Energy at top = $mgh + (1/2)mv^2 + \text{Heat energy}$

Work done by Engine to lift cars

Potential energy largest = mgh

Kinetic energy largest = $1/2 mv^2$

Brakes convert remaining Kinetic energy to heat

Exercise: Cons. of Energy

- An automobile of mass 2000 kg goes from rest to 30 m/s on a level road.
- What is the change in kinetic energy?
- This kinetic energy is transformed from another form of energy. What is that form?

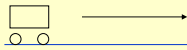
- The car moving at 30 m/s now starts up a hill. If no energy is supplied by the engine, what is the maximum height to which the car can coast.

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Exercise II: Cons. of Energy

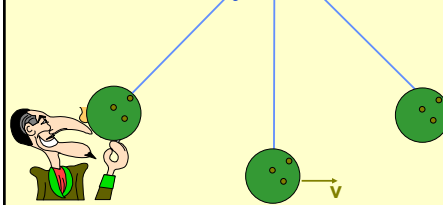
- If the speed of the car is doubled to 60 m/s, is the maximum height it can reach increased by:

- A factor of 2?
- A factor of 4?
- A factor of 8?



- If the car does not reach the maximum height, where does the energy go?
- If the car exceeds the maximum height, what will you say? Physics is wrong?

The Bowling-Ball Pendulum: Faith in Physics! Broken Nose?



- Demo: Hold bowling ball to nose and release
- What *should* happen?
 - Conservation of energy predicts no broken nose!
 - Ball should swing out, having maximum velocity at the low point of its swing.
 - Ball should have zero velocity when it returns to height of nose!
 - Secrets: don't move head and don't push!!!

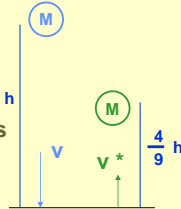
Exercise: Gravitational Energy

- A ball dropped on a hard floor bounces back to 4/9 of its original height.

- What fraction of its kinetic energy is lost during the bounce?

- Into what other forms is the energy transformed?

- What is the ratio of the speed just after the bounce to the speed just before?



Power

- Power = energy per unit time
- Unit: Watt = 1 Joule per second

- Light bulb - typical 100 watts = 100 joules/sec
- Heaters, etc, quoted in kilowatts

- Often Energy is quoted in kilowatt hours = 10^3 joules/sec x 3600 sec = 3.6×10^6 joules

- (Costs about \$0.10 – cost of 10 light bulbs for 1 hour)

Exercise related to Power

- A 100 watt light bulb is on 24 hours a day
- How much energy is used in a month?

- $E = 100 \text{ W} \times 24 \text{ hr/day} \times 30 \text{ days} = 0.1 \text{ KW} \times 720 \text{ hr} = 72 \text{ KW-hours}$

At \$0.10/ 72 KW-hours, this costs \$7.20.

- (Also $E = 100 \text{ J/s} \times 60 \text{ s/min} \times 60 \text{ min/hr} \times 24 \text{ hr/day} \times 30 \text{ days} = 100 \times 60 \times 60 \times 24 \times 30 \text{ J} = 2.592 \times 10^8 \text{ J}$ (not a practical scale!)

Summary

- Conservation Laws are the most powerful laws of physics

- Important conclusions with no details
- We considered them in the context of Newton's laws
- Really more general. These will still apply in the new revolutions of physics

- Conservation of Momentum (Vector)

- For an isolated system (no external forces) the momentum is conserved, i.e., the magnitude and direction never changes!

- Conservation of Energy

- Energy comes in many forms. One form can be converted into another, but the total never changes!
- Can apply to an isolated system
- If system is not isolated, the change of energy exactly equals the energy added from the rest of the world (e.g. work)
- No free lunch!

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Next Time

- The second Law of Thermodynamics
 - Entropy
 - The “arrow of time”
- Read
 - Lightman, Ch. 2