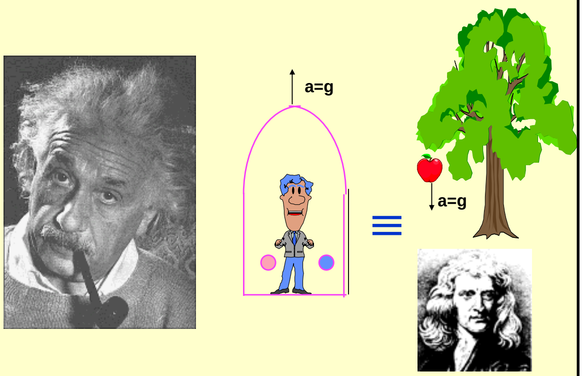
Lect. 16 General Relativity and Equivalence Principle

General Relativity



Announcements

- Schedule:
  - Today: General Relativity
    - March (Ch 12, p. 130- 140) "Did God have any choice?" (Rest of Chapter 12 about the universe covered later.)
  - Next Time: Continue General Relativity
    - March (Ch 13)
- Homework 7: Due Mon. Nov. 3
- Exam II: Wed. Nov. 5

Introduction

- Last time: Relativistic mass & Energy
  - Existence of speed limit from principle of relativity
  - Enforcement of speed limit (relativistic mass)
  - Mass is energy ($E = mc^2$)
- Today and Next Time: General Relativity
  - Unification of theory of space, time, energy, mass, gravity!
  - Consequences for the universe – later in course

Status at this point for Einstein (and us)

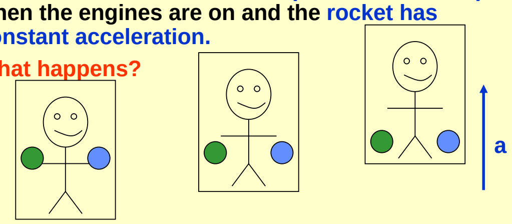
- Classical physics a la Newton:
  - Motion described in time and space
  - Newton's Laws:
    1. Inertia: Objects move in straight lines if there are no forces
    2. F= Ma
    3. Action/Reaction (Conservation of Momentum)
  - Forces come from other bodies (e.g., gravity)
  - Gravity is "action at a distance"
  - Remarkable fact: Inertial Mass = Gravitational Mass
- Conceptual Changes in Special Relativity
  - Time and Space related - Space time
  - Speed Limit = c = velocity of light
  - Must replace "action at a distance" by new laws for gravity
  - Mass redefined! Changes as function of velocity
  - What to do??

Gravitational & Inertial mass

- At this point, we have finished our presentation of Einstein's special theory of relativity. It is called special because it is restricted to physics described in inertial reference frames (constant velocity).
- It took Einstein 11 years to generalize relativity so that it applied to descriptions of physics in ANY reference frame.
- Starting question: Why do we need two kinds of mass?
  - Inertial mass: the measure of how hard it is to accelerate a body.
  - Gravitational mass: the measure of how big of a gravitational force the body exerts on other bodies.
- Experiment: measure the difference between these masses.
  - Eotvos (1909): no difference to 5 parts in 10^9
  - Dicke (1964) : no difference to 3 parts in 10^{11}

Einstein's "Happiest Idea" - I

- Consider a rocket ship far in space (gravitational forces are negligible).
- An astronaut releases 2 balls (of different mass) when the engines are on and the rocket has constant acceleration.
- What happens?



- From point of view of observer that is not accelerating (inertial reference frame): The rocket and astronaut continue accelerating but the balls do not accelerate. The balls do not "keep up" with the rocket, so the bottom of rocket "catches up" to meet the balls
- Rocket "catches up" to both of the balls at the same time, since each one continues to move at same velocity (law of inertia).

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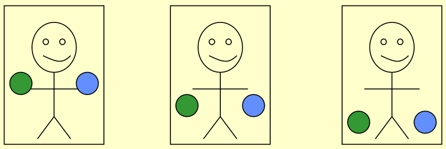
Einstein's "Happiest Idea" - II

- From point of view of Astronaut:
- The balls accelerate towards the bottom of the rocket

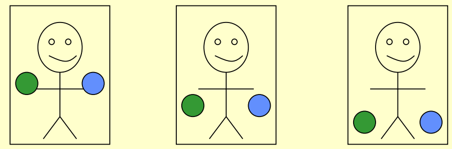


•When astronaut releases balls, they are not moving relative to the rocket. They accelerate relative to the rocket until they hit the bottom with a velocity.

•All objects (no matter what mass, or type) accelerate towards the bottom of the rocket with the same acceleration a.

Einstein's "Happiest Idea" - III

- From point of view of Astronaut
- The balls accelerate towards the bottom of the rocket, just as if they were in a gravitational field.

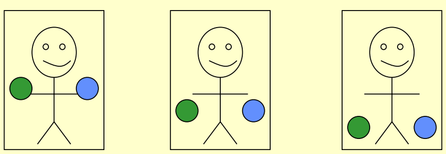


•All objects (no matter what mass, or type) accelerate towards the bottom of the rocket with the same acceleration a.

•Recall: this is exactly what happens in due to gravity (Galileo, Newton) !

Einstein's "Happiest Idea"

•Gravity and acceleration are the same thing!



•No experiment can detect a difference between acceleration and gravity!

•General Relativity!

The Principle of Equivalence

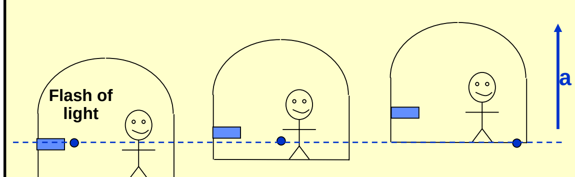
- Einstein: "No experiment performed in one place can distinguish a gravitational field from an accelerated reference frame"
- Example on the earth: Galileo's observation that all bodies fall at the same rate in the Earth's gravitational field must be equivalent to being in an accelerating system.
- A gravitational field that causes all objects to fall downward with acceleration g is exactly equivalent to being in a rocket with upward acceleration g!

The Principle of Equivalence - continued

- Einstein: "No experiment performed in one place can distinguish a gravitational field from an accelerated reference frame"
- This is a strong statement about the nature of gravitation! - New predictions!
- Example: Light must bend in a gravitational field:
- Why? Light must fall just like anything else!
- Next slides

The Principle of Equivalence - continued

- Light must bend in a gravitational field:
- Why? From point of view of person in an inertial frame (not accelerating):
  - The rocket is accelerating
  - Light travels in a straight line



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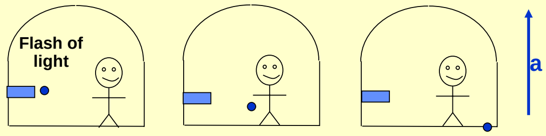
The Principle of Equivalence - continued

- Light must bend in a gravitational field:
- Why? Consider what astronaut must see when he shines a light in an accelerating rocket.
  - Light must fall just like anything else!



Both observers (the astronaut and the person who is not accelerating) agree:

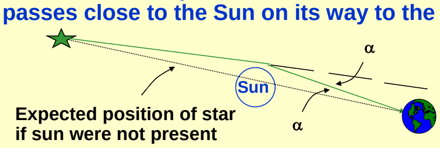
The light hits the floor, even though it started parallel to floor

How Much Does Light Bend?

- Not much (in our ordinary experience)!
  - Consider a distance of 30m (this room):
  - Light takes $t = 30\ m / 3X10^8\ m/s = 10^{-7}$ sec to cross the room
  - How far does it "fall" in this time t?
  - The same amount anything falls in time t
  - $\Delta y = 1/2\ gt^2 = 5X10^{-14}\ m$ (Very small distance!)
- How can we make this bigger?
  - Increase the flight path length.
  - Increase the gravitational field strength.
- Example on next slide:
  - Deflection of light of stars that passes close to the Sun.

First Experimental Test of General Relativity

- Measurement of positions of stars whose light passes close to the Sun on its way to the Earth.

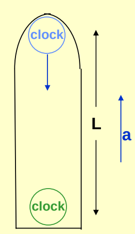


Expeditions organized to Brazil & Africa in 1919 to make measurements during solar eclipse.
(Great fanfare and anticipation - first joint scientific expedition of the countries who had just concluded World War I in 1918.)
Results: measure α =1.64" arc in agreement with Einstein's prediction of 1.75" arc.
More precision recently with radar.

Does Gravity Also Affect Clocks?

- We have just seen that the equivalence principle predicts that light bends in a gravitational field. What are the consequences of the equivalence principle for time?

- Consider a clock at the top of the rocket which sends light pulses to a clock at the bottom of the rocket at a definite frequency f_0.
- If the rocket is accelerating in the direction of the top clock, the bottom clock will receive the pulses at a frequency $f > f_0$.
- Why? Since the clock at the bottom will be moving at a different speed when it receives the pulses, it will see the light. Doppler shift!
- In the time it takes the pulses to travel to the bottom clock, the rocket has increased its velocity by an amount: $v = at = aL/c$ $\beta = v/c = aL/c^2$



$\Delta t_{bot} = \Delta t_{top} - (v\Delta t_{top})/c \Rightarrow \Delta t_{bot} = \Delta t_{top}\ (1-\beta) \Rightarrow$ $f = f_0 / (1 - \beta)$

Gravitational "Red Shift" (Slow down)

- Apply the equivalence principle to this result: the same effect must occur in a gravitational field!
- Light emitted from a height H will be observed from height 0 to have a higher frequency than that with which it was emitted.

$$f = f_0 / (1 - \beta), \quad \beta = v/c = aL/c^2$$

- Note: different frequencies $\Rightarrow$ different times!
- Clocks at bottom run slow compared to those at top!
  - "Run slow" because the same light beam is measured to have higher frequency (shorter time period) compared to clock at top
- Clocks at top run fast compared to those at bottom!
  - Example: light emitted from a star with a large gravitational field will appear at a lower frequency ("Red Shift") when observed by an observer in a small gravitational field (e.g. on earth.)

Experimental Test of Equivalence Principle and Gravitational "Red Shift"

- How big is the effect on earth?
- The fractional change in frequency of light emitted from a height L and observed at height 0 is:

$$\Delta f / f = \beta / (1 - \beta) - \beta = g L / c^2$$

- How big is this? If $L = 10m \Rightarrow \Delta f / f = 10^{-15}$
- Amazingly, this experiment was done first in 1960 by Pound & Rebka at Harvard by exploiting the then recently discovered Mossbauer effect!
- By taking data with emitter at both top and bottom of Jefferson Tower (height = 74 ft), they verified the shift was gravitational. Results:
  - Measured: $\Delta f / f = (5.13 +/- 0.15)\ X\ 10^{-15}$
  - Theoretical prediction: $\Delta f / f = 4.92\ X\ 10^{-15}$

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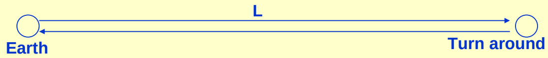
Twin Paradox Revisited

- Now we can understand better the twin paradox?
- The twin that left on the rocket and returned had to have very large accelerations! These affected his "clocks" relative to the one that stayed on earth



- Here is one way to see the magnitude of the effect:
  - The effects on clocks due to the earth's gravity is very small
  - The earth's gravity is equivalent to acceleration $g = 10\ m/s^2$
  - But for an astronaut to reach a speed approaching c and then turn around requires enormous acceleration a (or long times!)
  - Applying the formula $\Delta f / f = a L / c^2$ for the slow down of his clock during the turn around, the astronaut twin concludes there will be a large affect since L is the large distance to the star!
  - The astronaut twin really does age much less than the earth twin when they meet after the rocket returns to earth!

No Need for "Force" of Gravity!

- Newton's Theory: Force determines motion. For example, the gravitational attraction between the Sun and a planet determines the curved orbit of the planet about the Sun.
- Einstein: No need for gravitational "force"! Motion is as if objects are in accelerating space-time.
- But what does this mean?
  - Curved Space-Time coupled to mass!
  - "Matter tells space how to curve and space tells matter how to move". All is geometry!
  - Leads to Black holes , . . .
  - More about this next time

No Need for "Force" of Gravity! Continued

- Einstein's theory is very mathematical and difficult to actually use.
- Newton's Theory is still very accurate for small gravitational fields and it is MUCH easier to use. Newton's theory is still used for "everyday" problems"
  - Falling Bodies, Projectiles, . . .
  - Moon going around the Earth
  - Planetary motion EXCEPT that very accurate descriptions require Einstein's theory of General Relativity
- General Relativity VERY important to understand the universe! More about this later!

Summary

- Principle of Equivalence
  - "No experiment performed in one place can distinguish a gravitational field from an accelerated reference frame"
  - Einstein's theory tested by experiments
  - Clocks run slow in presence of gravity (acceleration)
  - Extends Special theory of relativity to any reference frames
- General relativity
  - Took Einstein 16 years to extend special theory of relativity to any accelerating reference frame
- Gravitational mass is unified with inertial mass
  - Not a mysterious accident! A direct consequence of the theory!
- Replaces Newton's theory
  - No need for forces!
  - Replaced by curved space time coupled to matter! More next time.
- Newton's laws still work for "everyday problems"