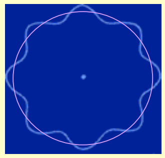
Lecture 21 Matter acts like waves!

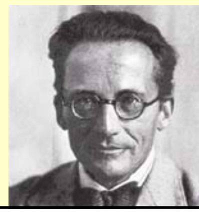
Particles Act Like Waves!

De Broglie's Matter Waves



$\lambda = h / p$

Schrodinger's Equation



Announcements

- Schedule:
  - Today: de Broglie and matter waves, Schrodinger's Equation
    March Ch. 16, Lightman Ch. 4
  - Next time: Does God play Dice? Probability Interpretation
    March Ch. 17, Lightman Ch. 4
- Homework
  - Today: Pass out last homework
- Essay/Report
  - Proposed topic due TODAY
  - Report/essay due Dec 8

Introduction

- Last time: Origins of Quantum Theory
  - Radiation from Hot Body: Max Planck (1900)
    - Introduction of Planck's constant h
    - Energy of light emitted in quanta with energy $E = h\nu$
  - Photoelectric effect: Albert Einstein (1905)
    - Light absorption transfers quanta with energy $E = h\nu$)
    - Photoelectric Effect
  - Atomic Model: Neils Bohr (1912)
    - Spectra from transitions between stable orbits given by quantization condition: radius = $n^2 a_0$, $L = n (h/2\pi)$, $E = E_0/n^2$
- Today: Matter Waves
  - Theory: de Broglie (1924) proposes matter waves
  - More General Theory: Schrodinger (1926) formulates the basic equation still used in quantum mechanics
  - Experiment: Davisson-Germer (1927) shows electrons act like waves -- show interference when scattering from crystals.

General Comments on Bohr's Theory

- Explains Balmer's formula for the frequencies of light emitted from Hydrogen.
- Picture in which laws of classical physics hold except only certain radii are allowed
- Explains stability of atoms but is only a first step - not correct in fact
- Cannot be extended to other atoms or other effects

Louis de Broglie

- An unlikely participant?
  - A member of the French nobility .. was Prince when he wrote his PhD thesis, later became Duke.
  - Initial humanist education
  - Finished his physics PhD in 1924 at age of 32.
  - First physicist to receive Nobel Prize for his thesis!

•Brilliant Idea
•If light (which is a wave) is quantized (like particles)
•Then particles should also like waves!

Louis de Broglie

- Approach: unify ideas of Planck and Einstein (light is quantized) with those of Bohr for the atom.
  - We know light is a wave (inteference effects) which sometimes acts like a particle (Planck's quanta, Einstein and the photoelectric effect).
  - If light (manifestly a wave) can sometimes be also viewed as a particle, why cannot electrons (manifestly a particle) be sometimes viewed as a wave?
- Additional motivation: Quantization rules occur naturally in waves. Perhaps Bohr's quantization rule might be understood in terms of "matter waves".

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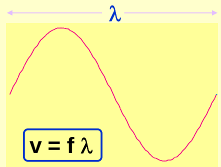
Waves and Quantization

- Recall in our earlier study of waves:

λ = wavelength = distance it takes for pattern to repeat

f = frequency = how many times a given point reaches maximum each second

v = velocity of wave



- Standing waves as example of a "quantization rule":
  - Suppose both ends of a string are fixed so that they can't move.
  - For a fixed length of string, only waves with certain wavelengths will survive to make standing waves... those wavelengths which have zeroes at the ends of the string.

$L = \lambda / 2$, $L = \lambda$, etc. ➡ Quantization rule: $L = n \lambda / 2$, n=1,2,...

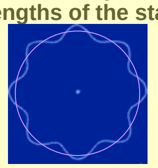
The de Broglie Wavelength

- Big question: How can we quantify deBroglie's hypothesis that matter can sometimes be viewed as waves? What is the wavelength of an electron?
- de Broglie's idea: define wavelength of electron so that same formula works for light also, when expressed in terms of momentum!
  - What is momentum of photon? This is known from relativity:
    - $p = E / c$ (plausible since: $E = mc^2$ and $p = mc \Rightarrow E = pc$)
  - How is momentum of photon related to its wavelength?
    - from photoelectric effect: $E = h\nu \Rightarrow pc = h\nu$
    - change frequency to wavelength: $c = \lambda\nu \Rightarrow c/\nu = \lambda$

➡ $p \lambda = h$ ➡ $\lambda = h / p$

de Broglie Waves & the Bohr Atom

- de Broglie's wave relation ($\lambda = h/p$) can now be used to "derive" Bohr's quantization rule for the hydrogen atom ($L = n (h/2\pi)$).
- How? if an electron is to be viewed as a wave whose wavelength is determined by its momentum, then in the H atom, the electron can have only certain momenta, namely those that correspond to the wavelengths of the standing waves on the orbit.



Standing wave condition: Circumference of circle = integral number of wavelengths

$2 \pi R = n \lambda = n h / p$

$p R = n h / 2\pi$

$L = n h / 2\pi$

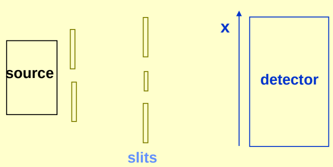
Conclusion: Bohr's quantization rule is just the requirement that the electron wave be a standing wave on the circular orbit!

The Significance of λ = h/p

- The de Broglie wave hypothesis "explains" the previously arbitrary quantization rule of Bohr ($L=n(h/2\pi)$). But this hypothesis is not restricted to electrons in the Hydrogen atom! Can we find any more evidence for the wave nature of matter?
- Where to look? Interference phenomena:
  - The key property of waves is that they show interference. Recall the interference patterns made by visible light passing through two slits or sound from two speakers.
  - The problem with electrons - typical wavelengths are very small and one must find a way to observe the interference over very small distances

The Two-Slit Experiment

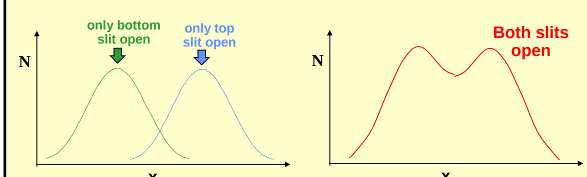
- We will first examine an experiment which Richard Feynman says contains "all of the mystery of quantum mechanics".
- The general layout of the experiment consists of a source, two-slits, and a detector as shown below;



The idea is to investigate three different sources (a classical particle (bullets), a classical wave (water), and a quantum object (electron or photon)). We will study the spatial distribution (x) of the objects which arrive at the detector after passing through the slits.

Classical Particles

- Classical particles are emitted at the source and arrive at the detector only if they pass through one of the slits.
- Key features:
  - particles arrive "in lumps". ie the energy deposited at the detector is not continuous, but discrete. The number of particles arriving per second can be counted.
  - The number which arrive per second at a particular point (x) with both slits open N_{12} is just the sum of the number which arrive per second when only the top slit is opened (N_1) and the number which arrive per second when only the bottom slit is opened (N_2).



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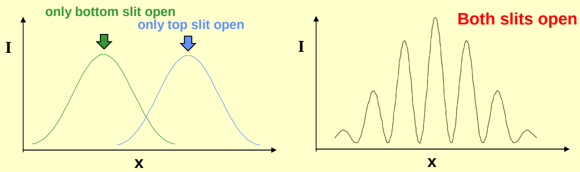
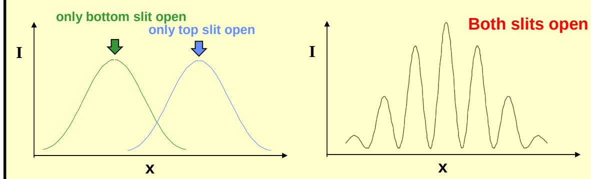
Classical Waves

- Classical waves are emitted at the source and arrive at the detector only if they pass through the slits.
- Key features:
  - detector measures the energy carried by the waves. eg for water waves, the energy at the detector is proportional to the square of the height of the wave there. The energy is measured continuously.
  - The energy of the wave at a particular point (x) with both slits open (I_{12}) is NOT just the sum of the energy of the wave when only the top slit is opened (I_1) and the energy of the wave when only the bottom slit is opened (I_2). An interference pattern is seen, formed by the superposition of the piece of the wave which passes through the top slit with the piece of the wave which passes through the bottom slit.



Quantum Mechanics

- Particles act like waves!
- Experiment shows that particles (like electrons) also act like waves!



Davisson-Germer Experiment (1927)

- Details: Actual experiment involve electrons scattered from a Nickel crystal.
- Done at Bell Labs -- where Davisson and Germer studying electrons in vacuum tubes

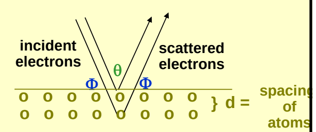
- For fixed energy of the incident electrons, **(E = 54 eV, λ = 1.65Å)**, we expect to see an interference peak in the scattered electrons if the angle Φ is such that the path difference is an integral number of wavelengths.

- Alternatively, for a fixed scattering angle (Φ = 65°), we expect to see the scattering rate to be large for incident electron energies which correspond to de Broglie wavelengths which are equal to the path difference between layers divided by an integer.

Davisson-Germer Experiment (1927)

- Idea: Interference effects can be seen by scattering electrons scattered from a crystal.

Interference occurs when the path difference between scatterings from different layers is an integral number of wavelengths.

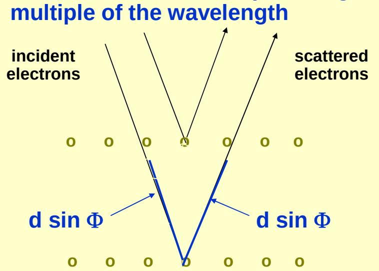


- Since the momentum p = mv of the electrons can be measured, the wavelength predicted by de Broglie is known - **λ = h / p = h / mv.** One can test for interference by changing the speed (i.e. the kinetic energy mv^2) of the electrons.

- **Result:** Electrons show interference like waves with wavelength **λ = h / p**

Davisson-Germer Experiment (1927)

- Interference Condition - same as we have given before for waves: The path length difference is a multiple of the wavelength



Constructive Interference:
$2\,d\,\sin\Phi = n\,\lambda$

Destructive Interference:
$2\,d\,\sin\Phi = (n + 1/2)\,\lambda$

Demonstration

- Interference of light through slits
  - Visible to the eye
- Light acts like a wave
- Interference of electrons going through graphite (carbon) crystal
  - Visible on the fluorescent screen (just like in TV tube)
- Electrons act like waves!

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What Next?

- The wave like nature of particles proposed by de Broglie is verified. The wavelength depends on the momentum ($\lambda = h/p$).
- Also explains Bohr's rule for the hydrogen atom using the same idea: electron bound to atom is like a standing wave.
- Two types of questions now suggest themselves:
  - What is a matter wave? What is waving? What is the right question to ask?
  - Given that a wave is associated with an electron, what determines the form of this wave? What is the new master equation which determines the wave from external conditions? What plays the role of Newton's equation F=ma for the matter wave?
- Schrodinger (1926) finds the solution

The Schrodinger Equation

- In 1926 Erwin Schrodinger proposed an equation which describes completely the time evolution of the matter wave Ψ:

$$(- (h^2 / 2m) \nabla^2 + V) \Psi = i h (d\Psi /dt)$$

where m = characteristic mass of "particle"
V = potential energy function to describe the forces

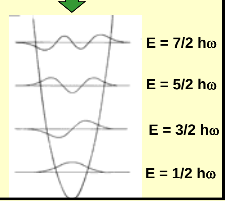
Newton:	Schrodinger
Given the force, find motion	Given potential, find wave
$F = ma = m (d^2x/dt^2)$	$(- (h^2 / 2m) \nabla^2 + V) \Psi = i h (d\Psi /dt)$
solution: $x = f(t)$	solution: $\Psi = f(x,t)$

Note: Schrodinger's equation is more difficult to solve, but it is just as well-defined as Newton's. If you know the forces acting, you can calculate the potential energy V and solve the Schrodinger equation to find Ψ.

(Extra) Example: Harmonic Oscillator

- Classical situation: Mass attached to a spring.
  - The spring exerts a force on the mass which is proportional to the distance that the spring is stretched or compressed. This force then produces an acceleration of the mass which leads to an oscillating motion of the mass. The frequency of this oscillation is determined by the stiffness of the spring and the amount of mass.
- Quantum situation: suppose F is proportional to distance, then potential energy is proportional to distance squared. Solutions to Schrodinger Eqn:

What is shown here?
Possible wave functions $\Psi(x)$ at a fixed time t!
How does this change in time?
They oscillate with the classical frequency!
What distinguishes the different solutions?
The Energy! (Classically this corresponds to the amplitude of the oscillation) Note: not all energies are possible! They are quantized!

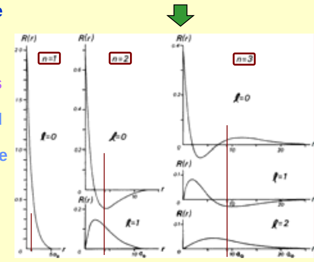


(Extra) Example: Hydrogen atom

- Potential Energy is proportional to 1/R (since Force is proportional to 1/R^2). What are the solutions to Schrodinger's equation and how are they related to Bohr's orbits?

Radial Wavefunctions for the Hydrogen Atom
(vertical lines ↔ Bohr radii)

- The Bohr orbits correspond to the solutions shown which have definite energies.
- The energies which correspond to these wave functions are identical to Bohr's values!
- For energies above the ground state (n=1), there are more than one wave function with the same energy.
- Some of these wave functions "peak" at the value for the Bohr radius for that energy, but others don't!



Key Results of Schrodinger Eq.

- The energy is quantized
  - Only certain energies are allowed
  - Agrees with Bohr's Idea in general
  - Predicts the spectral lines of Hydrogen exactly
  - Applies to many different problems - still one of the key equations of physics!
- The wavefunction is spread out
  - Very different from Bohr's idea
  - The electron wavefunction is not at a given radius but is spread over a a range of radii.

What is Ψ ?

- Our current view was fully developed by Bohr from an initial idea from Max Born.
- Born's idea: Ψ is a probability amplitude wave! Ψ^2 tells us the probability of finding the particle at a given place at a given time.
- More on this next time -- leads to indeterminancy in the fundamental laws of nature
  - Uncertainty principles
  - Not just a lack of ability to measure a property - but a fundamental impossibility to know some things
- Einstein doesn't like it:
  - "The theory accomplishes a lot, but it does not bring us closer to the secrets of the Old One. In any case, I am convinced that He does not play dice."

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One Practical Consequence: Leads to Description of All of Chemistry

- Key Idea: Pauli Exclusion principle - each of the states predicted by the Schrodinger Equation can hold only two electrons
  - Electrons have "spin"
  - Each state hold one electron of spin up and one of spin down
- Atoms - the entire periodic table is described by filling up the states adding one electron for each successive element
- Molecules - from simple molecules like H_2 to DNA and crystals - can be understood from these simple rules

Summary

- De Broglie (1924) has the idea: Particles act like waves! → $\lambda = h / p$
- Waves show interference effects: Standing waves: $L = n \lambda / 2$, n=1,2,... (like a piano or guitar)
- Demonstrated by Davisson-Germer experiment (1927): electrons act like waves!
- Explains the quantized energies of the Bohr atom
- Schrodinger (1926): Equation for wave function $\Psi(x,t)$ for a particle --- Still Today the Basic Eq. of Quantum Mechanics
- $(\Psi(x,t))^2$ is probablity of finding the particle at point x and time t. More about this later.
- Explains all of Chemistry!