Lecture 14 Space-Time

The Wedding of Time and Space

Moving clocks appear to run slow.
Moving objects appear to shrink along line of motion and appear distorted.
Order of events can differ for different observers.

Announcements

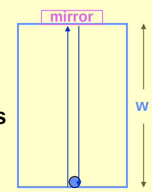
- References:
  - Today: Wedding of Space and Time: Time dilation, Length Contraction
    - March (Ch 10), Lightman (Ch 3)
  - Next time: Energy and Mass: $E = mc^2$
    - March (Ch 11)
- Homework 6 due Wednesday

Introduction

- Last time: Birth of Relativity
  - Einstein's two postulates for special relativity (special in the sense that it is restricted to descriptions in inertial reference frames moving at constant velocity).
  - Explored consequences of these two postulates within the framework of thought (gedanken) experiments.
  - Conclusion: We must give up idea that time is the same at different places.
- Today: Time and Space
  - Einstein's postulates let us calculate what different observers will measure for the time interval between the same two events. Note: they won't get the same answer!!
  - Given that time is different for different observers, we will see that space must be different as well!!
  - Today we will give explicit formulas for the apparent slowdown of time and change of length of moving objects

Time Measured "At Rest"

- Consider a system at rest
- That means "at rest with respect to the observer"



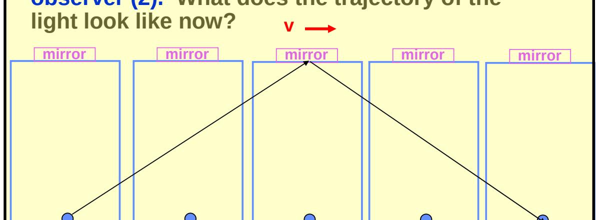
- A light pulse is emitted, travels to a mirror, is reflected and returns to its source.
- What does the clock read for the elapsed time between emission and return of pulse?

$$\boxed{T = 2w/c}$$

- This is the time measured in the "rest frame" called the "proper time"

Time Dilation

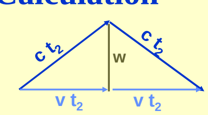
- Now, let the same source and mirror move to the right with velocity v with respect to a different observer (2). What does the trajectory of the light look like now?



- The distance the light pulse travels according to this observer is longer than it is when observed in the rest frame, but the speed of light is the same in all frames $\Rightarrow$ the times are different!

Time Dilation Calculation

- Use Pythagorean Theorem:



$$(c\,t_2)^2 = (v\,t_2)^2 + w^2$$

$$t_2 = w / \mathrm{sqrt}(c^2 - v^2) \Rightarrow t_2 = \gamma\, w/c \qquad \gamma = \frac{1}{\mathrm{sqrt}(1 - v^2/c^2)}$$

Total time: $T_2 = 2\,t_2 \Rightarrow \boxed{T_2 = \gamma\, 2w/c}$

Compare this with the proper time measured in the rest frame: $\boxed{T = 2w/c}$

- Result: Moving clocks run slow! $\boxed{T_2 = \gamma\, T}$

But wait! According to observer in rest frame, observer 2 is moving! Do we have a problem?

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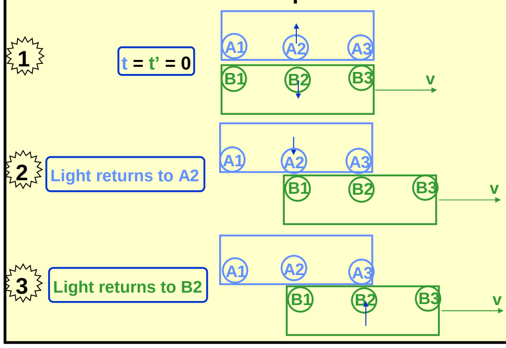
The Principle of Relativity: How can each observer say the other's clock runs slow?

- Consider the "Double Experiment" shown below:



The Results!

- What do the clocks read for the time intervals?

1 → 2: A2 → A2: $2w / c$; B2 → B1: $\gamma\, 2w / c$

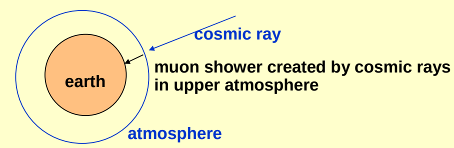
1 → 3: A2 → A3: $\gamma\, 2w / c$; B2 → B2: $2w / c$

•For Events 1-2, the blue frame clock (A2) "ran slow"
•For Events 1-3, the orange frame clock (B2) "ran slow"

• What's the difference?? The smallest time interval is measured in the frame in which only one clock is needed! This frame is called the "proper frame" for the time interval - Time in this frame is the "Proper Time".

• Correct equation: $\boxed{T_{improper} = \gamma\, T_{proper}}$

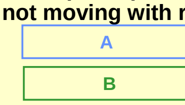
Is the clock slowing real? YES!

- Example of muons, sub-atomic particles that last about 1.5 µsec = 1.5×10^{-6} sec when observed at rest.
- 

- Traveling near c, a muon would only go about 3×10^{8} (m/s)x 1.5×10^{-6} sec = about 500 m. VERY few muons would make it to Earth's surface.
- But about 70% actually do reach the earth.
- How can this be? Because time appears to run slow by a factor of γ as measured by observer on earth.

What about Lengths??

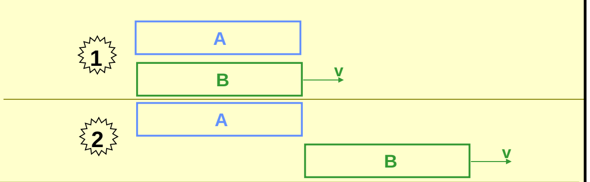
- Operational Definition of a Moving Length: Suppose the two sticks (A & B) have the same length L (when they are not moving with respect to each other).



• What does A say the length of B is when B is moving relative to A?

What about Lengths??

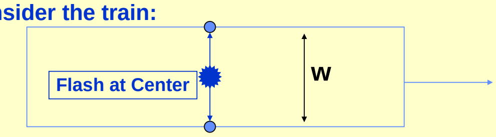
- If B is moving at velocity v relative to A, what is the length of B measured by A?



•L_B (measured by A) = v t_A where t_A is the time for 1-2 as measured by A.
• But from B's point of view, a point (front edge of A) moved a distance L in a time t_B at speed v ⇒ $L = v\, t_B$
• But according to A, B's clocks run slow: $t_B = \gamma\, t_A$
Therefore: $L_B = v\, t_A = v\, t_B / \gamma = L / \gamma$ $\boxed{L_B = L / \gamma}$

Length perpendicular to the motion does NOT change!

- How do we know the length w is measured to be the same for the two observers??
- Consider the train:



- Both observers must agree on the facts. Either the moving train fits on the stationary tracks or not.
- "Gedanken Experiment" shows that two observers shown above would get the same result for w if both are on the train or both are on the ground
- Different from measurement of length along track!

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Conclusions on Time and Lengths

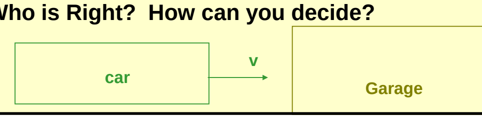
- The Consequences of Einstein's 2 Postulates:
- Time Dilation: Moving clocks appear to run slower (by factor of γ.)
- Length Contraction: Moving objects appear contracted along the direction of motion (by factor of γ.)
- Lengths perpendicular to the direction motion are unchanged.

$$T_{improper} = \gamma \, T_{proper}$$
$$L_{parallel}(moving) = L_{parallel}(rest) / \gamma$$
$$L_{perpendicular}(moving) = L_{perpendicular}(rest)$$

$\gamma > 1$

Garage "Paradox"

- Question: If a long car goes fast enough, will it fit into a short garage?
  - Assume: Length of car in rest frame of car = L_0 = 20 ft
  - Length of garage in rest frame of garage = G_0 = 20 ft
  - Speed of car wrt garage = v = 0.6 c (γ = 1.25)
- Answer from Garage Attendant: You bet !!
  - L = 20/γ = 20/(1.25) = 16 ft.
    Your 16 ft car should fit easily into my 20 ft garage!
- Answer from Car Driver: No way!!
  - G = 20/γ = 20/(1.25) = 16 ft.
    My 20 ft car can't fit into your 16 ft garage!
- Who is Right? How can you decide?



Garage "Paradox" - continued

- Let Sue be at center of car; Joe, at center of garage
- At the moment Sue and Joe pass each other, each one sees the front bumper of car as half way to the front door of garage. (see text, p. 120-122)
  - Joe sees front bumper at 1/2 (10 ft) = 5 ft
  - Sue sees front door at 2 (10 ft) = 20 ft
- But each knows this is "old news" because it took light time to reach them. They calculate:
  - Joe: Bumper must now be at 5 ft + 0.6 ft/ns x 5 ns = 5 + 3 = 8 ft
  - Sue: Door must now be at 20 ft - 0.6 ft/ns x 20 ns = 20 - 12 = 8 ft
- Sue and Joe agree, but Sue says door must be open, Joe says door is still closed.
- They disagree on the time the door opens.
- There is no paradox. Both are right!

Garage "Paradox" - continued

- Order of events is different for Sue and Joe
- Sue: Front door opens before back door closes so my 20 ft car easily passes through your 16 ft garage
- Joe: Front door opens after back door closes and your 16 ft car easily fits into my 20 ft garage
- There is no real paradox. No disagreement on observation. The car can go through the garage! But there is a difference in the perceived order of events.

Exercise

- When Sue passes Joe she thinks to herself: "I see a closed door ahead at 20 ft. I calculate that by now my bumper is past the door.
- Which statements below are correct?
- A. The door must have opened OK since there was no crash.
- B. I hope that the door really opened as promised.
  I cannot know know for sure, because there has not been time for light to reach me.

Space-Time: 4 dimensions

- The difficulties of different appearances to different observers can be dealt with by considering "space-time" as one 4-dimensional space.
- Define "length in space-time" by S given by
  $$S^2 = L^2 - (ct)^2 = L_0^2$$
  (Note negative sign. This is the difference from usual Pythagorian Theorem)
- c is the conversion factor between time and space
- Even embedded in our international standards for length! The meter is now defined in terms of the speed of light!

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International Standard of Length

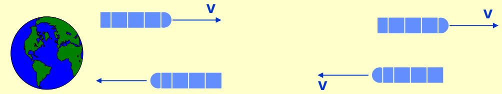
- History of standards, modern definitions
  - http://physics.nist.gov/cuu/Units/index.html
- "The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second." (adopted 1983)

What is a second? – More later.

- History:
  - "The meter was intended to equal 10^{-7} or one ten millionth of the length of the meridian through Paris from pole to the equator. However, the first prototype was short by 0.2 millimeters because researchers miscalculated the flattening of the earth due to its rotation. Still this length became the standard. .... In 1889, a new international prototype was made of an alloy of platinum with 10 percent iridium, to within 0.0001, that was to be measured at the melting point of ice. In 1927, the meter was more precisely defined as the distance, at 0°, between the axes of the two central lines marked on the bar of platinum iridium kept at the International Bureau of Weights and Measures (BIPM, Bureau International des Poids et Mesures) located outside Paris .....

Rocket Twin Paradox

- Imagine twins, one of which takes a ride on a rocket at relativistic speeds. When the rocket twin returns, is he younger, older, or the same age as his twin?

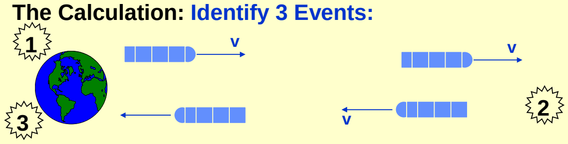


- **Question:** Is this like the garage paradox or is it different? That is, does this question have an absolute answer or not?

- This must be different in some way! Since they are reunited, each cannot be older than the other!

The Rocket Twin is Younger!

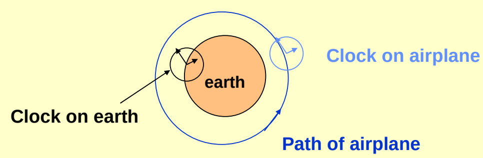
- Answer: The rocket twin returns younger! But why? Why can't we use principle of relativity to say they are the same age?
- The key is: The Rocket Twin accelerated while the Earth Twin didn't! The acceleration distinguishes the two twins and prevents us from applying the principle of relativity.
- The Calculation: Identify 3 Events:



- The Rocket Twin measures proper time for both time intervals: 1-2 and 2-3. Therefore the Rocket Twin measures the smallest time interval from 1-3!

The Rocket Twin is Younger!

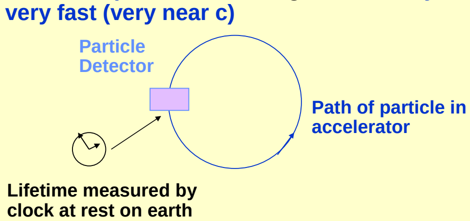
- The "Twin Paradox" has been tested experimentally!
- Two identical very accurate "atomic" clocks have been compared: one on the earth, one in airplane circling the earth



The "Airplane Clock" runs slower!

Other tests of the "Twin Paradox"

- Subatomic particles are accelerated in the giant accelerators (like at Fermilab, Batavia, Illinois)
- The particles get very close to the speed of light c - but never reach c!
- The same particles live longer when they are going very fast (very near c)



Summary

- Special relativity describes motion of objects at constant velocity.
  - Essential for speeds approaching c -- Tested Experimentally
- Moving objects appear to have:
  - Clocks that run slow (ALL physical processes "run slow")
  - Lengths that contract along the line of motion
- The order of events at different places in space can be different for different observers.
- Space & time intertwined: 4-dimension "Space-Time"
  - c is the conversion factor between time and space
  - International standard
- Two type of paradoxes
  - Car Garage: Not a paradox. The only difference is how Sue and Jo described the order of events
  - Twin: The twin who leaves the earth and returns really is younger! Tested experimentally by accurate clocks and subatomic particles