Physics 211 understand world using \( \sum \vec{F} = m\vec{a} \)

(technically describes acceleration of center of mass where \( \vec{x}_{cm} \equiv \frac{\sum m_i\vec{x}_i}{\sum m_i} \).)

**Forces:** Gravity + Contact

**Known magnitude and direction:**

- **Gravity:** \( F = G \frac{M_1 M_2}{R^2} \) Note: \( G \frac{M_{\text{earth}}}{R_{\text{earth}}^2} = 9.81 \text{ m/s}^2 \)
- **Springs:** \( F = -kx \) (parallel to spring toward \( x=0 \))
- **Kinetic friction** \( f = \mu_k N \) (parallel to surface opposes relative motion)
- **Buoyant force** \( F_{\text{buoyant}} = \rho_{\text{fluid}} V_{\text{displaced}} \) (opposite direction of gravity—up)

**Known direction:**

- **Normal force** (pushes perpendicular to surface)
- **Tension** (pulls parallel to string)

**Unknown:**

- **Static friction** \( f \leq \mu_s N \) (parallel to surface, but magnitude/direction unknown!)

**Kinematics:** (Remember true in \( x, y, z \) and \( \theta \) directions)

**Definitions:**

\[
\vec{v} \equiv \frac{d\vec{x}}{dt} \quad \quad \vec{a} \equiv \frac{d\vec{v}}{dt} \quad \quad \vec{\omega} \equiv \frac{d\vec{\theta}}{dt} \quad \quad \vec{\alpha} \equiv \frac{d\vec{\omega}}{dt}
\]

**Relative motion**

\( \vec{v}_{\text{plane,ground}} = \vec{v}_{\text{plane,air}} + \vec{v}_{\text{air,ground}} \)

**When constant \( \vec{a} \):**

\[
\vec{v}_f = \vec{v}_0 + \vec{a}t \quad \vec{x}_f = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2
\]

**When constant \( \vec{a} \):**

\[
\vec{\omega}_f = \vec{\omega}_0 + \vec{a}t \quad \vec{\theta}_f = \vec{\theta}_0 + \vec{\omega}_0 t + \frac{1}{2} \vec{\alpha} t^2
\]

**Uniform circular motion:** \( a = \frac{v^2}{R} \) towards center of circle

**Rolling without slipping:** \( x = R \theta : v = R \omega : a = R \alpha \)

**Statics**

**Definitions:**

\[
\vec{r} \equiv \vec{R} \times \vec{F} : I \equiv \sum m_i R_i^2
\]

**No acceleration:** \( \sum \vec{F} = 0 \) in \( x, y, z \) directions

**No angular acceleration:** \( \sum \vec{\tau} = 0 \) about any axis
Conservation Laws (from integrating $\sum \vec{F} = m\vec{a} : \sum \vec{\tau} = I\vec{\omega}$)

Work- Kinetic Energy : $\int \vec{F} \cdot d\vec{x} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 : \int \vec{\tau} \cdot d\vec{\theta} = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

$W_{\text{gravity}} = mg(y_f - y_i)$ near earth surface

$W_{\text{gravity}} = Gm\left(\frac{1}{r_f} - \frac{1}{r_i}\right) : W_{\text{spring}} = \frac{1}{2}k(x_i^2 - x_f^2)$

Impulse - Momentum :

Definitions $\vec{p} \equiv m\vec{v} : \vec{L} \equiv I\vec{\omega}$

Linear momentum $\int \vec{F} dt = \vec{p}_f - \vec{p}_i$ (Often choose system so $F_{\text{net}} = 0$)

Angular momentum $\int \vec{\tau} dt = \vec{L}_f - \vec{L}_i$ (Often choose system so $\tau_{\text{net}} = 0$)

Applications

Simple Harmonic Motion (linear restoring force) :

Springs: $(t) = x_{\text{max}} \cos(\omega t + \varphi) : v(t) = -v_{\text{max}} \sin(\omega t + \varphi) : a(t) = -a_{\text{max}} \cos(\omega t + \varphi) : \omega = \sqrt{\frac{k}{m}}$

Pendula: $(t) = \theta_{\text{max}} \cos(\omega t + \varphi) : \omega = \sqrt{\frac{mgL \cos\theta}{I}}$

The wave equation (transverse wave on string) :

$y(x,t) = y_{\text{max}} \cos(kx - \omega t + \varphi) : v_{\text{wave}} = \sqrt{\frac{T}{\mu L}} = \frac{\omega}{k} = f\lambda$

Standing waves $y(x,t) = y_{\text{max}} \cos(kx + \varphi) \cos(\omega t)$

Fluids

$Pressure \equiv \frac{F}{A}$ : Direction is perpendicular to surface

$Pressure = \rho gh$ : In a fluid, pressure only depends on “weight” of fluid above.

Skills

Draw FBD

Decompose vectors

Choose system to make problem easier (conservation of energy/momentum)

Calculate moment of Inertia using parallel axis theorem

Right Hand Rules-----objects spinning and cross products