Lecture 22 Heisenberg Uncertainty Relations

Does God play Dice? Heisenberg's Uncertainy Principle

$\Delta p\ \Delta x \geq \hbar/2$

$\Delta E\ \Delta t \geq \hbar/2$

$\hbar$ = "hbar"
= $h/2\pi$

Announcements

- Schedule:
  - Last Time: Matter waves : de Broglie, Schrodinger's Equation March (Ch 16), Lightman Ch. 4
  - Today: Does God play Dice? Probablity Interpretation, Uncertainty Principles
    March (Ch 17) Lightman Ch 4
  - Next time: Measurement and Reality - Does observation determine reality? - Meaning of two-slit experiment - Schrodinger's Cat
    March (Ch 18), Lightman Ch 4
- Essay/Report
  - Last time: Short statement of subject your essay due
  - Monday, December 8: Essay due

Introduction

- Last Time: Matter Waves
  - Theory: de Broglie (1924) proposes matter waves
    - assumes all "particles" (e.g. electrons) also have a wave associated with them with wavelength determined by its momentum, $\lambda = h/p$.
    - Bohr's quantization follows because the electron in an atom is described by a "standing electron wave".
  - Experiment: Davisson-Germer (1927) studies electron scattering from crystals - see interference that corresponds exactly to the predicted de Broglie wavelength.
  - The Schrodinger equation: Master Equation of Quantum Mechanics: like Newton's equation F=ma in classical mechanics.
  - But what waving?
- Today: Probability is intrinsic to Quantum Mechanics; Heisenberg Uncertainty Principle

The Nature of the Wave function Ψ

- Max Born proposed:
  Ψ is a probability amplitude wave!

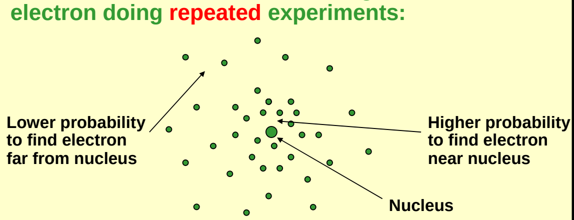
  Ψ^2 tells us the probability of finding the particle at a given place at a given time.

  Ψ is well-defined at every point in space and time

- But Ψ cannot be measured directly - Its square gives the probability of finding a particle at any point in space and time

Probability interpretation for Ψ^2

- The location of an electron is not determined by Ψ. The probability of finding it is high where Ψ^2 is large, and small where Ψ^2 is small.
- Example: A hydrogen atom is one electron around a nucleus. Positions where one might find the electron doing repeated experiments:



The Uncertainty Principle

- Werner Heisenberg proposed that the basic ideas on quantum mechanics could be understood in terms of an Uncertainty Principle

$\Delta p\ \Delta x \geq \sim h$ where Δp and Δx refer to the uncertainties in the measurement of momentum and position.

(~ h means "roughly equal to h" -- will give exact factors later)

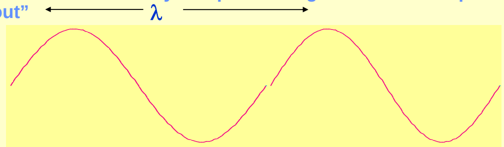
Since p = mv, this also means $\Delta v\ \Delta x \geq \sim h/m$

(Neglecting relativistic effects - OK for v << c)

Lecture 22 Heisenberg Uncertainty Relations

Uncertainty Principle and Matter Waves

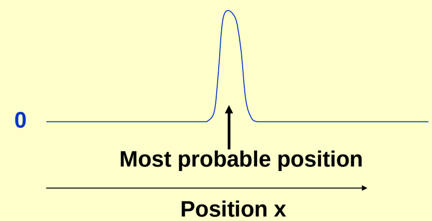
- The uncertainty principle can be understood from the idea of de Broglie that particles also have wave character
- What are properties of waves
  - Waves are patterns that vary in space and time
  - A wave is not in only one place at a give time - it is "spread out" ← λ →



- Example of wave with well-defined wavelength λ and momentum p = h/ λ, but is spread over all space, i.e., its position is not well-defined

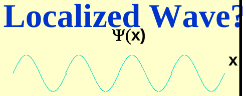
The Nature of a Wave - continued

- Example of wave with well-defined position in space but its wavelength λ and momentum p = h/ λ is not well-defined , i.e., the wave does not correspond to a definite momentum or wavelength.

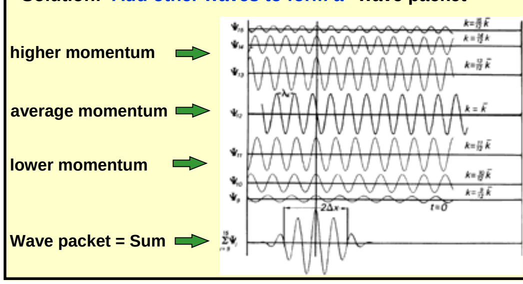


How can one construct a Localized Wave?

- An extended periodic wave is a state of definite momentum



- Note: This wave is not localized!
- Problem: How to describe a localized wave?
- Solution: Add other waves to form a "wave packet"



Localized Wave Packet

- In order to have a wave localized in a region of space Δx, it must have a spread of momenta Δp
- The smaller Δx, the larger the range required
- Leads to the Heisenberg Uncertainty Principle:

$$\Delta p\ \Delta x \geq \sim h$$

- Can understand from de Broglie's Equation

$\lambda = h / p$ or $p\,\lambda = h$

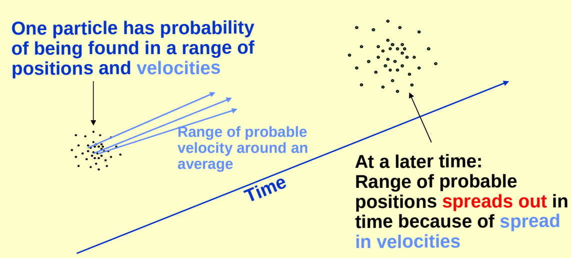
- The minimum range Δx is of order the wavelength λ which requires a range of momenta Δp at least as large as $\Delta p\ \Delta x \geq \sim h$

Is This Like a Classical Wave?

- Yes --- And No!
- A classical wave also spreads out. The more localized the region in which the wave is confined, the more the wave spreads out in time.
- Why isn't that called an "uncertainty principle" and given philosophical hype?
- Because nothing is really "uncertain": the wave is definitely spread out. If you measure where it is, you get the answer: "It is spread out."
- This is different in quantum mechanics where each particle is not spread out. Only the probability of where the particle will be found is spread out.

Time Evolution of the Wave Packet

- Suppose one measures the position and velocity of a particle at one time - each has some uncertainty
- What happens at later times?
- The wave packet spreads out!



Lecture 22 Heisenberg Uncertainty Relations

Examples of Uncertainty Principle

- The more exact form of the uncertainty principle is

$$\Delta p \, \Delta x \geq (1/2)\, h/2\pi = (1/2)\, \hbar$$

- The constant "h-bar" has approximately the value

$$\hbar = 10^{-34} \text{ Joule seconds}$$

So in SI units: $2m \, \Delta x \, \Delta v \geq 10^{-34}$

- Examples: (See March Table 17-1) (atom size)
  - electron: $m \sim 10^{-31}$ Kg, $\Delta x \sim 10^{-10}$ m, $\Delta v \sim 10^{7}$ m/s
    Can predict position in future for time ~ $\Delta x / \Delta v \sim 10^{-17}$ s
  - pin-head $m \sim 10^{-5}$ Kg, $\Delta x \sim 10^{-4}$ m, $\Delta v \sim 10^{-25}$ m/s
    Can predict position in future for time ~ $\Delta x / \Delta v \sim 10^{21}$ s
    (greater than age of universe!)

Uncertainty Principle in Energy & Time

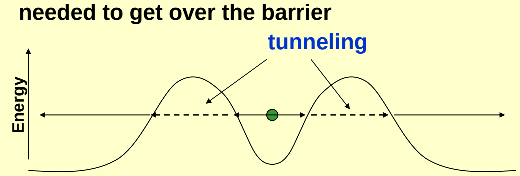
- Similar ideas lead to uncertainty in time and energy

$$\Delta E \, \Delta t \geq (1/2)\, h/2\pi = (1/2)\, \hbar$$

- In SI units: $2\, \Delta E \, \Delta t \geq 10^{-34}$
- In quantum mechanics energy is conserved over long times just as in classical mechanics
- But for short times particles can violate energy conservation!
- Particles can be in Virtual States for short times
- Things that are impossible in classical mechanical are only improbable in quantum mechanics!

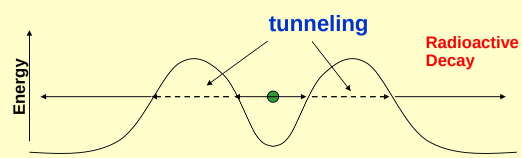
Quantum Tunneling

- In classical mechanics an object can never get over a barrier (e.g. a hill) if if does not have enough energy
- In quantum mechanics there is some probablility for the object to "tunnel through the hill"!
- The particle below has energy less than the energy needed to get over the barrier



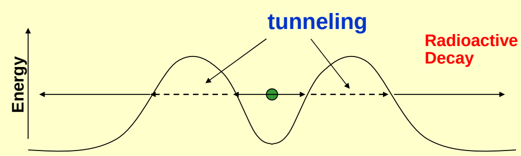
Example of Quantum Tunneling

- The decay of a nucleus is the escape of particles bound inside a barrier
- The rate for escape can be very small.
- Particles in the nucleus "attempt to escape" 10^{20} times per second, but may succeed in escaping only once in many years!



Example of Probability Intrinsic to Quantum Mechanics

- Even if the quantum state (wavefunction) of the nucleus is completely well-defined with no uncertainty, one cannot predict when a nucleus will decay.
- Quantum mechanics tells us only the probability per unit time that any nucleus will decay.
- Demonstration with Geiger Counter



Not everything is uncertain! I

- The uncertainly principle only says that the product of the uncertainties in two quantities must exceed a minimum value

$$\Delta p \, \Delta x \geq (1/2)\, \hbar \qquad \Delta E \, \Delta t \geq (1/2)\, \hbar$$

- Momentum p can be measured to great accuracy - but only if one measures over a large region - i.e., one does not know the position accurately
- Energy E can be measured to great accuracy - but only if one measures over a long time - i.e., one does not know the time for an event accurately

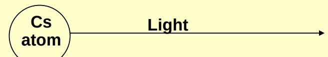
Lecture 22 Heisenberg Uncertainty Relations

Not everything is uncertain! II

- The Schrodinger wavefunction Ψ for a particle is precisely defined for each quantum state.
- The function $\Psi^2(r)$, the probability to find the particle a distance r from the nucleus, is well-defined.
- The energies of quantum states of atoms are extremely well-defined and measured to great precision - often measured to accuracies of 1/1,000,000,000 % = 10^{-12}
- But any one measurement will find an electron in the atom at some particular point - the theory only predicts the probability of finding the electron at any point

Not everything is uncertain! III

- The most accurate clocks in the world are "atomic clocks" that produce light of frequency ν which is precisely defined beacuse energies in atoms are quantized to definite values and E = h ν.
- Time standard for the United States is a clock the uses Cesium atoms

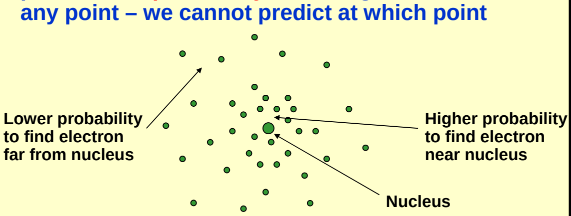


Uncertainty less than 2 x 10^{-15}, which means it would neither gain nor lose a second in 20 million years!

History of Atomic Clocks at NIST
http://www.boulder.nist.gov/timefreq/cesium/atomichistory.htm

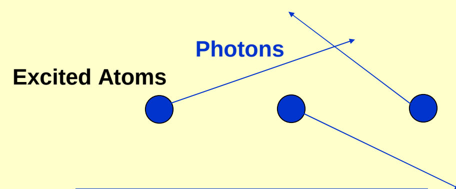
Not everything is uncertain! IV

- Because the energy is so certain it means something else must be very uncertain
- Example: the location of an electron in the atom. Any one measurement will find an electron in the atom at some particular point - the theory only predicts the probability of finding the electron at any point – we cannot predict at which point



Important Quantum Effects in Our World
I Lasers

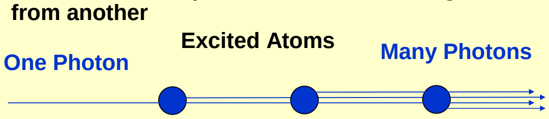
Usually light is emitted by an excited atom is in a a random direction - light from many atoms goes in all directions – direction and energy have uncertainty for light emitted from any one atom



What is special about a Laser??

Important Quantum Effects in Our World
I Lasers - continued

Lasers work because of the quantum properties of photons -- one photon tends to cause another to be emitted – one photon cannot be distinguished from another

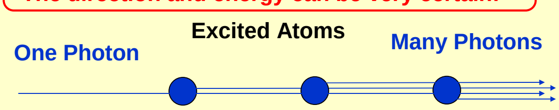


If there are many excited atoms, the photons can "cascade" -- very intense, collimated light is emitted forming a beam of precisely the same color light

Important Quantum Effects in Our World
I Lasers - continued

Since photons cannot be distinguished, which atom emitted a given photon is completely uncertain

But that means:
The direction and energy can be very certain!



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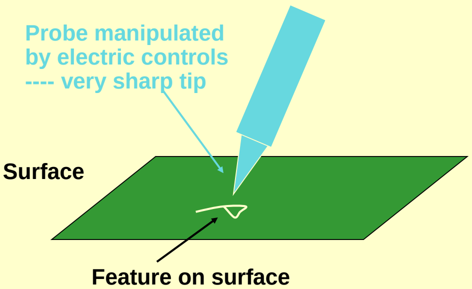
Lecture 22 Heisenberg Uncertainty Relations

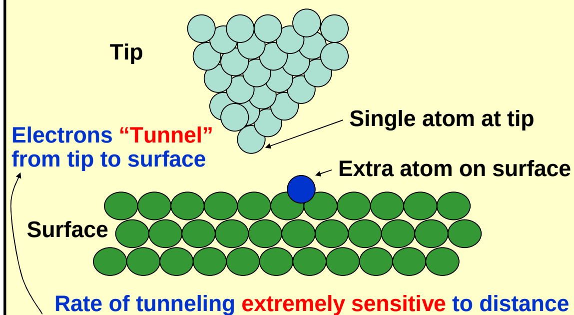
"Seeing" Quantum Effects in Our World

"Scanning Tunneling Microscope"
Measures electric current from tip to surface
as tip is moved



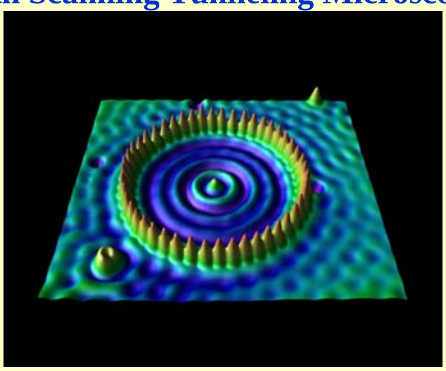
"Seeing" Quantum Effects in Our World

Scanning Tunneling Microscope -- Nobel Prize 1985



Rate of tunneling extremely sensitive to distance
of tip from surface due to quantum effects

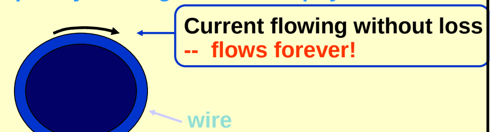
Observation of atoms, electron waves with Scanning Tunneling Microscope



Important Quantum Effects in Our World

Superconductivity

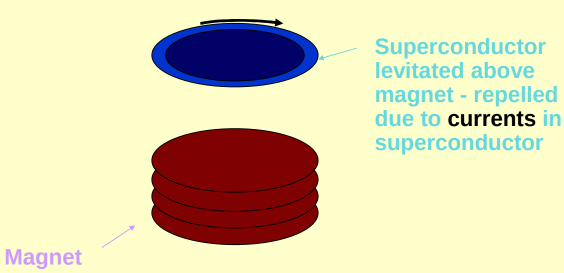
Discovered in 1911 by K. Onnes
Completely baffling in classical physics



Explained in 1957 by Bardeen, Cooper And Shrieffer at the Univ. of Illinois. (Bardeen is the only person to win two Nobel Prizes in the same field!)
Due to all the electrons acting together to form a single quantum state -- electrons flow around a wire like the electrons in an atom!

Demonstration

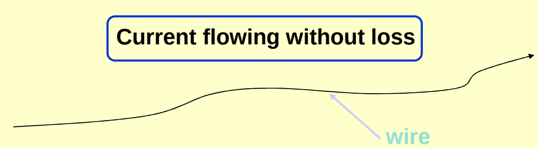
"High - Temperature Superconductors"
Discovered in 1987 (Nobel Prize)
(Still not understood!)



Important Quantum Effects in Our World

Superconductivity

Completely baffling in classical physics



Electric Power lines could carry electricity from California to New York with no loss of power!

Possible now, but not economically feasible

Lecture 22 Heisenberg Uncertainty Relations

Summary

- Particles have wave character!
- Schrodinger's Equation predicts the wave function Ψ with complete certainty
  - Agrees with all experiments up to now
- The meaning of Ψ^2 is the probability that the particle will be found at a given place and time
- Heisenberg showed that quantum mechanics leads to uncertainty relations for pairs of variables

  $\Delta p \, \Delta x \geq \hbar/2$ $\Delta E \, \Delta t \geq \hbar/2$
- Quantum Theory says that we can only measure individual events that have a range of possibilities
  - We can never predict the result of a future measurement with certainty
  - More next time on how quantum theory forces us to reexamine our beliefs about the nature of the world