Lecture 15 The Ultimate Speed Limit and E=mc²

Relativistic mass and Relation of Mass and Energy

- How is speed limit enforced?
- The faster you go, the heavier you get!
- Energy and Mass are equivalent
- Rest Mass: 1 Kg = Power plant for 1 year
- What about Conservation of Energy? Conservation of Momentum?

Announcements

- Today: E = mc², The faster you go the heavier you get
  - March (Ch 11)
- Next Time: General Relativity
  - March (Ch 12)
- Homework 6 due TODAY
- Give out Homework 7
- Exam II -- Wed. Nov. 5

Introduction

- Last Time: Time Dilation, Space Contraction, Speed Limit, “Paradoxes”
  - Moving Clocks run slow
  - Moving objects shrink along the line of motion
  - Simultaneity and the “garage paradox” – not really a paradox
  - Real world: supports conclusion of “twin paradox”
- Today: Mass is Energy, Energy is Mass
  - Recall: Existence of speed limit from principle of relativity
  - Enforcement of speed limit (relativistic mass)
  - Mass is energy (E = mc²)
  - Einstein’s own words: http://www.aip.org/history/einstein/voice1.htm

The Speed Limit

- Review of the idea that nothing can travel faster than the speed of light.
- The example below shows directly, from the principle of relativity, that c is the ultimate speed limit.
  - (This is a version of the example from the text, page 108.)

- Light pulses (A & B) are emitted at O, travel to mirrors, are reflected and return to O.
- Now suppose O is moving (with respect to us) to the right at a speed which is greater than the speed of light.
- Whether B returns to O or not cannot depend on the reference frame. Therefore, O cannot move at speeds greater than light!!

How is the Speed Limit Enforced?

- We have now seen that if things could travel faster than the speed of light, the Principle of Relativity would be violated.
- Question: How is this speed limit enforced? Why can’t we just keep adding energy to the object which will cause its velocity to keep increasing??
- Answer: As we add energy to the object, its mass increases also which makes it harder to accelerate!
- How can this be?? Isn’t mass a property of the object, an absolute quantity? It is in classical physics, but . . . .

Einstein’s postulates also force us to reconsider meaning of mass

- Newtonian (Classical) Physics: Mass is an absolute quantity for each object in Newton’s laws (i.e. it is conserved and it never changes for each object). This is a central idea for Newton used in 2nd law: F = ma
- In Newton’s time (and in our everyday experience), it seems to be verified that mass never changes.
  - Einstein: Mass is what we measure it to be. We must define mass by an operational measurement. Einstein did “Gedanken” experiments (which were later supported by real experiments) that show that the apparent mass of of an object depends on how fast the object is moving with respect to us.
A Gedanken Experiment

- Consider a glancing collision of two equal mass objects which are moving at relativistic speeds.
- From the initial rest frame of B one sees:

![Diagram showing initial motion of objects A and B]

- After the collision B moves slowly at right angles to the initial direction of A.
- From the initial rest frame of A one sees:

![Diagram showing motion perpendicular to original direction]

- Consider motion ⊥ to original direction of motion
- The time it takes (in each object's rest frame) to travel a fixed ⊥ distance must be the same (since problem is the same for both A and B).

A Gedanken Experiment - Continued

- Why choose the “Gedanken” experiment in this way?
- So the velocity v can be very large (near the speed of light where relativistic effects are important), and yet the motion perpendicular (⊥) to the original velocity is much slower.
- Recall from last time, the lengths ⊥ to the velocity are not modified.

A Gedanken Experiment - Continued

- The time measured (in each object’s rest frame) to travel a fixed ⊥ distance must be the same (since problem is the same for both A and B).
- But A and B each think the other’s clock is running slow! Therefore each says that the other is moving at a slower ⊥ velocity!
- From the point of view of B, the time measured for A to travel that distance is larger by the factor of γ.
- Thus, the ⊥ velocity of A must be smaller than the ⊥ velocity of B by a factor of γ
- Similarly, from point of view of A, B moves slower.

E = mc²

- How can we understand the relation E = mc²?
- Light transmits momentum p and energy E from one end of box to the other
- Box recoils with momentum opposite to the light exactly as if mass is transferred by the light
- What is the relation? Maxwell had shown that light has E = pc, and using p = mv, with v=c, we find E = mc²

How It Works

- This increase in mass then makes it impossible to accelerate an object beyond the speed of light.
- When energy is added to object at rest, it accelerates.
- As object accelerates, its speed increases, but as speed increases, so does its mass which in turn resists further acceleration. The rate of increase of speed therefore gets smaller!
Lecture 15 The Ultimate Speed Limit and E=mc²

E = mc² - continued
• We see from this consideration of conservation of momentum that as the energy of an object increases, its mass increases.

• Einstein showed in 1905 that this is the whole story! Energy is mass -- mass is energy. In his words, “the mass of a body is a measure of its energy content”.

• The mass in E=mc² is the relativistic mass, i.e. \( \gamma \) times the rest mass \( m_0 \).

• The most famous equation in modern science! Energy and mass are equivalent!

E = mc²
• Einstein’s own words
  * http://www.aip.org/history/einstein/voice1.htm

E = mc² - continued
• Einstein’s own words

E = mc² applies to all mass and energy
• Light transmits energy (mass) from one end of box to the other
• Energy goes into heat when it is absorbed. Heat causes end of box to get (slightly) heavier!
• Applies to ALL kinds of energy
• In our ordinary life, the change of mass usually too small to detect (because \( c \) is so large!)
• \( E = mc^2 \rightarrow 1 \text{ Kg} \times (3 \times 10^8)^2 = 9 \times 10^{16} \text{ Joules} \)
• 1 Joule = 10⁻¹⁷ Kg
• About 1Kg mass = entire energy output of large power plant in one year

• The rest mass also is energy!
• When particles are bound together the energy is lower. Examples:
  - Earth bound to sun by gravity
  - Nucleus of atoms (discussed later in course) is made of particles bound together
• If a large weakly bound nucleus can be broken into smaller strongly bound nuclei, energy is released
  - Rest mass of nuclei converted to other forms of energy
  - Einstein’s prophetic statement: “It is not impossible that with bodies whose energy content is variable to a high degree (e.g. with radium salts) the theory may be successfully put to the test.”
  - The Bomb: conversion of rest mass to kinetic energy:
    - Chain reaction
    \[ n + U^{235} \rightarrow La^{139} + Mo^{95} + 2n + \text{kinetic energy} \]
  - 1 gram of mass = energy released in an atomic bomb

E = mc² - continued
• Result of Einstein’s Postulates: Mass and energy are equivalent!
• Completely different from Newtonian ideas!
• Now the ideas of mass and energy are unified — two things which appeared to be completely unrelated in the old paradigm (classical physics) are the same in the new paradigm (special relativity)!

E = mc² - continued
• If this is completely different from Newtonian ideas, how can it be that Newton's laws were widely used - and still are?
• The relativistic expression is \( E = mc^2 = \gamma m_0 c^2 \)
  - Now recall: \( \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \)
• For small \( v/c \) one can show \( \gamma \) is very nearly \( 1 + (1/2)(v/c)^2 \)
  - (Try this on a calculator!)\n• Thus \( E = mc^2 = \gamma m_b c^2 = m_b c^2 + (1/2) m_b v^2 \)
  - Rest Energy Newton’s Kinetic Energy!
Lecture 15 The Ultimate Speed Limit and \(E=mc^2\)

### Intergalactic Space Travel??

- How much energy would be required to accelerate you and your rocket ship \((m = 1000 \text{ Kg})\) to 0.99c?

\[
E_{\text{added}} = mc^2 - m_0 c^2 = (\gamma - 1) m_0 c^2
\]

\[
\gamma = 7.09 \quad E_{\text{added}} = 6.09 m_0 c^2
\]

\(E_{\text{added}} \sim 6000\times\text{annual output of large power plant}\)

- How much energy would be required to accelerate you and your rocket ship \((m = 1000 \text{ Kg})\) to 0.99c?

- How long would it take?

- If your body can withstand acceleration only up to around \(a \sim 10g \sim 100\text{m/s}^2\), it will take about

\[
\text{Time} \sim \frac{c}{a} \sim 3,000,000 \text{ s} \sim 0.1\text{ years}
\]

- Most of the energy must be supplied far from the earth - how to do this?????

### Special Relativity

**Consequences of Einstein’s 2 postulates**

- Consequences in Form of Equations - Summary

\[
T_{\text{improper}} = \gamma T_{\text{proper}}
\]

\[
L_{\text{parallel}}(\text{moving}) = L_{\text{parallel}}(\text{rest}) / \gamma
\]

\[
L_{\text{perpendicular}}(\text{moving}) = L_{\text{perpendicular}}(\text{rest})
\]

\[
m(\text{moving}) = \gamma m(\text{rest})
\]

\[
E = mc^2
\]

\[
\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} > 1
\]

### Summary

- Nothing can go faster than the speed of light.

- How is this speed limit enforced?
  - The faster you go, the more massive you become!
  - A force causes a body to accelerate: momentum changes
  - As the speed increases, more and more of the the energy goes into increased mass and less and less into increased velocity
  - Never reach the speed of light!

- Energy and mass are equivalent: \(E = mc^2\)
  - Unifies two concepts that were totally previously disconnected
  - Nevertheless, agrees with Newton’s formulas for small \(v\)
  - Applies to ALL forms of energy and mass
    - Usually too small to be detected
    - Output of large power plant for 1 year \(\sim 1\text{ Kg}\)
    - Nuclear energy involves larger changes in energy
      - Rest mass of nuclei converted into kinetic energy
  - 1 gram of mass \(\sim\) energy released in an atomic bomb